Promises, Prices, and Yields

Valuing bonds, measuring rate risk, and matching liabilities

Summer 2026

Computational Finance / FinTech Summer School

Day 2 | final-year undergraduate / introductory master's level

Two gilts repay £100 in 2032 – so why are they not alike?

UK government bond	Annual coupon	Redemption	Coupon dates
1% Treasury Gilt 2032	£1.00	31 Jan 2032	31 Jan / 31 Jul
4.25% Treasury Stock 2032	£4.25	7 Jun 2032	7 Jun / 7 Dec

- Which security has the higher price? The higher return? The greater rate risk?
- Which one better funds a payment due on **31 January 2032**?

A gilt is a sterling bond issued by the UK government. "£100 nominal" fixes contractual cash flows; it does not state today's market price.

Day 2 roadmap: six questions about one bond contract

1. **Contract:** what exactly has the issuer promised?
2. **Price:** how does the curve turn those promises into a price?
3. **Yield:** what does one quoted rate summarize – and conceal?
4. **Rate risk:** how much does price move when rates move?
5. **Credit:** which promised cash flows might not arrive?
6. **Decision:** which bond fits a stated liability?

The sequence follows the contract: promised cash flows first, valuation second, quotations third, and risk only after the priced object is clear.

By the end, you should be able to defend both price and risk

- **Reconstruct** a bond's cash flows from its term sheet.
- **Price** each dated payment with a matching discount factor.
- **Solve** yield to maturity and explain its assumptions.
- **Compute** accrued interest, clean price, and dirty price.
- **Measure** duration and test its approximation by exact repricing.
- **Separate** promised yield from expected return under default risk.
- **Recommend** a security using the liability date and stress results.

YTM = yield to maturity: the constant discount rate that reproduces a stated bond price under a declared compounding convention.

Day 1 supplies the engine; Day 2 supplies the traded contract

Term sheet → Dated cash flows → Discount curve → Price and risk

- Day 1 established $P = \sum_i CF_i D(t_i)$.
- Bonds add quotation conventions, an inverse problem called yield, and price sensitivity.

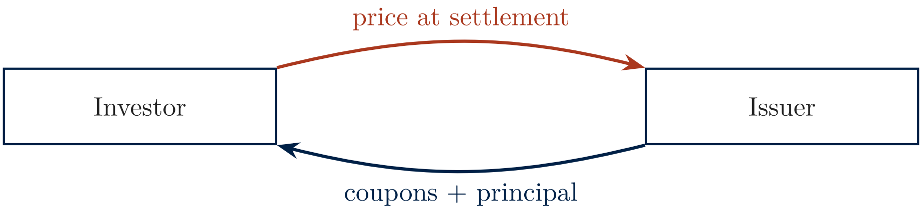
CF_i: cash flow paid at date t_i. $D(t_i)$: price today of one certain pound delivered at that date.

Read the contract

What exactly has the issuer promised?

Translate legal terms into a complete dated cash-flow table before asking for price or return.

A bond is one financing contract seen from two balance sheets



- The issuer receives funds now and creates a fixed-income liability.
- The investor pays now and acquires a contractual asset.
- Signs reverse; dates and states do not.

Settlement date = date on which the buyer pays and ownership transfers. Principal = contractual amount repaid at maturity.

Appendix A1 · Sign audit

The term sheet generates the cash-flow table

Contract field	Valuation question
Nominal / face value	Which amount determines coupons and redemption?
Coupon rate and dates	How much is paid, and at which dates?
Maturity	When is principal due?
Currency / indexation	What unit is promised, and can it change?
Seniority / security	Who is paid first after distress?
Embedded option	Can issuer or investor alter the payment rule?

Face value and nominal value are used interchangeably here. Redemption is repayment of principal at maturity.

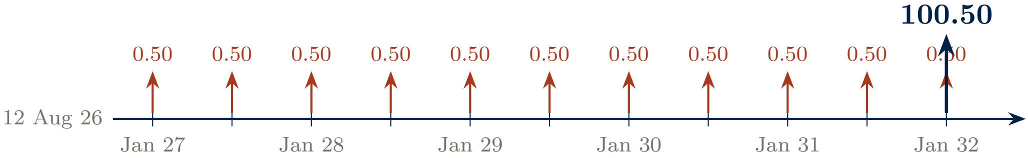
The coupon rate determines promised cash – not today's return

$$\text{semi-annual coupon} = \frac{\text{face value} \times \text{annual coupon rate}}{2}.$$

Security	Face	Annual coupon	Each payment
1% Treasury Gilt 2032	£100	£1.00	£0.50
4.25% Treasury Stock 2032	£100	£4.25	£2.125

Coupon rate is fixed at issue for a conventional bond. Yield changes with market price; the two must not be used as synonyms.

Write every remaining payment before compressing the pattern

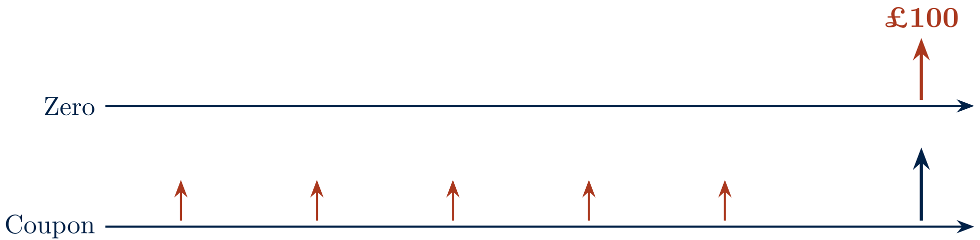


- Ten coupons arrive before the final £100.50 payment.
- The final payment contains the last coupon and principal.

Timeline shown per £100 nominal. Dates, not merely payment numbers, govern discount factors and accrued interest.

Appendix A2 · Full schedule Python P1 · Cash-flow table

A zero-coupon bond changes timing, not the valuation principle



- A zero-coupon bond promises one principal payment at maturity.
- A coupon bond distributes part of its value before maturity.

Zero-coupon bond = discount bond: no interim coupon; issued below face when its yield is positive.

Conventional and index-linked gilts promise different units

- **Conventional gilt:** fixed sterling coupons and fixed sterling principal.
- **Index-linked gilt:** coupons and principal adjust with the UK Retail Prices Index.
- Inflation protection changes the cash-flow rule; it is not created by selecting a different nominal discount rate after the fact.

> The rest of the main lecture uses conventional gilts to isolate bond-pricing and rate-risk mechanics.

RPI = Retail Prices Index. UK index-linked gilt payments use lagged RPI indexation; the detailed convention belongs in the appendix.

Appendix A3 · Indexation mechanics

Nominal amount, price, and redemption value are three numbers

Number	Meaning for a conventional gilt
Nominal amount	Scale used to calculate coupons and principal
Market price	Amount paid per £100 nominal at settlement
Redemption value	Principal returned at maturity, normally £100

- Buying £10,000 nominal does not mean paying £10,000.
- At a clean price of 82.82, the quoted amount is approximately £8,282 before accrued interest and fees.

Prices in this deck are quoted per £100 nominal, the standard unit for comparison.

Market conventions determine which cash flows belong to the buyer

1. Declare the **settlement date**.
2. Identify the previous and next coupon dates.
3. Check whether settlement is inside the ex-dividend period.
4. Apply the market's day-count and compounding conventions.

> UK gilts normally settle on the next business day after trade. Coupon entitlement follows the settlement date and the ex-dividend rule.

Ex-dividend period = interval in which a buyer does not receive the imminent coupon. Actual/Actual = accrued-interest fraction based on actual days elapsed and actual days in the coupon period.

Checkpoint: can the term sheet be turned into data?

> **Security:** 3% bond, £100 face, semi-annual coupons on 15 March and 15 September, maturity 15 September 2029. Settlement: 20 September 2026.

1. What is each coupon?
2. Which is the next payment date?
3. What is the final payment?
4. Which additional convention is needed before calculating accrued interest?

Answer: £1.50; 15 March 2027; £101.50; the day-count rule and ex-dividend treatment.

Price the promises

How does the curve turn promised cash flows into a price?

Treat each payment as its own zero-coupon claim, then add values under the Law of One Price.

Begin with the smallest claim: one certain payment at one date

$$P_0 = F\,D(T).$$

- F is the face payment delivered at maturity T.
- $D(T)$ is the time-0 price of one certain pound delivered at T.
- The product is the price of a zero-coupon bond with face F.

Time 0 means the valuation/settlement date. The claim is treated as default-free in this section.

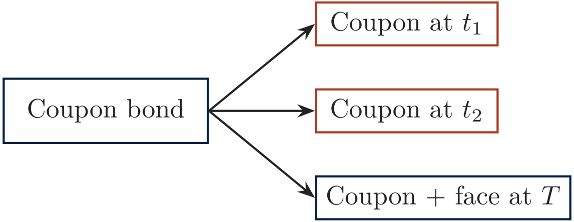
A zero-coupon yield is another quotation of the same price

$$D(T) = e^{-z(T)T} \qquad \Longrightarrow \qquad P_0 = Fe^{-z(T)T}.$$

- $z(T)$ is the continuously compounded spot rate for maturity T.
- Rate and discount factor are equivalent only after maturity and compounding are declared.

