Forwards, Futures, and Hedging

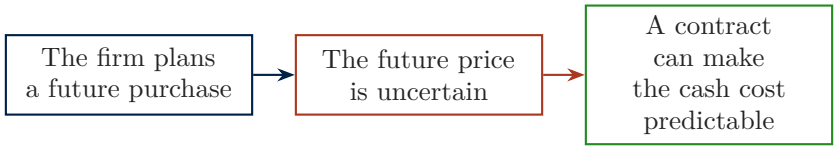
A first course in fixing an uncertain future price

Summer 2026

Computational Finance / FinTech Summer School

Day 5 | introductory derivatives

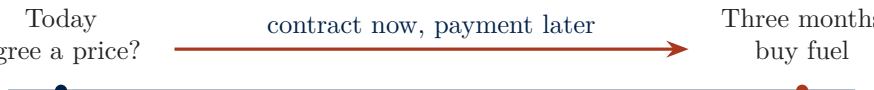
Day 4 priced risk; Day 5 manages a commercial price



- Investors may choose which risks to hold; firms also choose which operating risks to retain.
- Today the problem is concrete: an airline knows the quantity it expects to buy, but not its future fuel price.
- We will learn the contract from its cash flows, not from a pricing formula.

Commercial exposure = a future operating cash flow whose amount changes with a market price.

Can the airline stabilise cost without buying fuel today?



No fuel needs to be stored today
but the future trade becomes an obligation

> **Central question: what uncertainty does the contract remove, and what obligations does it create?**

Roadmap: six questions build one hedge

1. **Exposure:** what physical cash flow is at risk?
2. **Forward:** what future trade is promised today?
3. **Futures:** how does an exchange organise that promise?
4. **Margin:** when does cash move before expiry?
5. **Hedge:** which direction and how many contracts?
6. **Judge:** which risks disappear, and which remain?

The order is deliberate: identify the exposure before selecting the instrument.

By the end, you should be able to build and explain a simple hedge

- **Identify** whether a future buyer or seller is hurt by a price rise or fall.
- **Draw and calculate** long and short forward payoffs.
- **Explain** standardisation, clearing, convergence, and daily settlement.
- **Calculate** futures profit or loss, a margin call, and a whole-contract hedge.
- **Distinguish** a successful hedge from a profitable speculative trade.
- **Recognise** basis, quantity, timing, and liquidity risk.

No forward-pricing derivation is required today. The emphasis is contract logic, cash flows, and risk management.

One airline and three prices will carry the entire lecture

Decision	Buy 20,000 barrel-equivalent units of jet fuel in three months
Price uncertainty	USD55, USD75, or USD95 per unit at the purchase date
Exact-forward benchmark	Agree today to pay USD75 per unit in three months
Later futures proxy	WTI futures, 1,000 barrels per standard contract
Simplifying convention	Ignore fees, taxes, discounting, and quantity uncertainty unless introduced

WTI = West Texas Intermediate crude oil. Barrel-equivalent = a common volume unit used here to keep the arithmetic transparent. All case prices are illustrative inputs.

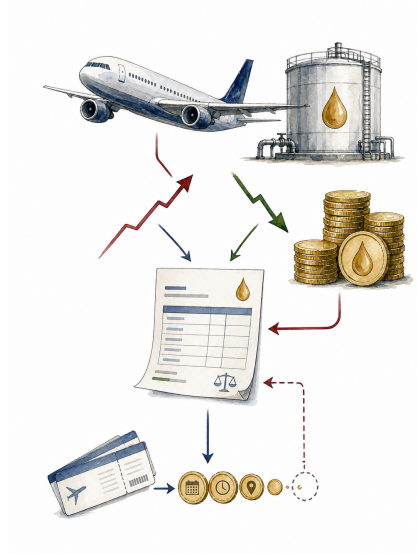
Python P1 · Payoff table

PART 1 OF 6

Identify the exposure

What cash flow changes when the market price moves?

Start with the physical purchase. The derivative comes later.



The physical cash flow is quantity multiplied by the future price

$$\text{fuel cost at } T = Q \times S_T.$$

- The airline is a **future buyer**: it must pay the market price when fuel is needed.
- A higher price raises its cash outflow; a lower price reduces it.
- Before any hedge, the airline is economically exposed to a price rise.

T = purchase date; Q = quantity purchased; S_T = spot price at T. Spot price = price for immediate purchase and delivery.

Without a hedge, the three price states create three different costs

Future fuel price	Quantity	Physical fuel cost
USD55	20,000	USD1.10m
USD75	20,000	USD1.50m
USD95	20,000	USD1.90m

> A USD40 price range becomes a USD800,000 range in the airline's cash cost.

USD1.10m = $20{,}000 \times 55$. The other rows use the same calculation.

Appendix A3 · Cost identity Python P2 · Hedge states

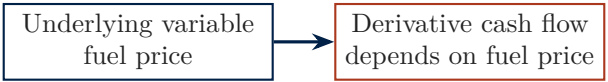
The airline has several responses; a derivative is only one

Response	What it changes	Main limitation
Remain exposed	Nothing before purchase	Cost remains uncertain
Buy and store now	Timing of the physical purchase	Storage and financing are costly
Pass cost to fares	Revenue response	Demand and competition may limit pass-through
Agree a future price	Price paid or offset later	Creates a contractual obligation

Pass-through = the ability to transfer a cost increase into the price charged to customers.