Spot rate = rate beginning on the valuation date and ending at one stated maturity. Continuous compounding uses the growth factor e^{zT}.

A coupon bond is a portfolio of zero-coupon claims



$$\{C, C, \ldots, C + F\} = \{C \text{ at } t_1\} + \cdots + \{C + F \text{ at } T\}.$$

Stripping = separating a coupon bond into tradable claims on individual coupon and principal payments.

Law of One Price makes strip values add up to the bond price

$$P_0 = \sum_{i=1}^{n} CF_i D(t_i).$$

- The bond and its strip portfolio deliver identical cash at every date.
- If they had different prices, buy the cheaper cash-flow package and sell the dearer one.
- No view about the average rate is needed.

Law of One Price = identical state-by-date cash flows must have the same price in an arbitrage-free market.

Appendix A6 · Algebra and trade

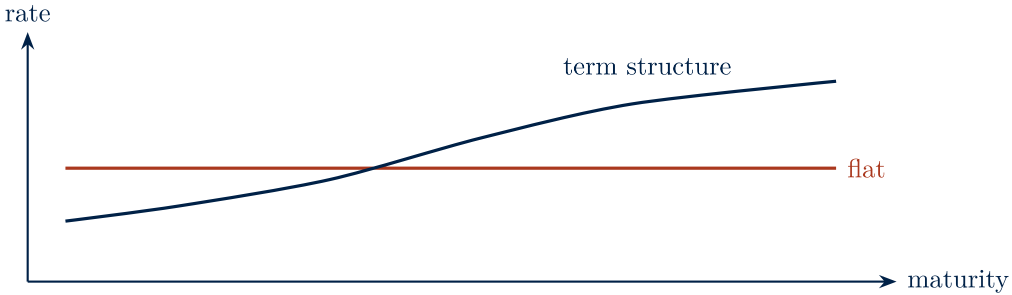
Curve pricing is a date-aligned dot product

$$P_0 = \underbrace{\begin{bmatrix} CF_1 & CF_2 & \cdots & CF_n \end{bmatrix}}_{\text{cash-flow vector}} \underbrace{\begin{bmatrix} D(t_1) \\ D(t_2) \\ \vdots \\ D(t_n) \end{bmatrix}}_{\text{discount-factor vector}} .$$

- Each amount is multiplied by the price of a pound at the same date.
- Sorting and date matching are economic operations, not formatting details.

A dot product multiplies aligned entries and adds the products. It does not repair missing or mislabelled dates.

A flat yield is a compression, not the primitive pricing object



- Flat-rate pricing assigns the same rate to every payment date.
- Curve pricing allows the market price of future pounds to vary by maturity.

Term structure = relationship between rates and maturity for otherwise comparable claims.

Worked bond: price three cash flows with three spot rates

Year	Cash flow	Spot rate	Present value
1	5.00	3.00%	$5/1.03 = 4.85$
2	5.00	3.50%	$5/1.035^2 = 4.67$
3	105.00	4.00%	$105/1.04^3 = 93.34$
		Price	**102.86**

Convention on this slide: annual coupons and annually compounded spot rates. The point is date alignment, not a UK gilt quotation.

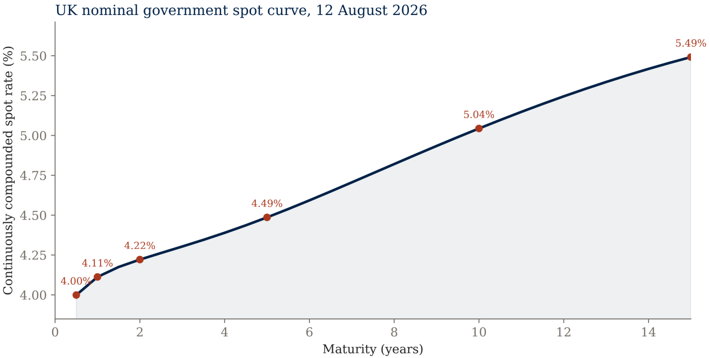
Appendix A7 · Full arithmetic

Replication can be audited date by date

Portfolio	Year 1	Year 2	Year 3
Coupon bond	5	5	105
1-year strip position	5	0	0
2-year strip position	0	5	0
3-year strip position	0	0	105
Difference	0	0	0

Replication is stronger than matching maturity or total undiscounted cash: every relevant date and state must match.

The UK curve assigns a different price to each payment date

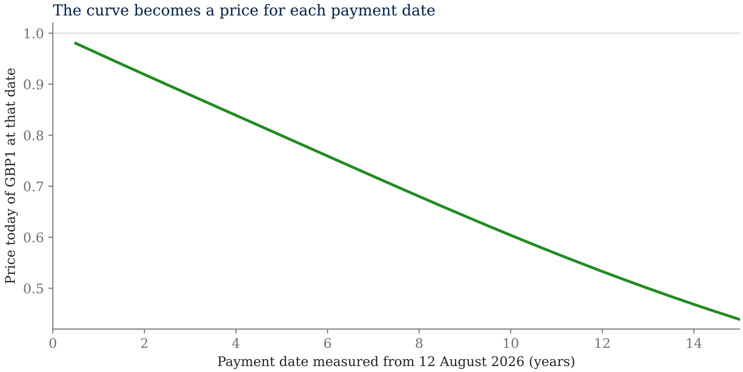


BoE = Bank of England. The displayed government spot rates are continuously compounded annual quotations, estimated from gilt and repo-market inputs.

Bank of England, UK nominal government spot curve, 12 August 2026.

Appendix A9 · Data and convention Python P2 · Interpolate the curve

Discount factors translate the rate curve into cash-flow prices



At positive rates, a pound promised farther away generally has a lower present price. Curve shape determines how quickly that price falls.

The 1% 2032 gilt prices at 82.86 under the observed curve

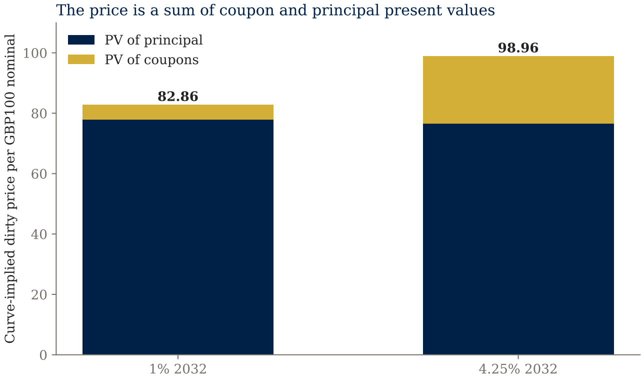
Component	Nominal cash	Present value
Eleven remaining coupons	5.50	4.94
Principal on 31 Jan 2032	100.00	77.92
Curve-implied dirty price	105.50	**82.86**

$$P = \sum_i CF_i \exp[-z(t_i)t_i].$$

Dirty price includes accrued interest. This is a curve-implied illustrative value, not an executable market quote.

Appendix A10 · Full cash-flow audit Python P3 · Vector pricing

Coupon level changes where today's value sits in time



The high-coupon gilt returns more value before redemption. That affects price, reinvestment exposure, and duration even when maturity years look similar.

A model price is conditional, not an observed fact

- **Cash-flow rule:** coupon, maturity, indexation, options, default states.
- **Discount curve:** currency, maturity, credit quality, and compounding.
- **Trading convention:** settlement, accrued interest, clean/dirty quote.
- **Market frictions:** liquidity, bid–ask spread, taxes, funding, and fees.

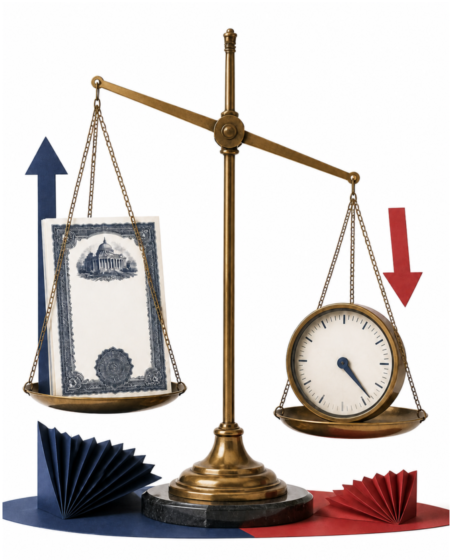
> A reproducible valuation reports its inputs and conventions beside the number.

Bid–ask spread = difference between prices at which dealers buy and sell; it is not captured by a single mid-curve valuation.

Interpret yield

What does yield summarize – and what does it conceal?

Invert price into one rate, then separate that quotation from coupon income and realised return.



Yield to maturity is the constant rate that reproduces price

$$P_{\text{dirty}} = \sum_{j=0}^{n-1} \frac{CF_j}{(1 + y/2)^{j+w}}.$$

- y is the annual nominal YTM with semi-annual compounding.
- w is the fraction of a coupon period from settlement to the next coupon.
- Every cash flow is discounted with the same y.

This one-rate representation reproduces price; it does not claim the spot curve is flat.

Appendix A11 · Semi-annual equation

Yield is an inverse problem solved by a numerical root

$$f(y) = P(y) - P_{\text{observed}} = 0.$$

1. Choose a bracket in which the price residual changes sign.
2. Solve for y with a robust root-finding method.
3. Reprice at the solution and verify the residual is near zero.

Root = input value that makes a function equal zero. Bisection and Brent's method retain a valid bracket and do not require a guessed derivative.

Appendix A12 · Bracket and residual Python P4 · Solve YTM

Coupon, current yield, YTM, and spot rates answer different questions

Measure	Question answered
Coupon rate	What contractual coupon is paid per unit of face?
Current yield	Coupon divided by clean price; how large is coupon income relative to price?
YTM	Which one constant rate reproduces the dirty price if held to maturity?
Spot curve	Which date-specific rates price the separate payments?

Current yield ignores capital gain/loss to redemption and the time value of individual coupons.

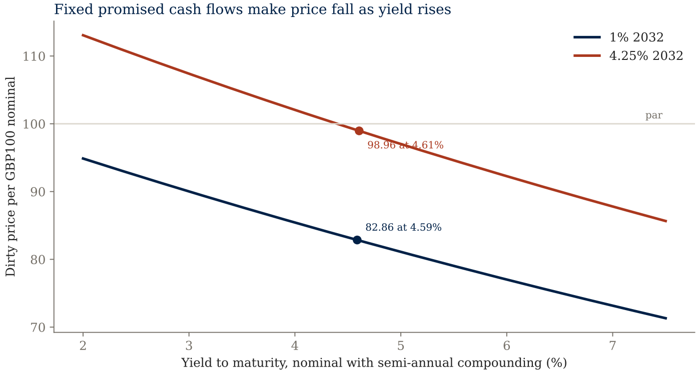
Par, premium, and discount compare coupon with YTM

Coupon rate < YTM	Coupon rate = YTM	Coupon rate > YTM
Discount bond	Par bond	Premium bond
Price < 100	Price = 100	Price > 100
Capital gain to par	No capital change	Capital loss to par

The classification assumes a standard fixed-coupon bond, no default, and the same compounding convention for coupon frequency and YTM.

Appendix A13 · Derivation

Fixed cash flows make price fall when yield rises



Both points are curve-implied prices converted into their constant-rate YTMs. The nearby yields do not imply nearby prices because coupon cash flows differ.

The price–yield relation is curved, not linear

- A 100 bp rise and a 100 bp fall do not produce equal-and-opposite price changes.
- Price gains from a yield fall exceed price losses from an equal yield rise, locally.
- The curvature becomes important for large shocks and long-duration bonds.

$$\frac{dP}{dy} < 0, \qquad \frac{d^2P}{dy^2} > 0 \quad \text{for a standard option-free bond.}$$

Option-free = no contractual call, put, or conversion feature that changes cash flows as rates move.

Pull to par is a conditional price path

- A discount bond approaches £100 as maturity nears if YTM is unchanged.
- A premium bond approaches £100 under the same condition.
- Coupon payments create predictable price drops around payment dates.
- Market yield changes can dominate the mechanical path.

Pull to par is not a forecast of tomorrow's price; it is a holding-everything-else-fixed valuation result.

Appendix A14 · Constant-yield path

YTM embeds a reinvestment assumption

- Holding to maturity fixes the redemption date, not the return earned on interim coupons.
- Realising the quoted YTM requires coupons to be reinvested at that same yield.
- If reinvestment rates fall, terminal wealth is lower than the YTM calculation implies.
- Zero-coupon bonds remove coupon reinvestment risk for a matched maturity payment.

Reinvestment risk = uncertainty about the rates available when interim cash flows are invested.

Holding-period return depends on what actually happens before sale

$$\text{HPR} = \frac{P_1 - P_0 + \text{coupon income} + \text{reinvestment income}}{P_0}.$$

- A sale before maturity introduces an uncertain future sale price P_1.
- YTM at purchase need not equal the realised holding-period return.

HPR = holding-period return over a declared investment horizon. It is ex post when all components are realised.

Appendix A15 · Return decomposition

Clean price separates trading quotation from accrued coupon

$$P_{\text{dirty}} = P_{\text{clean}} + AI.$$

- **Clean price:** quoted bond value excluding accrued interest.
- **Accrued interest:** coupon earned by the seller since the previous coupon date.
- **Dirty price:** settlement amount per £100 nominal before fees.

AI = accrued interest. Clean quotes avoid a mechanical saw-tooth caused by coupon accrual, while settlement uses the dirty amount.

Appendix A16 · Formula and edge case

Twelve accrued days add 0.0326 to the 1% gilt's clean price

$$AI = £0.50 \times \frac{\text{12 days since 31 July}}{\text{184 days in coupon period}} = £0.0326.$$

Curve-implied dirty price	82.8555
Less accrued interest	0.0326
Curve-implied clean price	**82.8229**

Settlement date is 12 August 2026. Actual/Actual uses the actual 184-day coupon period from 31 July 2026 to 31 January 2027.

Appendix A17 · DMO convention audit

The two 2032 gilts have close yields but different economic shapes

Curve-implied measure	1% 2032	4.25% 2032
Dirty price	82.86	98.96
Clean price	82.82	98.19
Current yield	1.21%	4.33%
YTM	4.59%	4.61%

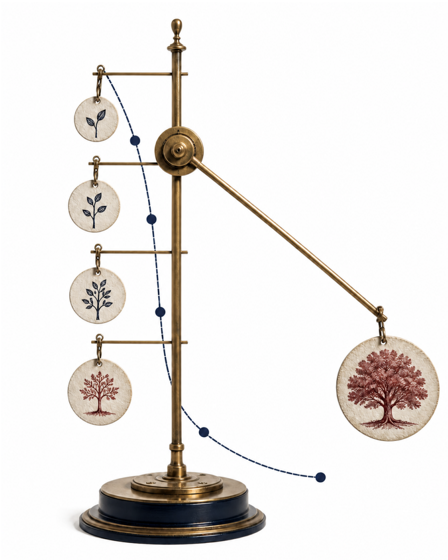
> A lower price is not automatically “better value”: the cash-flow amounts and dates are different.

Both YTMs are nominal annual rates with semi-annual compounding, solved from curve-implied dirty prices.

Measure rate risk

How much does price move when rates move?

Turn the timing of present value into duration, test the approximation, and confront non-parallel curve shifts.



Maturity alone does not measure when value is received

Each marker is a cash flow’s share of dirty price. Both bonds mature in 2032, but the high-coupon bond returns more present value earlier.

Three features increase price sensitivity, other things equal

1. **Longer maturity:** more value is exposed for longer.
2. **Lower coupon:** less value returns before principal.
3. **Lower yield:** distant cash flows receive larger present-value weights.

> These are comparative-statics statements: change one feature while holding the others fixed.

Comparative statics = analysis of how an outcome changes when one input changes under stated ceteris-paribus assumptions.

The Penn century bond makes the horizon effect visible

$$P(y) = \sum_{t=1}^{100} \frac{4.67}{(1+y)^t} + \frac{100}{(1+y)^{100}}.$$

Annual YTM	Price per $100	Change from par
3.67%	126.51	+26.51%
4.67%	100.00	0.00%
6.67%	70.06	-29.94%

- Berk and DeMarzo use a $300m Penn bond: 100 years, issued at par, with a 4.67% annual coupon and YTM.
- Macaulay duration is 22.18 years: shorter than maturity, yet rate changes still create large capital gains or losses.

Source: Berk and DeMarzo, Ch. 6, Alternative Example 6.7.

Appendix A21 · Duration and convexity

Present-value weights connect cash-flow timing to price

$$w_i = \frac{\mathrm{PV}(CF_i)}{P}, \qquad \sum_{i=1}^{n} w_i = 1.$$

- A large nominal payment can have a smaller weight when it is distant.
- Weights depend on both cash flows and the discount environment.
- Duration will average payment dates using these economic weights.

The symbol w_i now means present-value weight; it is unrelated to the coupon-period fraction w used in the YTM equation.

Macaulay duration is the PV-weighted average payment date

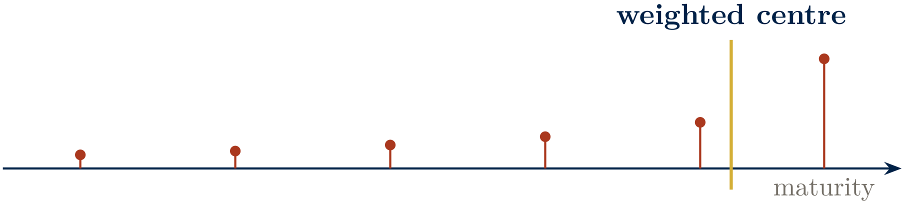
$$D_{\mathrm{Mac}} = \sum_{i=1}^{n} t_i w_i = \frac{\sum_i t_i \, \mathrm{PV}(CF_i)}{P}.$$

- Units are years.
- A zero-coupon bond's Macaulay duration equals its maturity.
- A coupon bond's duration is shorter than its final maturity.