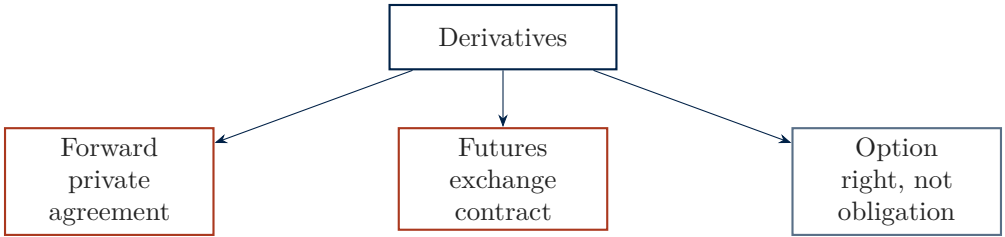
A derivative gets its value from another market variable

> A **derivative** is a contract whose value depends on an **underlying** price, rate, index, or event.



- The contract is not the physical fuel.
- Its cash flow is designed to offset some movement in the physical purchase cost.

Forwards and futures are binding; options add a choice



- Today's focus is the **obligation** in forwards and futures.
- Options return on Day 6; swaps are repeated exchanges over several dates.

Binding = both sides must perform according to the contract unless the position is closed or replaced.

The participant's objective matters more than the contract label

Participant	Starting point	Main objective
Hedger	Existing or expected commercial exposure	Reduce an unwanted price risk
Speculator	No offsetting commercial exposure	Profit from a view on price direction
Arbitrageur	Inconsistent prices across related trades	Lock in a price discrepancy

Arbitrage = a set of offsetting trades designed to secure a profit without net market-price exposure, subject to implementation assumptions.

The same long contract can hedge one position and speculate for another

Person or firm	Existing exposure	Meaning of a long fuel contract
Airline	Must buy fuel later	Hedge: gain can offset a higher purchase price
Trader with no fuel need	None	Speculation: gains only if the price rises
Fuel producer	Will sell fuel later	Adds to price-rise exposure rather than hedging it

> A contract is not inherently a hedge. It becomes a hedge only relative to an offsetting exposure.

Checkpoint: start from the adverse price movement

Commercial situation	Adverse move	Derivative direction
Airline will buy fuel	Price rises	Long
Farmer will sell wheat	Price falls	Short
Importer will pay euros	Euro rises	Long euro
Investor has no exposure but expects oil to rise	None to offset	Long speculation

Long = benefits from a rise in the underlying price. Short = benefits from a fall. These words describe the derivative position.

PART 2 OF 6

Understand the forward promise

What future trade is agreed today?

Read the terms, build the payoff, and combine it with the physical purchase.

A forward fixes the terms of a future trade today

> A **forward contract** is a private agreement to buy or sell an underlying asset at a future date for a delivery price agreed today.

- No fuel changes hands when the contract is signed.
- Both sides accept an obligation for the future settlement date.
- At inception, a fairly negotiated forward usually requires no payment apart from credit arrangements.

Delivery price = the fixed price written into the contract. In this example it is $K = \text{USD}75$ per unit.

Five terms are enough to read the contract

Contract term	Airline example
Underlying	Jet-fuel price
Quantity	20,000 units
Settlement date	Three months from today
Delivery price	USD75 per unit
Settlement method	Physical delivery or equivalent cash settlement

Cash settlement = pay the contract's value in money instead of delivering the physical asset. The payoff logic is the same in this simplified example.

The agreement is made now; its economic result is known later

K = delivery price fixed in the contract; S_T = spot price at settlement.

Long means committed to buy; short means committed to sell

Position	Contract obligation	Benefits when
Long forward	Buy at K on date T	$S_T > K$
Short forward	Sell at K on date T	$S_T < K$

The airline chooses the long side because it is a future buyer and a price rise is adverse.

The long payoff is asset value minus the price paid

$$\text{long payoff per unit} = S_T - K.$$

- If the market price is USD95, buying for USD75 creates a USD20 gain.
- If the market price is USD55, the obligation to pay USD75 creates a USD20 loss.
- If $S_T = K$, neither side has a payoff.

Payoff = contract cash flow at settlement, before any transaction costs. Positive is a gain to the stated position.

Appendix A1 · Payoff audit

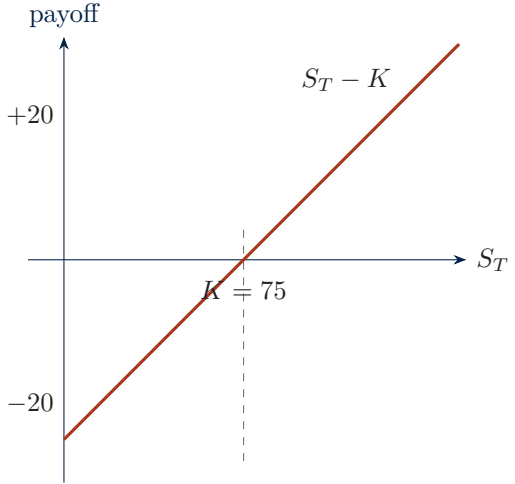
Three states make the long payoff visible

S_T	K	Long payoff: $S_T - K$
USD55	USD75	−USD20
USD75	USD75	USD0
USD95	USD75	+USD20

> The long position exchanges lower-price benefit for protection against a higher price.

Python P1 · Payoff table

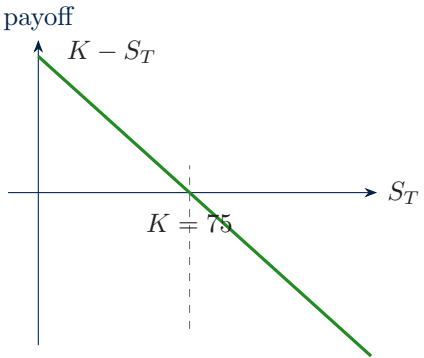
The payoff diagram is a picture of the same subtraction



Slope $+1$ means a one-dollar rise in S_T increases the long payoff by one dollar per unit.