Macaulay duration is named after Frederick Macaulay. It is a timing measure under the yield-based discounting convention.

Appendix A18 · Weights and bounds

Duration is a timing statistic before it is a sensitivity formula



- Moving coupon value earlier shifts the weighted centre left.
- Moving principal farther away shifts it right.

The centre is not a payment date that must occur; it summarizes the full distribution of discounted cash flows.

The low-coupon 2032 gilt carries slightly greater duration

Curve-implied measure	1% 2032	4.25% 2032
Time to maturity	5.47 years	5.82 years
Macaulay duration	**5.32 years**	5.17 years
Modified duration	**5.20 years**	5.05 years

- The high-coupon gilt matures later but returns enough value early to have lower duration.
- This is exactly why maturity alone can mis-rank rate exposure.

Time to maturity uses Actual/365 from 12 August 2026. Duration uses the solved semi-annual YTM convention.

Modified duration converts timing into local price sensitivity

$$D_{\text{mod}} = \frac{D_{\text{Mac}}}{1 + y/2}, \qquad \frac{\Delta P}{P} \approx -D_{\text{mod}} \Delta y.$$

- The minus sign encodes the inverse price–yield relation.
- For the 1% gilt, $D_{\text{mod}} = 5.196$.
- A small $+1$ bp shift implies roughly -0.052% in price.

$1 \text{ bp} = 0.01$ percentage point $= 0.0001$ in decimal rate units. The approximation is local and assumes a parallel yield shift.

Appendix A19 · Derivative

A 50 bp rise: approximate first, then reprice exactly

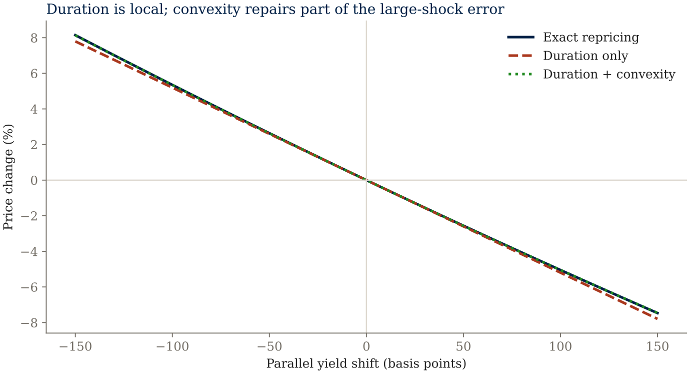
$$\frac{\Delta P}{P} \approx -5.196(0.005) = -2.598\%.$$

1% Treasury Gilt 2032	Result
Starting dirty price	82.8555
Duration-implied price	80.7028
Exact repriced value	**80.7336**
Difference	0.0308

The exact repricing holds contractual cash flows fixed and adds 50 bp to YTM.

Appendix A20 · Full audit   Python P5 · Approximate and reprice

Approximation error grows as the yield shock moves away from zero



Duration matches the slope at the starting yield. Convexity adds curvature but exact repricing remains the benchmark when cash flows and the shock are known.

Convexity explains the bend in the price–yield curve

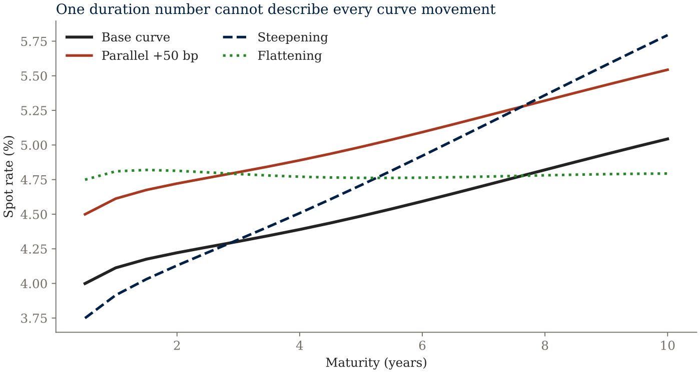
$$\frac{\Delta P}{P} \approx -D_{\text{mod}}\Delta y + \frac{1}{2}\mathcal{C}(\Delta y)^2.$$

- $\mathcal{C}$ measures the second-order curvature of price with respect to yield.
- The squared shock contributes positively for a standard option-free bond.
- Convexity matters more for large rate moves and long-duration positions.

Convexity has units of years squared under this yield convention. Detailed differentiation is deferred to the appendix.

Appendix A21 · Second-order derivation

One duration number assumes one shape of curve movement



Parallel shift = every maturity rate changes by the same amount. Steepening raises long rates relative to short rates; flattening does the reverse.

Appendix A22 · Shock vectors

Key-rate thinking asks which part of the curve moved

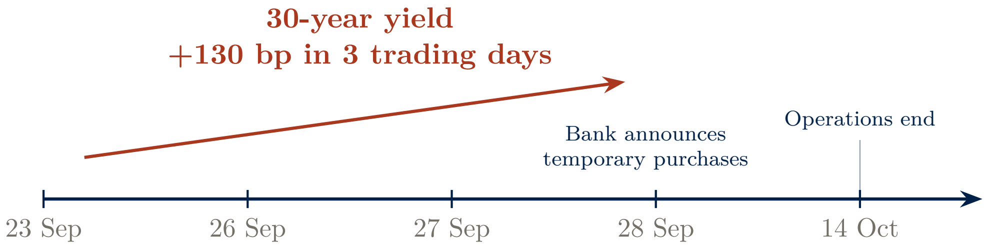
- Reprice after a small shock to one maturity segment while holding others fixed.
- Repeat at selected key maturities to build a vector of sensitivities.
- A bond concentrated near 5 years should respond differently from one concentrated near 30 years.

$$KRD_k \approx -\frac{P(z_k + h) - P(z_k - h)}{2hP}.$$

KRD = key-rate duration. z_k is the spot rate at key maturity k; interpolation rules determine how neighbouring dates move.

Appendix A23 · Finite difference

In September 2022, long gilt yields moved by a historical amount

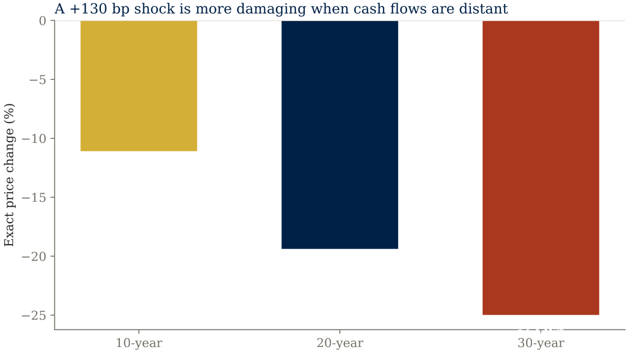


- The move was about three times any previous comparable historical move.
- Long-duration portfolios suffered large mark-to-market losses before any default question arose.

Bank of England, Financial stability buy/sell tools: a gilt market case study, 2023.

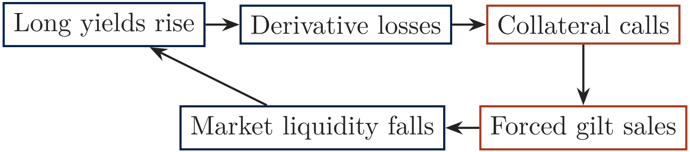
Appendix A24 · Chronology and sources

A +130 bp rate stress shows why distant cash flows matter



This scenario is not the realised price path of specific gilts: 1% annual coupon, initial yield 4.0%, shocked yield 5.3%.

Duration losses became a system problem through leverage and liquidity



LDI = liability-driven investment. Collateral call = demand for additional assets or cash after a leveraged position loses value. Duration identifies exposure; leverage and forced selling amplify it.

A rate-risk memo reports more than duration

- **Position:** clean/dirty value, cash-flow dates, and currency.
- **Local sensitivity:** modified duration and the yield convention.
- **Stress:** exact repricing under small, large, and curve-shape shocks.
- **Horizon:** intended sale date or liability date.
- **Funding:** leverage, collateral, and liquidity needs under stress.

> Duration is an input to risk judgement; it is not the whole risk report.

Add credit and frictions

Which promised cash flows might not arrive?

Move from one certain cash-flow path to state-contingent payments, recovery, and spread interpretation.

Government-bond pricing isolated rate risk; firms add states

- A conventional gilt was treated as delivering one fixed sterling cash-flow path.
- A corporate bond promises fixed payments but may default before maturity.
- Valuation must therefore specify cash flows in each relevant state.

$$P_0 = \sum_s q_s \, CF_s,$$

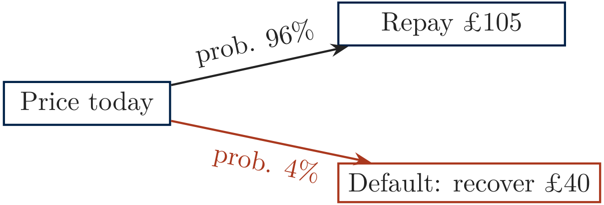
s indexes states of the world. q_s is a state price, not an ordinary probability; it incorporates time and risk pricing.

Promised cash flow and expected cash flow are different objects

Object	Meaning
Promised payoff	Contractual payment if the issuer performs fully
Recovery payoff	Amount received after default or restructuring
Expected payoff	Probability-weighted average across possible payoffs
Market price	Risk-adjusted, discounted value of state-contingent payoffs

Expected payoff is not automatically discounted at a risk-free rate when default covaries with bad economic states.

A one-period default tree separates promise from recovery



$$\mathbb{E}[CF] = 0.96(105) + 0.04(40) = \pounds 102.40.$$

Recovery = payment received after default. The example isolates expected cash flow; pricing also requires a risk adjustment and time discounting.

Appendix A25 · Expected loss audit

A high promised YTM need not be a high expected return

- YTM solves the price using the **promised** £105 payoff.
- Default states replace that promise with a lower recovery payoff.
- Expected return depends on default probability, recovery, and the purchase price.

$$\text{promised return} = \frac{105}{P_0} - 1, \qquad \text{expected return} = \frac{102.40}{P_0} - 1.$$

The gap between promised and expected return is not a free premium; part compensates for expected loss.

Concept adapted from Berk and DeMarzo, *Corporate Finance*, Ch. 6.4.

A credit spread requires a matched benchmark

$$\text{credit spread} = y_{\text{corporate}} - y_{\text{benchmark}}.$$

- Match currency, maturity, and quotation convention.
- A government or swap curve may serve as benchmark depending on the question.
- Comparing a 10-year corporate yield with today's policy rate mixes horizons.

Credit spread = yield difference over a declared reference curve; it is quoted in basis points.

Spread compensates for more than expected default loss

- Expected default loss and uncertainty about recovery.
- Systematic risk: defaults tend to rise when wealth is already scarce.
- Liquidity and dealer-balance-sheet capacity.
- Tax treatment, embedded options, funding, and model/convention differences.

> It is usually invalid to read a spread as a default probability without a recovery assumption and a pricing model.

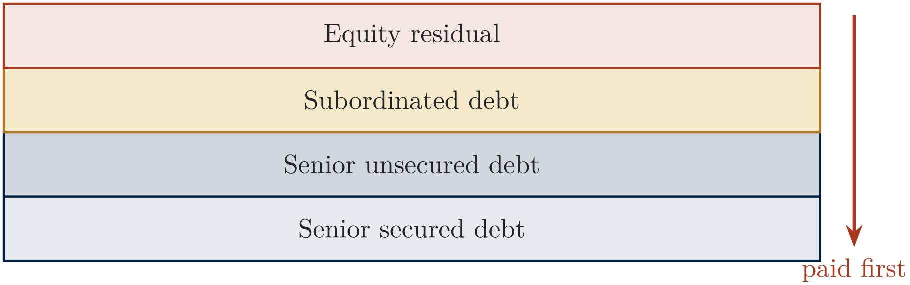
Appendix A27 · Matched benchmark

Ratings summarize credit categories; they do not set prices

- Rating agencies classify relative credit quality using ordered categories.
- Bonds with the same rating can trade at different spreads.
- Ratings can change after new information; they are not guarantees.
- Market price also reflects maturity, liquidity, optionality, and recovery expectations.

Investment grade commonly denotes the higher rating categories; precise symbols differ across rating agencies.

Seniority and security govern recovery after default



Secured debt has a claim on specified collateral. Subordinated debt ranks behind senior claims in the contractual payment waterfall.

Embedded options change the cash-flow rule itself

- **Callable bond:** issuer may repay early, often when rates fall.
- **Puttable bond:** investor may demand early repayment under stated terms.
- **Convertible bond:** investor may exchange debt for equity.
- The option affects which cash flows occur in each rate or firm-value state.

> Do not value an option-bearing bond by mechanically adding a spread to an option-free cash-flow stream.

Embedded option = contractual right contained inside the bond rather than traded separately.

Credit analysis begins with a five-part map

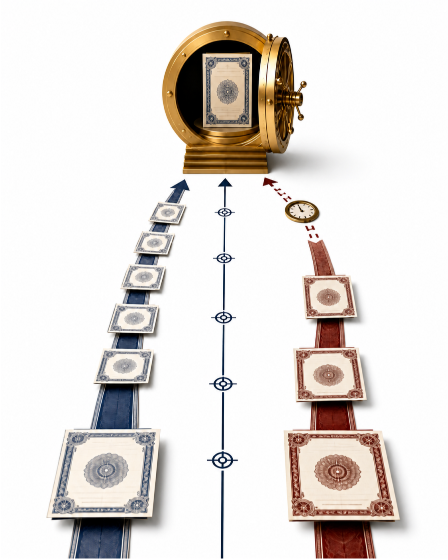
1. List promised cash flows and possible default dates.
2. Specify recovery by seniority and collateral.
3. Choose a matched benchmark curve.
4. Separate expected loss from risk, liquidity, and option premia.
5. Stress both issuer outcomes and market exit conditions.

The main lecture establishes the map. Estimating default intensities and full structural/reduced-form credit models lies beyond Day 2.

Make the decision

Which bond fits the liability?

Combine contract, price, yield, duration, and stress results around one dated funding obligation.



Decision brief: fund £10m due on 31 January 2032

> A university endowment has a certain sterling payment of £10 million due on 31 January 2032. It is considering two conventional UK gilts as the core matching asset.

- Objective: reduce uncertainty in the funded amount at the liability date.
- Constraints: today's purchase cost, interim coupon reinvestment, and rate stress.
- Valuation date: 12 August 2026.

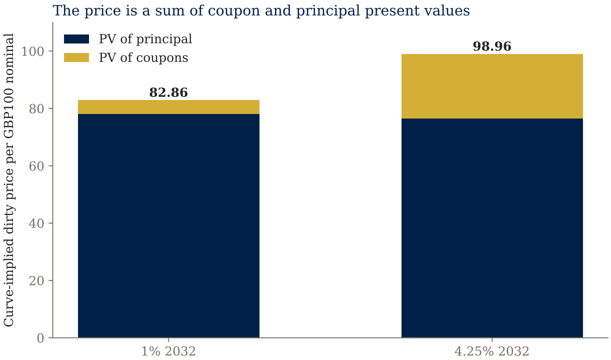
Liability matching evaluates assets relative to a specified payment obligation; it is not a generic search for the highest quoted yield.

The candidates have similar labels but different contracts

Contract feature	1% 2032	4.25% 2032
Final maturity	31 Jan 2032	7 Jun 2032
Coupon dates	31 Jan / Jul	7 Jun / Dec
Semi-annual coupon	0.50	2.125
ISIN	GB00BM8Z2T38	GB0004893086

ISIN = International Securities Identification Number, a unique identifier for a security. Contract terms are from the UK Debt Management Office.

Price decomposition shows what is being purchased



The 4.25% gilt's higher price buys more coupon value before maturity; it is not evidence that the bond is intrinsically expensive.

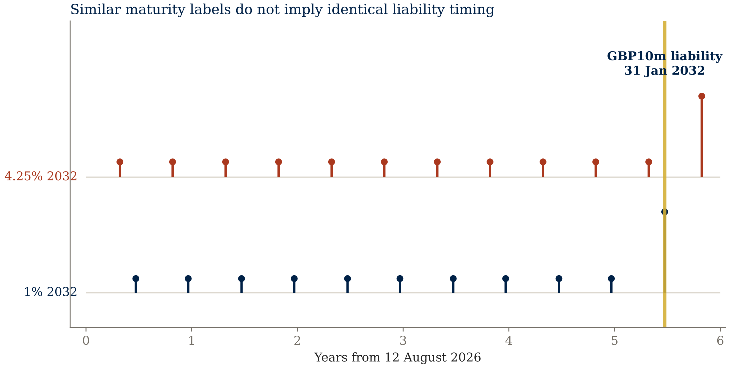
Yield ranking does not answer the liability question

Curve-implied measure	1% 2032	4.25% 2032
Dirty price	82.86	98.96
YTM	4.59%	4.61%
Current yield	1.21%	4.33%

- The 2 bp YTM difference is economically small relative to timing mismatch.
- Current yield rewards coupon size but ignores redemption and liability date.

A 2 bp difference equals 0.02 percentage point; transaction costs and model assumptions may be comparable in magnitude.

Cash-flow fit reveals reinvestment and timing risk



The 1% gilt redeems on the liability date. The 4.25% gilt redeems later, so its larger earlier coupons must be reinvested and its principal is unavailable on 31 January without a prior sale.

Duration quantifies rate exposure, not exact liability fit

Measure	1% 2032	4.25% 2032
Macaulay duration	5.315	5.171
Modified duration	5.196	5.055
+50 bp exact price change	-2.561%	-2.490%

> The lower-duration bond is not automatically the better hedge: the target liability itself has a date and sensitivity.