The short payoff is the mirror image

$$\text{short payoff per unit} = K - S_T = -(S_T - K).$$



The short has slope -1: it loses when the underlying price rises and gains when it falls.

One side's gain is the other side's loss

$$(S_T - K) + (K - S_T) = 0.$$

S_T	Long payoff	Short payoff	Sum
55	-20	$+20$	0
75	0	0	0
95	$+20$	-20	0

Zero-sum refers to contract payoffs between the two counterparties; each may also have a separate physical exposure.

Appendix A2 · Zero-sum audit

The airline combines two cash flows, not two stories

$$\underbrace{QS_T}_{\text{physical fuel cost}} - \underbrace{Q(S_T - K)}_{\text{long-forward gain}} = QK.$$

- A higher spot price raises the fuel invoice.
- The same higher price increases the long-forward gain.
- With an exact quantity and exact underlying match, the two price effects cancel.

The forward gain is subtracted from cost. If the payoff is negative, subtracting it correctly adds the contract loss.

Appendix A3 · Hedge identity

The exact full hedge locks the case cost at USD1.50 million

S_T	Physical cost	Forward payoff	Net fuel cost
55	1.10m	−0.40m	1.50m
75	1.50m	0	1.50m
95	1.90m	+0.40m	1.50m

> Net cost = physical cost − derivative payoff. The hedge removes both the upside surprise and the downside windfall.

Values are USD millions. Exact match means the derivative and purchase use the same price, quantity, and date.

Appendix A3 · Full arithmetic Python P2 · Hedge states

A private forward is flexible, but performance depends on the counterparty

Advantage	Cost or risk
Match the precise fuel grade	Less liquid than a standard exchange contract
Match the precise quantity	Exiting early requires renegotiation or an offset
Match the precise settlement date	A valuable contract creates counterparty credit exposure

> **Counterparty credit risk** is the risk that the other side fails to make a payment or delivery it owes.

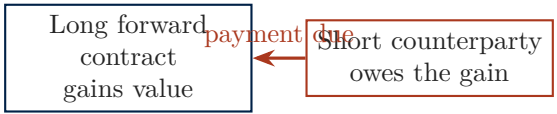
Appendix A4 · Exposure before settlement

Move the promise onto a futures market

How can many traders use a common contract?

Standardisation and clearing make positions easier to trade and control credit exposure.

The forward leaves one unresolved question: will both sides perform?



- As market prices move, one side builds an amount receivable from the other.
- Waiting until the final date allows that unpaid exposure to accumulate.
- A futures market responds with standard contracts, a clearing house, collateral, and daily settlement.

Amount receivable = money owed to a party because the contract has positive value to it.

A futures contract is a standardised, exchange-traded obligation

> A **futures contract** is an exchange-traded agreement to buy or sell a standard quantity under standard settlement rules at a specified future date.

- The economic direction remains long or short.
- The exchange defines the contract size, quality, delivery months, and price quotation.
- The clearing system collects collateral and settles gains and losses each day.

Exchange-traded = positions are entered through an organised market using a common contract specification.

Forwards customise the promise; futures standardise its use

Dimension	Forward	Futures
Trading venue	Bilateral / over the counter	Organised exchange
Terms	Negotiated	Standardised
Credit arrangement	Between counterparties	Central clearing and margin
Cash-flow timing	Usually at settlement	Gains and losses settled daily
Exit	Negotiate or offset privately	Usually take an opposite exchange position

Over the counter (OTC) = privately negotiated rather than traded as a standard exchange contract.

One WTI futures contract represents 1,000 barrels

NYMEX WTI field	Contract specification
Product code	CL
Contract unit	1,000 barrels
Price quotation	U.S. dollars and cents per barrel
Minimum price move	USD0.01 per barrel
Settlement	Physical delivery under the exchange rules

CME Group, Light Sweet Crude Oil Futures contract specifications; accessed August 2026.

NYMEX = New York Mercantile Exchange, part of CME Group. CL is the exchange product code.

A small quoted move can be a material cash amount

$$\underbrace{\text{USD}0.01}_{\text{minimum price move}} \times \underbrace{1{,}000}_{\text{barrels per contract}} = \underbrace{\text{USD}10}_{\text{tick value}}.$$

- A USD1 per barrel move changes one contract by USD1,000.
- A USD1 move across 20 contracts changes the position by USD20,000.
- Always convert the quoted price into the contract's cash scale before discussing risk.

Tick = the smallest permitted quoted price change. Tick value = that move multiplied by the contract unit.

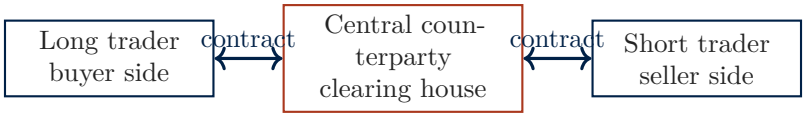
Appendix A5 · Tick audit

A futures quote identifies a contract month, not “the oil price”

Quote field	Example	Interpretation
Contract	CL Dec 2026	WTI contract settling under December rules
Last price	75.10	USD per barrel for this contract month
Change	+0.35	Increase from the prior settlement
Volume	182,400	Contracts traded during the session
Open interest	301,700	Contracts still open after matching long and short positions

The quote is synthetic and used only to read fields. Contract month determines the settlement and delivery timetable.

The clearing house becomes the contractual middle



margin and daily settlement
support both obligations

Central counterparty (CCP) = an institution that becomes buyer to each seller and seller to each buyer through novation. Novation = replace the original bilateral contract with contracts against the CCP.