A stress matrix separates approximation from exact repricing

Bond	Shock	Exact price	Exact change	Duration estimate
1% 2032	-50 bp	85.04	+2.636%	+2.598%
1% 2032	+50 bp	80.73	-2.561%	-2.598%
1% 2032	+130 bp	77.46	-6.508%	-6.755%
4.25% 2032	-50 bp	101.50	+2.565%	+2.527%
4.25% 2032	+50 bp	96.49	-2.490%	-2.527%
4.25% 2032	+130 bp	92.70	-6.324%	-6.571%

Shock is applied to each bond's YTM. Exact price is the benchmark; duration is retained because it explains the direction and first-order magnitude.

Appendix A28 · Case audit Python P6 · Stress and match

Recommendation: begin with liability fit, then control residual risks

1. Prefer the **1% Treasury Gilt 2032** as the cleaner core match because principal arrives on 31 January 2032.
2. Size the nominal holding so principal plus managed coupon proceeds meet the £10m obligation.
3. Reinvest interim coupons in date-matched low-risk instruments.
4. Report residual basis, reinvestment, transaction-cost, and operational risks.

This is the liability-matching conclusion under a certain sterling liability and curve-implied values; it is not investment advice or an executable trade instruction.

Day 2 leaves one reproducible bond workflow

Contract → Cash flows → Curve price → Yield → Sensitivity → Decision

- Price comes from dated cash flows and matching discount factors.
- Yield summarizes price; duration summarizes local rate sensitivity.
- Credit, liquidity, options, and the investor's liability determine whether the number is useful.

A calculation is complete only when its contract, data date, convention, and decision objective are visible.

References and data sources

Core textbooks

- Brealey, Myers, Allen and Edmans, *Principles of Corporate Finance*, Ch. 3: present value, bond valuation, term structure, and interest-rate risk.
- Berk and DeMarzo, *Corporate Finance*, Ch. 6: bond contracts, zero- and coupon-bond yields, dynamic prices, strip replication, the Penn century-bond case, promised versus expected return, and corporate and sovereign credit risk.

References, market sources, and data

- MIT OpenCourseWare 15.401, *Fixed-Income Securities*; MIT 18.642, *Bond Mathematics*; NYU Stern fixed-income notes.
- UK Debt Management Office: *Gilts in Issue*, *About Gilts*, glossary, and *Formulae for Calculating Gilt Prices from Yields*, 4th ed. (2024).
- Bank of England: UK nominal government spot curve dated 12 August 2026; yield-curve documentation; 2022 gilt-market case studies.

The deck integrates the cited textbook sequences and cases, harmonises notation, and independently reproduces every numerical calculation from the declared data and conventions.

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Technical appendix map

← **Return to the Day 2 synthesis**

Practical map

Appendix A1: Perspective and sign audit

Event	Investor	Issuer
Settlement price	$-P_0$	$+P_0$
Each coupon	$+C$	$-C$
Principal at maturity	$+F$	$-F$

- Market value is an asset to the investor and a liability to the issuer.
- A valuation sign error can reverse a return or NPV conclusion while leaving arithmetic internally consistent.

← **Return to the two perspectives**

Appendix map

Appendix A2: Full 1% Treasury Gilt 2032 schedule

Payment date	Cash flow	Payment date	Cash flow
31 Jan 2027	0.50	31 Jul 2029	0.50
31 Jul 2027	0.50	31 Jan 2030	0.50
31 Jan 2028	0.50	31 Jul 2030	0.50
31 Jul 2028	0.50	31 Jan 2031	0.50
31 Jan 2029	0.50	31 Jul 2031	0.50
		31 Jan 2032	100.50

Per £100 nominal; settlement/valuation date 12 August 2026. The 31 July 2026 coupon is already past and therefore absent.

← **Return to the cash-flow timeline**

Appendix map

Appendix A3: Index-linked gilt mechanics

- A reference Retail Prices Index (RPI) level at issue establishes the base index.
- A lagged index ratio scales coupon and principal payments.
- For a 3-month-lag index-linked gilt, the real clean price is multiplied by the relevant index ratio before accrued interest is added.
- Unlike some international inflation-linked bonds, index-linked gilts do not have a generic par-value deflation floor.

$$\text{inflation-adjusted cash flow} = \text{real contractual cash flow} \times \text{index ratio}.$$

RPI data and interpolation lags make an index-linked cash-flow table date-dependent; it must be generated before valuation.

← **Return to conventional and index-linked gilts**

Appendix map

Appendix A4: Settlement, entitlement, and accrued days

1. A gilt trade normally settles on $T+1$, the next business day.
2. Coupon entitlement is determined by the settlement date relative to the ex-dividend date.
3. Outside the ex-dividend period, the buyer receives the next coupon and compensates the seller through accrued interest.
4. Inside the ex-dividend period, accrued interest becomes negative rebate interest because the seller receives the imminent coupon.

$$AI = C_{\text{period}} \frac{\text{actual days since previous coupon}}{\text{actual days in coupon period}}.$$

T+1 = settlement one business day after trade. UK gilt ex-dividend periods begin seven business days before payment.

← **Return to the settlement checklist**

Appendix map

Appendix A5: Zero price/yield conversions

$$\begin{aligned}
\text{Continuous:}\quad & D(T) = e^{-z_c T}, & z_c &= -\frac{\ln D(T)}{T}, \\
\text{Annual effective:}\quad & D(T) = (1 + z_a)^{-T}, & z_a &= D(T)^{-1/T} - 1, \\
\text{Nominal, } m \text{ times:}\quad & D(T) = \left(1 + \frac{z_m}{m}\right)^{-mT}.
\end{aligned}$$

- Equivalent quotations produce the same $D(T)$.
- Mixing a continuous spot rate with a periodic discount formula changes price.

← **Return to the zero-coupon price**

Appendix map

Appendix A6: Replication algebra for a coupon bond

$$\text{Bond cash flows: } (C, C, \ldots, C+F),$$
$$\text{Strip portfolio: } C\mathbf{e}_1 + C\mathbf{e}_2 + \cdots + (C+F)\mathbf{e}_n.$$

$$P_{\text{bond}} = CD(t_1) + CD(t_2) + \cdots + (C+F)D(t_n).$$

- $\mathbf{e}_i$ denotes one pound delivered only at date t_i.
- Price additivity follows from a date-by-date payoff identity, not from averaging yields.

← **Return to the Law of One Price**

Appendix map

Appendix A7: Worked three-year spot-rate arithmetic

$$\begin{aligned}
\mathrm{PV}(C_1) &= \frac{5}{1.03} = 4.8544, \\
\mathrm{PV}(C_2) &= \frac{5}{1.035^2} = 4.6676, \\
\mathrm{PV}(C_3 + F) &= \frac{105}{1.04^3} = 93.3380, \\
P_0 &= 4.8544 + 4.6676 + 93.3380 = 102.8600.
\end{aligned}$$

> Audit identity: recalculating every present-value component independently must reproduce the reported total.

← **Return to the worked bond**

Appendix map

Appendix A8: Arbitrage if the bond and strips disagree

Suppose the strip portfolio costs 102.86 but the bond trades at 104.00.

Trade	Today	Years 1–2	Year 3
Short expensive bond	+104.00	-5 each	-105
Buy replicating strips	-102.86	+5 each	+105
Net cash flow	**+1.14**	0	0

The example abstracts from short-sale constraints, bid–ask spreads, taxes, funding, and settlement differences.

← **Return to the date-by-date audit**

Appendix map

Appendix A9: Bank of England curve construction and provenance

- Dataset: UK nominal government liability spot curve, 12 August 2026.
- Inputs: gilt prices and general-collateral repo rates used by the Bank's estimation process.
- Published points: half-year maturities; selected bond payment dates use linear interpolation in quoted spot rates.
- Compounding: continuously compounded, quoted annually.
- Discounting in this deck: $D(t) = \exp[-z(t)t]$, with t measured Actual/365.

Repo = repurchase agreement, a collateralised financing transaction. Curve estimates are model outputs and may differ from executable zero-coupon prices.

← **Return to the observed curve**

Appendix map

Appendix A10: Full curve-price audit for the 1% gilt

Date	t	Cash flow	$z(t)$	Present value
Jan 2027	0.47	0.50	3.99%	0.491
Jul 2027	0.97	0.50	4.11%	0.480
Jan 2028	1.47	0.50	4.17%	0.470
Jul 2028	1.97	0.50	4.22%	0.460
Jan 2029	2.47	0.50	4.26%	0.450
Jul 2029	2.97	0.50	4.30%	0.440
Jan 2030	3.47	0.50	4.34%	0.430
Jul 2030	3.97	0.50	4.39%	0.420
Jan 2031	4.47	0.50	4.43%	0.410
Jul 2031	4.97	0.50	4.48%	0.400
Jan 2032	5.47	100.50	4.54%	78.40
			Total	**82.86**

Displayed row values are rounded; the total uses unrounded interpolated rates and discount factors from the data file.