A futures position can end in three different ways

Ending	What the trader does	Typical use
Offset	Enter the opposite position before expiry	Most financial users close this way
Physical delivery	Follow the contract's delivery procedure	Relevant to deliverable commodity users
Cash settlement	Exchange the final value in cash	Used by contracts whose rules specify cash

> Buying a futures contract does not normally mean the trader intends to receive the commodity.

Offset = close an open position by taking an equal opposite position in the same contract.

At expiry, futures and spot prices must converge

$$F_T \longrightarrow S_T \qquad \text{at the contract's settlement point.}$$

- If a deliverable futures price stayed well above the eligible spot price, selling futures and sourcing delivery would be attractive.
- If it stayed well below, buying through the futures mechanism would be attractive.
- The settlement rules therefore link the final futures price to the relevant spot market.

F_T = futures price at expiry; the relevant S_T must match the contract's grade, location, and delivery rules. Convergence does not mean earlier futures prices equal spot prices.

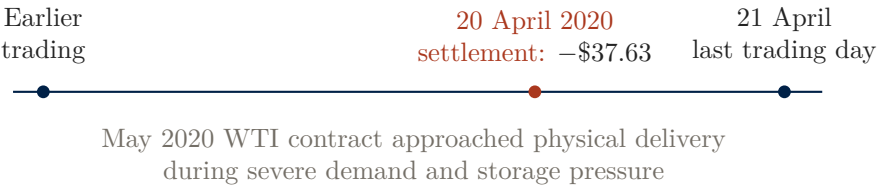
Volume counts trading; open interest counts positions still open

Event	Daily volume	Change in open interest	Reason
A opens long; B opens short	1	+1	New matched contract
A sells to close; C buys to open	1	0	Position changes owner
C sells to B; both close	1	−1	Matched contract disappears

Volume = number of contracts traded during a period. Open interest = number of matched contracts that remain outstanding. One open contract has both a long and a short side.

Appendix A6 · Position ledger

Negative WTI showed why the exact contract and exit date matter



- The negative price was for a particular expiring futures contract, not every barrel everywhere.
- Contract month, delivery location, storage conditions, and planned exit date are part of the risk.

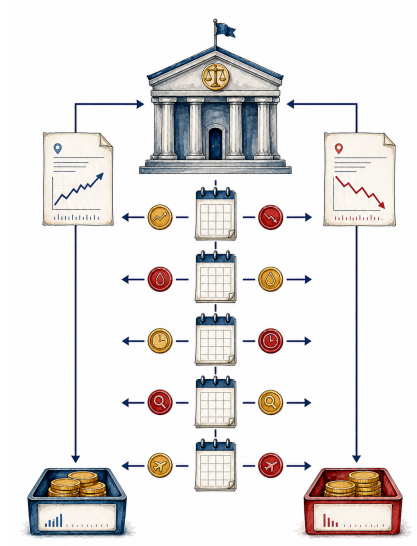
U.S. Commodity Futures Trading Commission, Interim Staff Report, November 2020.

Appendix A11 · Source distinction

Follow the cash through margin

Why can cash move before the hedge finishes?

Daily settlement controls credit exposure but creates funding needs.



Margin is collateral, not the futures price

> **Margin** is collateral posted to support a futures obligation. It is not a purchase price for the underlying asset.

- A long futures position does not pay USD75,000 to buy one 1,000-barrel contract at USD75.
- Instead, both long and short post margin and receive or pay daily gains and losses.
- The required collateral is only part of the contract's economic exposure.

Four margin terms describe the cash process

Term	Meaning
Initial margin	Collateral deposited when the position is opened
Maintenance margin	Minimum account balance before replenishment is required
Variation margin	Daily cash transfer arising from that day's gain or loss
Margin call	Request to restore the account after it falls below the required level

The example uses synthetic round numbers. Actual exchange and broker requirements vary over time and by participant.

Our margin ledger uses two contracts and disclosed assumptions

Field	Illustrative input
Position	Long 2 futures contracts
Contract unit	1,000 barrels each
Entry futures price	USD75 per barrel
Initial margin	USD7,000 per contract; USD14,000 total
Maintenance margin	USD6,000 per contract; USD12,000 total
Settlement prices	75, 74, 73, 76

These are illustrative values, not current CME or broker margin requirements.

Python P4 · Margin ledger

Daily futures profit or loss follows the price change

$$\text{daily P/L} = N \times q \times (F_t - F_{t-1}).$$

- N is positive for a long position and negative for a short position.
- q converts the quoted price change into the contract's cash scale.
- The day's P/L is credited to or debited from the margin account.

P/L = profit or loss; N = signed number of contracts; q = units per contract; F_t = today's settlement price.

Day 1: a one-dollar fall costs the long position USD2,000

$$2 \times 1{,}000 \times (74 - 75) = -\text{USD}2{,}000.$$

Opening margin balance	USD14,000
Day 1 variation margin	−USD2,000
Closing balance	USD12,000

The balance equals maintenance margin, so the stated rule produces no call yet.

Day 2: another fall takes the balance below maintenance

$$\text{USD12,000} - \text{USD2,000} = \text{USD10,000} < \text{USD12,000}.$$

- The account is now below the USD12,000 maintenance level.
- Under the stated rule, the trader receives a USD4,000 margin call.
- The payment restores the account to the USD14,000 initial-margin level.

A margin call is a collateral payment. It is not an additional derivative loss; the loss already appears in the account balance.

The complete ledger separates trading P/L from collateral deposits

Day	Settlement	Daily P/L	Before call	Margin call	After call
0	75	0	14,000	0	14,000
1	74	−2,000	12,000	0	12,000
2	73	−2,000	10,000	4,000	14,000
3	76	+6,000	20,000	0	20,000

Total trading P/L is $-2{,}000 - 2{,}000 + 6{,}000 = +$USD2,000.

All account amounts are USD. “Before call” includes the day’s P/L; “after call” adds any collateral deposit.

Appendix A7 · Full ledger Python P4 · Reproduce it

The ending account reconciles with the entry-to-exit gain

$$2 \times 1{,}000 \times (76 - 75) = +\text{USD}2{,}000.$$

Total collateral deposited	USD18,000
Ending margin account returned	USD20,000
Net cash from futures position	+USD2,000

The collateral deposits are returned with the accumulated futures gain or loss. They must not be counted again as trading losses.

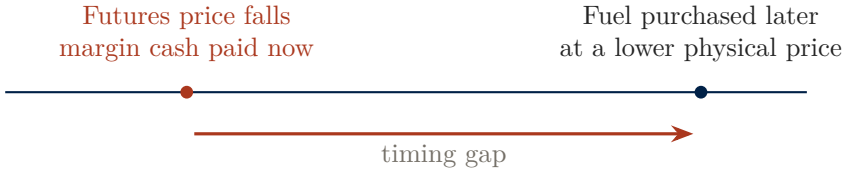
For a short position, reverse the sign of every daily price change

$$\text{short daily P/L} = Nq(F_{t-1} - F_t), \qquad N > 0.$$

Price move	Long 2 contracts	Short 2 contracts
75 to 74	−USD2,000	+USD2,000
73 to 76	+USD6,000	−USD6,000

The clearing house transfers the daily loss from one side to the daily gain of the other, after applying its risk controls.

A hedge can work economically and still create a cash squeeze



- The lower eventual fuel bill may offset the futures loss in economic terms.
- The margin payment arrives before that lower bill is realised.
- Liquidity planning is therefore part of hedge design, not an afterthought.

Liquidity risk = the risk that cash is unavailable when a payment is due, even if the overall hedge is economically sensible.

Checkpoint: four cash concepts must stay separate

Concept	Correct interpretation
Futures loss	Change in contract value caused by the price move
Margin balance	Collateral account after daily settlement
Margin call	Deposit that restores collateral after the balance falls too far
Funding need	Cash the firm must make available at the required time

> A margin call is evidence of adverse daily price movement, not proof that the commercial hedge has failed.

PART 5 OF 6

Build a simple hedge

Which side, which month, and how many contracts?

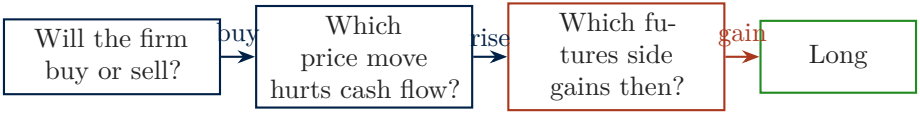
Translate the commercial exposure into a transparent futures position.

The direction rule follows the price movement that hurts

Commercial exposure	Adverse price move	Simple futures hedge
Must buy later	Price rises	Long futures
Must sell later	Price falls	Short futures

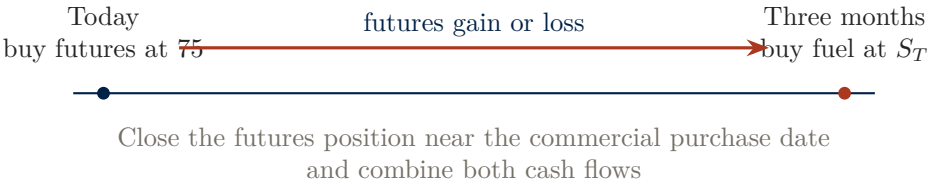
> Hedge direction is chosen so that the futures position gains in the state that harms the physical cash flow.

For the airline, three questions determine the long side



Do not start from "I think prices will rise." A hedge starts from how the firm's existing cash flow responds to price.

The physical purchase and futures position meet at the hedge date



Hedge date = date when the futures position is closed and the commercial transaction is priced. It need not be the futures contract's expiry date.

Quantity divided by contract size gives the benchmark count

$$N^* = \frac{\text{20,000 units}}{\text{1,000 units per contract}} = 20 \text{ contracts.}$$

- The result is exact only because the exposure quantity is a multiple of the contract unit.
- In practice, the count must be a whole number unless a different instrument is added.
- Rounding leaves a small unhedged or overhedged quantity.

N^* = benchmark number of contracts under an exact quantity match; it is not an optimal hedge-ratio estimate.

Appendix A9 · Quantity audit Python P3 · Whole contracts

Twenty long contracts offset the full exposure

$$\text{futures gain} = 20 \times 1{,}000 \times (S_T - 75).$$

- At $S_T = 95$, the futures gain is USD400,000.
- At $S_T = 55$, the futures loss is USD400,000.
- Under an exact price match, this amount offsets the same USD400,000 movement in the physical fuel bill.

This benchmark assumes the futures price at hedge close equals the physical fuel price. Basis risk is introduced after the benchmark is understood.

The full benchmark hedge removes the price-state spread

S_T	Physical cost	Long-futures payoff	Net cost
55	1.10m	−0.40m	1.50m
75	1.50m	0	1.50m
95	1.90m	+0.40m	1.50m

> An exact hedge converts an uncertain market price into the agreed benchmark price.

Amounts are USD millions. Net cost = physical cost minus futures payoff.