← **Return to the curve-implied price**

Appendix map

Appendix A11: Semi-annual YTM convention

Let $w \in (0,1]$ be the fraction of the current coupon period remaining.

$$P_{\text{dirty}} = \frac{C}{(1+y/2)^{w}} + \frac{C}{(1+y/2)^{1+w}} + \cdots + \frac{C+F}{(1+y/2)^{n-1+w}}.$$

- Full coupon periods increment the exponent by one.
- Dirty price is used because the buyer pays for the complete post-settlement cash-flow claim.

The DMO formula handles quasi-coupon dates, ex-dividend cases, and rounding in greater detail; the main equation captures a regular conventional gilt outside the ex-dividend period.

← **Return to the YTM equation**

Appendix map

Appendix A12: Root-finding bracket and residual

For the 1% 2032 gilt, define $f(y) = P(y) - 82.85549697$.

Trial YTM	Price residual
0.00%	+22.64
4.00%	+2.61
4.587866%	-1.1×10^{-13}
6.00%	-5.75

- Price is monotone in yield for this option-free positive-cash-flow bond.
- A sign-changing bracket contains one economically relevant root.

← **Return to root finding**

Appendix map

Appendix A13: Why coupon relative to YTM determines price class

At a coupon date with n periods remaining,

$$P = C\frac{1-(1+r)^{-n}}{r} + F(1+r)^{-n},$$

$$P - F = (C - rF)\frac{1-(1+r)^{-n}}{r}.$$

Therefore,

- $C/F < r \Rightarrow P < F$ (discount),
- $C/F = r \Rightarrow P = F$ (par),
- $C/F > r \Rightarrow P > F$ (premium).

Here $r = y/2$ is the semi-annual yield and C/F is the semi-annual coupon rate.

← **Return to par, premium, and discount**

Appendix map

Appendix A14: Pull to par under an unchanged yield

Immediately after a coupon, with n payments remaining,

$$P_n = C\frac{1-(1+r)^{-n}}{r} + F(1+r)^{-n}.$$

One coupon period later, immediately after the next coupon,

$$P_{n-1} = C\frac{1-(1+r)^{-(n-1)}}{r} + F(1+r)^{-(n-1)}.$$

As $n \to 0$, the remaining ex-coupon value approaches F.

- The path is mechanical only if r is unchanged.
- Market repricing and default can move price away from this path.

← **Return to pull to par**

Appendix map

Appendix A15: Holding-period return decomposition

For a one-year horizon with coupons C_j reinvested to the horizon,

$$\text{HPR} = \frac{P_1 - P_0 + \sum_j C_j (1 + r_j)^{\tau_j}}{P_0}.$$

- $P_1 - P_0$: capital gain or loss at sale.
- r_j: realised reinvestment rate for coupon j.
- τ_j: remaining time from coupon receipt to horizon.

Purchase YTM predicts this return only under its holding-period and reinvestment assumptions.

← **Return to holding-period return**

Appendix map

Appendix A16: Clean, dirty, and ex-dividend treatment

Outside the ex-dividend period,

$$P_{\text{dirty}} = P_{\text{clean}} + C_{\text{period}} \frac{d_{\text{elapsed}}}{d_{\text{period}}}.$$

During the ex-dividend period,

- the seller retains the imminent coupon,
- the buyer receives a negative accrued-interest adjustment (rebate interest),
- the next coupon is omitted from the buyer's price/yield cash-flow set.

The clean/dirty distinction is a quotation convention; it does not change the underlying economic value transferred at settlement.

← **Return to clean and dirty price**

Appendix map

Appendix A17: DMO convention versus simplified equation

Layer	Treatment
Main simplified equation	Regular semi-annual cash flows; coupon-period fraction w; unrounded calculations
DMO market formula	Quasi-coupon dates, ex-dividend logic, irregular first/last periods, and specified rounding
Accrued interest	Actual/Actual gilt convention
Displayed price	Curve-implied illustrative value, not DMO/Tradeweb reference price

DMO = UK Debt Management Office. Quasi-coupon dates extend the regular coupon cycle for price/yield calculations even when a cash payment does not occur.

← Return to the accrued-interest calculation

Appendix map

Appendix A18: Macaulay weights sum to one

Let $PV_i = CF_i/(1+y/m)^{mt_i}$ and $P = \sum_i PV_i$. Then

$$w_i = \frac{PV_i}{P} \geq 0, \qquad \sum_i w_i = \frac{\sum_i PV_i}{P} = 1.$$

Therefore,

$$D_{\text{Mac}} = \sum_i t_i w_i$$

is a weighted average and must lie between the first and last payment dates.

- For one payment at T, the only weight is 1 and duration is T.
- Positive early coupons make duration less than final maturity.

← **Return to Macaulay duration**

Appendix map

Appendix A19: Modified duration from the price derivative

With $P(y) = \sum_i CF_i(1+y/m)^{-mt_i}$,

$$\frac{dP}{dy} = -\sum_i t_i CF_i(1+y/m)^{-mt_i-1},$$

$$-\frac{1}{P}\frac{dP}{dy} = \frac{\sum_i t_i PV_i}{P(1+y/m)} = \frac{D_{\text{Mac}}}{1+y/m}.$$

Hence

$$D_{\text{mod}} = -\frac{1}{P}\frac{dP}{dy}.$$

The derivative uses decimal yield units: a change from 4% to 5% is $\Delta y = 0.01$, not 1.

← **Return to modified duration**

Appendix map

Appendix A20: Exact 50 bp stress audit

$$y_0 = 4.587866\%, \qquad P(y_0) = 82.855497,$$
$$y_1 = 5.087866\%, \qquad P(y_1) = 80.733622.$$

$$\text{Exact change} = \frac{80.733622}{82.855497} - 1 = -2.560935\%.$$

Duration gives

$$-D_{\text{mod}}\Delta y = -5.196179(0.005) = -2.598090\%.$$

Approximation error: 0.037155 percentage point.

← **Return to the 50 bp stress**

Appendix map

Appendix A21: Convexity as the second derivative

A second-order Taylor expansion around y_0 gives

$$P(y_0 + \Delta y) \approx P(y_0) + P'(y_0)\Delta y + \frac{1}{2}P''(y_0)(\Delta y)^2.$$

Divide by $P(y_0)$, using $D_{\text{mod}} = -P'/P$ and $\mathcal{C} = P''/P$. Then

$$\frac{\Delta P}{P} \approx -D_{\text{mod}}\Delta y + \frac{1}{2}\mathcal{C}(\Delta y)^2.$$

Convexity measures the curvature of price with respect to yield. For the 1% gilt, numerical convexity is 30.04 years^2 under the semi-annual YTM convention.

← **Return to convexity intuition**

Appendix map

Appendix A22: Curve shocks are vectors

Let $\mathbf{z} = (z_{0.5}, z_1, \ldots, z_{10})$.

$$\text{Parallel: } \Delta\mathbf{z} = (50, 50, \ldots, 50) \text{ bp},$$

$$\text{Steepener: } \Delta z_t = -25 + \frac{t - 0.5}{9.5}(100) \text{ bp},$$

$$\text{Flattener: } \Delta z_t = 75 - \frac{t - 0.5}{9.5}(100) \text{ bp}.$$

- Reprice every cash flow using its shocked date-specific spot rate.
- A scalar duration is exact only for the local direction it represents.

z_t = spot rate for maturity t. bp = basis point; 100 bp equals one percentage point.

← **Return to curve-shape risk**

Appendix map

Appendix A23: Key-rate duration by finite difference

For key maturity k, construct two curves:

- $\mathbf{z}^{+k}$: add h near k, tapering to zero at neighbouring keys.
- $\mathbf{z}^{-k}$: subtract the same local shock.

Then

$$KRD_k = -\frac{P(\mathbf{z}^{+k}) - P(\mathbf{z}^{-k})}{2hP(\mathbf{z})}.$$

- Central differences reduce first-order numerical error.
- Interpolation/tapering is part of the risk definition and must be reported.

KRD = key-rate duration. h is the small finite-difference bump, stated in decimal rate units.

← **Return to key-rate thinking**

Appendix map

Appendix A24: 2022 gilt crisis chronology and scale

Date	Event
23 Sep 2022	UK fiscal announcement preceded an exceptional repricing in long gilts
By 28 Sep	30-year gilt yield rose 130 bp in three trading days
28 Sep	Bank announced temporary purchases of long-dated conventional gilts
11 Oct	Operations expanded to index-linked gilts
14 Oct	Temporary purchase operations ended

- The Bank purchased £19.3bn in total: £12.1bn conventional and £7.2bn index-linked gilts.
- Purchases targeted market functioning rather than a monetary-policy yield level.

← **Return to the 2022 case**

Appendix map

Appendix A25: Expected payoff and expected loss

Promise: £105; default probability: 4%; recovery: £40.

$$\begin{aligned} \mathbb{E}[CF] &= 0.96(105) + 0.04(40) = 102.40, \\ \text{EL} &= 105 - 102.40 = 2.60, \\ \text{loss given default} &= 105 - 40 = 65. \end{aligned}$$

Equivalently,

$$\text{EL} = \Pr(\text{default}) \times \text{loss given default} = 0.04(65) = 2.60.$$

EL = expected loss in currency units. Recovery rate conventions may use face, promised claim, or market value as denominator; the denominator must be declared.