A partial hedge covers only part of the forecast quantity

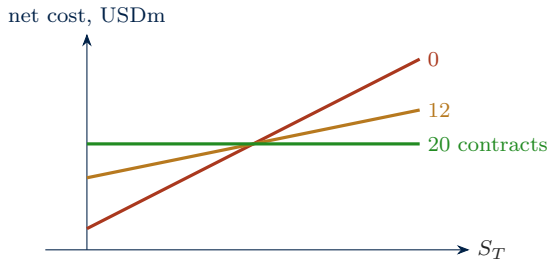
$$\text{coverage} = \frac{12 \times 1{,}000}{20{,}000} = 60\%.$$

- Twelve contracts cover 12,000 of the expected 20,000 units.
- The remaining 8,000 units continue to respond to the future market price.
- Partial coverage can be deliberate when quantity, timing, or liquidity is uncertain.

Coverage = derivative reference quantity divided by the physical exposure quantity. It describes size, not hedge quality.

More contracts reduce price sensitivity but increase obligation size

Position	Cost at 55	Cost at 75	Cost at 95	Cost range
0 contracts	1.10m	1.50m	1.90m	0.80m
12 contracts	1.34m	1.50m	1.66m	0.32m
20 contracts	1.50m	1.50m	1.50m	0



Appendix A9 · State table Python P2 · Compare coverage

A future seller uses the mirror-image short hedge

Wheat farmer	Price falls to 180	Price rises to 260
Physical sale	Lower revenue	Higher revenue
Short futures	Gain	Loss
Combined effect	Downside is offset	Upside is surrendered

> A short hedge protects a future selling price; it is not a prediction that the price will fall.

Choose a contract month close to, and usually after, the exposure date

- A contract expiring before the purchase may force an early replacement.
- A much later contract may move differently from the exposure price.
- Liquidity also matters: the nearest date is not useful if trading is thin.

Liquidity = ability to trade promptly in meaningful size without a large price concession.

WTI can be a useful proxy without being the same as jet fuel

> A **cross hedge** uses a futures contract on a related, but not identical, underlying because an exact contract is unavailable or illiquid.

- Jet fuel and WTI share broad crude-oil market forces.
- Refining margins, grade, location, and timing can still make their prices diverge.
- The proxy should be chosen for economic connection and usable liquidity, not familiarity alone.

Proxy = a related observable price used to represent part of the exposure. The remaining divergence is basis risk.

Checkpoint: a usable hedge statement has direction, count, and mismatch

> "The airline buys 12 WTI futures contracts, covering 60% of 20,000 barrel-equivalent units near the purchase date."

- **Direction:** long, because a price rise harms a future buyer.
- **Count:** 12, because each contract represents 1,000 barrels.
- **Known mismatch:** WTI is not the same product or location as jet fuel.

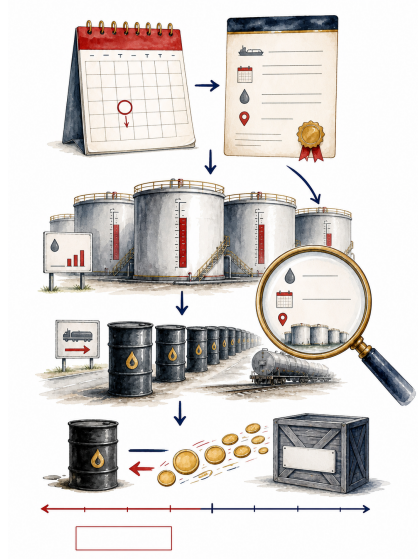
A recommendation without the exposure quantity, date, and residual mismatch is incomplete.

PART 6 OF 6

Judge the hedge

What risk was removed, and what remains?

Evaluate the combined position rather than celebrating one profitable leg.



Basis measures the gap between the exposure price and futures price

$$b_t = S_t - F_t.$$

- S_t is the price relevant to the physical exposure.
- F_t is the selected futures price at the same date.
- A hedge can remove broad market movement while changes in this gap remain.

Basis = spot or exposure price minus futures price. Basis risk = uncertainty about how the basis changes while the hedge is open.

Appendix A10 · Basis arithmetic Python P5 · Effective cost

Four common mismatches create residual risk

Mismatch	Airline example	Consequence
Asset	Jet fuel versus WTI crude	Refining spread can change
Quantity	Forecast volume versus contracts	Some units remain open or overhedged
Timing	Purchase date versus contract month	Prices need not move together
Location	Airport supply versus delivery hub	Transport and local scarcity matter

Residual risk = uncertainty that remains after the hedge. A hedge can reduce risk substantially without being exact.

A basis example converts futures gains into an effective purchase cost

$$\underbrace{790}_{\text{physical purchase price}} - \underbrace{(585-570)}_{\text{long-futures gain}} = \underbrace{775}_{\text{effective cost}} .$$

- The futures price rises by 15, so the long hedge gains 15 per unit.
- The physical exposure is priced at 790, not at the futures close of 585.
- The remaining gap, $790 - 585 = 205$, is the closing basis.

Numbers reproduce the introductory arithmetic in the SJTU derivatives sequence. Units are intentionally generic.

Appendix A10 · Identity Python P5 · Python

The hedge removes benchmark movement but retains mismatch and implementation risk

Risk reduced	Risk retained or introduced
Broad rise in the selected market benchmark	Jet-fuel versus WTI basis movement
Price uncertainty on the covered quantity	Uncovered or overhedged quantity
Exposure over the chosen hedge horizon	Timing and contract-month mismatch
None of the firm's cash needs	Margin and funding liquidity

> Hedging transforms the risk distribution; it does not make every source of uncertainty disappear.

A derivative loss can be evidence that the physical position benefited

Price outcome	Physical fuel purchase	Long hedge
Price rises	More expensive	Gains
Price falls	Cheaper	Loses

> Judge the combined cost against the risk objective set before the price was known.

Hindsight error = evaluating the hedge only by whether the derivative leg made money, after observing the market outcome.