← **Return to the default tree**

Appendix map

Appendix A26: Promised YTM versus expected return

Suppose the one-year risky bond costs £95.

$$\text{Promised return} = \frac{105}{95} - 1 = 10.526\%,$$
$$\text{Expected return} = \frac{102.40}{95} - 1 = 7.789\%.$$

- The 2.737 percentage-point gap is expected default loss relative to price.
- Neither number alone identifies the compensation for systematic default risk or illiquidity.

For a multi-period coupon bond, timing of default, missed coupons, recovery timing, and reinvestment make the distinction more complex.

← **Return to promised versus expected return**

Appendix map

Appendix A27: Spread decomposition and benchmark audit

A schematic decomposition is

$$\begin{aligned}\text{spread} \approx \text{expected loss} + \text{systematic risk premium}\\ + \text{liquidity premium} + \text{option/tax/model effects.}\end{aligned}$$

Before interpreting a spread, verify

- matched currency and maturity,
- the same compounding and clean/dirty convention,
- comparable optionality and seniority,
- whether the benchmark is a gilt, OIS, or swap curve.

OIS = overnight index swap. A yield spread is a quotation difference, not a structural decomposition unless a model supplies the mapping.

← **Return to spread components**

Appendix map

Appendix A28: Integrated case calculation audit

Verified result	1% 2032	4.25% 2032
Curve-implied dirty price	82.855497	98.958897
Accrued interest	0.032609	0.766393
Clean price	82.822888	98.192504
YTM	4.587866%	4.606513%
YTM repricing residual	-1.1×10^{-13}	-5.7×10^{-14}
Macaulay duration	5.315376	5.171136
Modified duration	5.196179	5.054713
+130 bp exact change	-6.507999%	-6.324285%

All values are per £100 nominal and use unrounded inputs. Root residuals verify that solved YTMs reproduce dirty prices numerically.

← **Return to the stress matrix**

Appendix map

Appendix A29: Sources, data, and scope

- Brealey, Myers, Allen and Edmans, *Principles of Corporate Finance*, Ch. 3; Berk and DeMarzo, *Corporate Finance*, Ch. 6.
- MIT OpenCourseWare 15.401, *Fixed-Income Securities*; MIT 18.642, *Bond Mathematics*; NYU Stern fixed-income notes.
- UK Debt Management Office: *Gilts in Issue*; *About Gilts*; glossary; *Formulae for Calculating Gilt Prices from Yields*, 4th ed. (2024).
- Bank of England: *Yield curves*; yield-curve terminology and concepts; nominal government spot curve dated 12 August 2026.
- Bank of England: *Financial stability buy/sell tools: a gilt market case study* (2023); *An anatomy of the 2022 gilt market crisis* (2023).

> Local data files and calculation outputs are stored with the deck. Market examples are educational; curve-implied prices are not executable quotes or investment advice.

Linked titles open the official sources. Accessed August 2026.

← **Return to the Day 2 synthesis**

Appendix map

Python practical map

1. P1 · Represent dated cash flows →
2. P2 · Read and interpolate the BoE curve →
3. P3 · Price by aligned vector operations →
4. P4 · Solve and verify YTM →
5. P5 · Compute duration and reprice →
6. P6 · Build the decision stress table →

Python is used where repetition, date alignment, interpolation, root finding, or scenario analysis adds value; it does not replace the contract logic.

← **Return to the Day 2 synthesis**

Appendix map

Practical P1: Represent the bond as a dated table

```
import pandas as pd

# Each row is one legal payment date.
dates = pd.to_datetime([
    "2027-01-31", "2027-07-31", "2028-01-31", "2028-07-31",
    "2029-01-31", "2029-07-31", "2030-01-31", "2030-07-31",
    "2031-01-31", "2031-07-31", "2032-01-31",
])
amounts = [0.50] * 10 + [100.50]
cashflows = pd.DataFrame({"date": dates, "cash_flow": amounts})

# Time is measured from the declared settlement date.
settlement = pd.Timestamp("2026-08-12")
cashflows["t"] = (cashflows["date"] - settlement).dt.days / 365
```

pandas = Python library for labelled tables. The final row combines the last 0.50 coupon with 100 principal.

← Return to the bond schedule

Practical map

Practical P2: Interpolate the curve at payment dates

```
import numpy as np
import pandas as pd

curve = pd.read_csv("data/cases/day02_boe_nominal_spot_curve_2026-08-12.csv")

# np.interp supplies a spot rate for every cash-flow maturity.
cashflows["spot_pct"] = np.interp(
    cashflows["t"],
    curve["maturity_years"],
    curve["spot_cc_pct"],
)

# BoE quotations are continuously compounded annual rates.
cashflows["df"] = np.exp(-(cashflows["spot_pct"] / 100) * cashflows["t"])
```

np is the conventional alias for NumPy, Python's numerical-array library. np.interp performs one-dimensional linear interpolation; requested maturities must lie inside the observed curve range.

Practical map

← Return to the BoE curve

Practical P3: Price by aligned multiplication

```python
# Present value is calculated row by row, then added.
cashflows["pv"] = cashflows["cash_flow"] * cashflows["df"]
dirty_price = cashflows["pv"].sum()

# Validation checks catch silent data errors.
assert cashflows["date"].is_monotonic_increasing
assert cashflows["date"].is_unique
assert (cashflows["df"] > 0).all()
assert abs(dirty_price - 82.85549697) < 1e-8
```

- Inspect the table before trusting the scalar total.
- `assert` raises an error when its condition is false, turning economic identities into executable checks.

← **Return to the curve-implied price**

Practical map

Practical P4: Solve YTM and verify its residual

```python
from scipy.optimize import brentq

# 184 is the number of days in the current coupon period.
w = (cashflows["date"].iloc[0] - settlement).days / 184
cash_flow = cashflows["cash_flow"].to_numpy()
powers = w + np.arange(len(cashflows))

# Price implied by one semi-annual-compounded annual YTM.
def price_from_ytm(y, cash_flow, powers):
    return np.sum(cash_flow / (1 + y / 2) ** powers)

def residual(y):
    return price_from_ytm(y, cash_flow, powers) - dirty_price
ytm = brentq(residual, -0.015, 0.25)
assert abs(residual(ytm)) < 1e-10
print(f"YTM = {100 * ytm:.4f}%")
```

scipy.optimize.brentq combines bracketing methods to solve a scalar root robustly. The bracket must contain a sign change.

← **Return to the inverse problem**

Practical map

Practical P5: Compute duration, then test it

```
pv = cash_flow / (1 + ytm / 2) ** powers
price = pv.sum()
times = powers / 2

macaulay = np.sum(times * pv) / price
modified = macaulay / (1 + ytm / 2)

shock = 50 / 10_000          # 50 basis points in decimal units
approx_pct = -modified * shock
exact_pct = price_from_ytm(ytm + shock, cash_flow, powers) / price - 1

print(macaulay, modified, approx_pct, exact_pct)
```

The code preserves units explicitly: powers are half-year periods; times are years; yield shocks are decimals.

← **Return to the 50 bp stress**

Practical map

Practical P6a: Declare the two bond cases

```
liability_date = pd.Timestamp("2032-01-31")

# w is the fraction of a coupon period to the next payment.
bonds = pd.DataFrame({
    "name": ["1% 2032", "4.25% 2032"],
    "coupon": [0.50, 2.125],
    "periods": [11, 12],
    "w": [172 / 184, 117 / 183],
    "ytm": [0.0458786624, 0.0460651257],
    "price0": [82.855497, 98.958897],
    "modified": [5.196179, 5.054713],
    "maturity": pd.to_datetime(["2032-01-31", "2032-06-07"]),
})
```

Every input is now visible: contract timing, solved YTM, base price, duration, and the liability date. The values come from the verified calculations in A28.

Continue to repricing and output →

Practical map

Practical P6b: Reprice and inspect the decision table

```python
rows = []
for bond in bonds.itertuples():
    cash_flow = np.full(bond.periods, bond.coupon)
    cash_flow[-1] += 100
    powers = bond.w + np.arange(bond.periods)
    for shock_bp in (-50, 50, 130):
        shock = shock_bp / 10_000
        exact = price_from_ytm(bond.ytm + shock, cash_flow, powers)
        rows.append({
            "bond": bond.name, "shock_bp": shock_bp,
            "exact_change_pct": 100 * (exact / bond.price0 - 1),
            "duration_change_pct": 100 * (-bond.modified * shock),
            "liability_date_gap_days": (bond.maturity - liability_date).days,
        })
stress = pd.DataFrame(rows)
print(stress.round(3))
```

Example row: 1% 2032 | +50 bp | exact change −2.561% | 0-day liability gap.

← **Return to the integrated stress matrix**

Practical map