Three avoidable errors can turn a hedge into a new exposure

Error	What goes wrong	Control
Wrong direction	Contract loses in the adverse physical state	Start from buyer/seller and adverse move
Too many contracts	Hedge exceeds the physical quantity	Reconcile contracts with current forecast
Insufficient cash	Margin call forces an untimely close	Reserve liquidity and stress cash timing

Overhedge = derivative reference quantity exceeds the exposure it is intended to offset.

A seven-question check catches most introductory hedge errors

1. What physical cash flow is exposed, in which units and on which date?
2. Does a price rise or fall harm it?
3. Which contract is economically related and sufficiently liquid?
4. Is the futures direction long or short?
5. How many whole contracts correspond to the intended coverage?
6. Which basis, quantity, timing, and location mismatches remain?
7. Can the firm fund plausible margin calls before the commercial benefit arrives?

Synthesis: hedging exchanges one uncertainty for a narrower set

- Exposure determines direction; contract unit determines count.
- Daily settlement changes cash timing, not the basic long/short payoff direction.
- Success means the combined position behaves as intended, with residual risks understood.

References and source notes

- Hull, J. C., *Options, Futures, and Other Derivatives*, Chapters 1–3: contract definitions, payoff logic, futures operations, margin, and introductory hedging.
- Kansoy, F., SJTU Derivatives Lectures 1–3 and associated activities, 2026: exposure-first sequence, numerical payoff construction, margin ledger, hedge direction, and basis arithmetic.
- CME Group, "Light Sweet Crude Oil Futures: Contract Specifications": 1,000-barrel CL contract and USD0.01-per-barrel minimum price move. cmegroup.com
- U.S. Commodity Futures Trading Commission, *Interim Staff Report on Trading in NYMEX WTI Crude Oil Futures Contract Leading up to, on, and around April 20, 2020*, November 2020. cftc.gov
- Brealey, Myers, Allen, and Edmans; Berk and DeMarzo: consulted for general risk-management context, but not used to impose forward-pricing theory on this introductory lecture.

All airline prices, margin requirements, and quote values are disclosed illustrative inputs unless a primary source is named.

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Technical appendix map

A1 Payoff audit | A2 Zero-sum | A3 Full-hedge identity

A4 Counterparty exposure | A5 WTI specification | A6 Open interest

A7 Margin ledger | A8 Reconciliation | A9 Partial hedge

A10 Basis arithmetic | A11 Negative WTI | A12 Provenance

> Main lecture frames carry links only where a derivation, audit trail, or source detail helps without interrupting the narrative.

Python practical map

A1. Long and short payoff audit

S_T	K	Long $S_T - K$	Short $K - S_T$	Long + short
55	75	-20	$+20$	0
75	75	0	0	0
95	75	$+20$	-20	0

$$\text{total long payoff on 20,000 units} = 20{,}000(S_T - K).$$

All table payoffs are USD per unit. Multiply by quantity only after the sign is correct.

← **Return to long payoff**

Appendix map

A2. The forward contract is zero-sum between its two sides

$$\Pi_T^{\text{long}} + \Pi_T^{\text{short}} = Q(S_T - K) + Q(K - S_T) = 0.$$

- For $Q = 20{,}000$, $S_T = 95$, and $K = 75$, the long gains USD400,000.
- The short loses the same USD400,000 under the contract.
- Their overall business profits need not sum to zero because each side may own a different physical exposure.

Π_T denotes the derivative payoff at settlement; transaction costs and default are excluded from the identity.

← **Return to zero-sum slide**

Appendix map

A3. Exact full-hedge identity

$$\begin{aligned} \text{physical cost} &= QS_T, \\ \text{long-forward payoff} &= Q(S_T - K), \\ \text{net cost} &= QS_T - Q(S_T - K) = QK. \end{aligned}$$

With $Q = 20{,}000$ and $K = 75$, $QK = \text{USD}1{,}500{,}000$ in every terminal price state.

Exact means same underlying price, quantity, and date. Fees, discounting, quantity error, and credit risk are excluded.

← **Return to full hedge**

Appendix map

A4. A valuable forward creates an amount receivable

$$20{,}000 \times (95 - 75) = \text{USD}400{,}000.$$

- At settlement, the long side is owed USD400,000 in the high-price state.
- If payment is postponed until the final date, the amount at risk can accumulate as prices move.
- Collateral, netting, credit limits, and central clearing are different ways of controlling this exposure.

Credit exposure is the positive amount that could be lost if the counterparty defaults; it is not the contract's full notional quantity multiplied by price.

← **Return to forward credit risk**

Appendix map

A5. WTI contract and tick-value audit

Field	Audit
Contract unit	1,000 barrels
Minimum price move	USD0.01 per barrel
Tick value	$1{,}000 \times 0.01 = \text{USD}10$ per contract
USD1 move	$1{,}000 \times 1 = \text{USD}1{,}000$ per contract
Twenty-contract USD1 move	$20 \times 1{,}000 = \text{USD}20{,}000$

CME Group, Light Sweet Crude Oil Futures contract specifications.

The reference hedge uses the 1,000-barrel WTI unit for sizing. It does not claim WTI and jet-fuel prices are identical.

← **Return to WTI specification**

Appendix map

A6. Open-interest ledger

Long action	Short action	Volume	Open-interest change
Open	Open	1	+1
Close	Open	1	0
Open	Close	1	0
Close	Close	1	−1

- Every trade contributes to volume.
- Open interest rises only when both sides create a new position.
- It falls only when both sides close an existing position.

← **Return to volume and open interest**

Appendix map

A7. Complete margin ledger

Day	F_t	ΔF_t	Daily P/L	Before call	Call	After call
0	75	0	0	14,000	0	14,000
1	74	-1	$-2,000$	12,000	0	12,000
2	73	-1	$-2,000$	10,000	4,000	14,000
3	76	$+3$	$+6,000$	20,000	0	20,000

$$\text{daily P/L} = 2 \times 1{,}000 \times \Delta F_t.$$

Initial margin is USD14,000; maintenance margin is USD12,000. A call restores the account to initial margin after a breach.

← **Return to margin ledger**

Appendix map

A8. Margin-account reconciliation

$$\text{entry-to-exit P/L} = 2 \times 1{,}000 \times (76 - 75) = \text{USD2,000},$$
$$\text{ending balance} - \text{deposits} = 20{,}000 - (14{,}000 + 4{,}000) = \text{USD2,000}.$$

- The two calculations must agree.
- The USD4,000 margin call changes deposited collateral, not total futures P/L.
- A failed reconciliation usually double-counts the call as a trading loss.

← **Return to reconciliation**

Appendix map

A9. Partial-hedge state table

S_T	Physical cost	0 contracts	12 contracts	20 contracts
55	1.10m	1.10m	1.34m	1.50m
75	1.50m	1.50m	1.50m	1.50m
95	1.90m	1.90m	1.66m	1.50m

$$\text{net cost} = 20{,}000S_T - N(1{,}000)(S_T - 75).$$

Amounts are USD millions. The table assumes an exact price match and varies only the number of contracts.

← **Return to coverage comparison**

Appendix map

A10. Basis identity for a long hedge

$$\begin{aligned}\text{effective purchase cost} &= S_2 - (F_2 - F_1)\\ &= F_1 + (S_2 - F_2)\\ &= F_1 + b_2.\end{aligned}$$

For $F_1 = 570$, $S_2 = 790$, and $F_2 = 585$: $b_2 = 205$ and effective cost $= 570 + 205 = 775$.

The hedge fixes the entry futures component F_1 but leaves the closing basis b_2 uncertain. Fees and quantity mismatch are excluded.

← **Return to basis example**

Appendix map

A11. What the negative WTI observation did and did not mean

Supported conclusion	Unsupported conclusion
May 2020 WTI settled at −$37.63 on 20 April 2020	Every crude-oil price was negative
The contract was one day from its last trading day	Oil had no long-run economic value
Demand, storage, delivery, and liquidity conditions mattered	A paper futures position can ignore delivery rules

CFTC, Interim Staff Report, November 2020, pp. 5–10.

The episode illustrates contract specificity; it is not a full model of the 2020 oil market.

← **Return to negative WTI**

Appendix map

A12. Source and assumption provenance

Item	Status	Provenance
20,000-unit airline exposure	Illustrative input	Repeated case designed for arithmetic continuity
USD55 / 75 / 95 states	Illustrative input	Symmetric terminal states around delivery price
Margin USD7,000 / 6,000	Illustrative input	Round values; not a current broker schedule
CL unit and tick	Contract specification	CME Group primary contract page
−$37.63 WTI settlement	Historical observation	CFTC November 2020 report
Basis example 570 / 790 / 585	Constructed example	SJTU derivatives sequence

The machine-readable assumptions are stored in `data/cases/day05_intro_metadata.json`.

← **Return to sources**

Appendix map

Python practical map

P1 Long and short payoff table

P2 Compare hedge coverage

P3 Whole-contract count

P4 Daily margin ledger

P5 Effective cost with basis

- The code reproduces calculations already understood in the lecture.
- Each example declares variables in financial language and comments the calculation.
- Data files are fixed case inputs, so results remain reproducible.

Technical appendix map

P1. Create a long and short payoff table

```python
import pandas as pd

# Prices that might prevail when the forward settles
spot_prices = pd.Series([55, 75, 95], name="spot_price")
delivery_price = 75

payoffs = pd.DataFrame({"spot_price": spot_prices})
payoffs["long_payoff"] = spot_prices - delivery_price
payoffs["short_payoff"] = delivery_price - spot_prices
payoffs["check_sum"] = payoffs["long_payoff"] + payoffs["short_payoff"]

print(payoffs)

```

Expected long payoffs: -20, 0, $+20$. Every value in `check_sum` should equal zero.

← **Return to payoff table**

P2. Compare no hedge, partial hedge, and full hedge

```python
import pandas as pd

# Run the notebook from the day05_intro_latex project folder
data = pd.read_csv("data/cases/day05_intro_hedge_states.csv")

# Put each coverage choice in a separate column for comparison
cost_table = data.pivot(
    index="spot_price", columns="coverage_label", values="net_cost"
)

print((cost_table / 1_000_000).round(2))  # report USD millions

```

Expected ranges: no hedge USD0.80m; partial hedge USD0.32m; full hedge USD0 under the exact-match assumption.

← **Return to coverage comparison**

P3. Convert exposure quantity into whole contracts

```python
exposure_units = 20_000
contract_units = 1_000
target_coverage = 0.60

# Translate the target percentage into a contract count
raw_contracts = target_coverage * exposure_units / contract_units
contracts = round(raw_contracts)

covered_units = contracts * contract_units
residual_units = exposure_units - covered_units

print(contracts, covered_units, residual_units)

```

Expected output: 12 contracts, 12,000 covered units, and 8,000 residual units. Inspect rounding whenever `raw_contracts` is not an integer.

← **Return to contract count**

Practical map

P4. Reproduce and verify the margin ledger

```python
import pandas as pd

ledger = pd.read_csv("data/cases/day05_intro_margin_ledger.csv")

# Check that daily profit/loss adds to the entry-to-exit profit
daily_total = ledger["daily_profit_loss"].sum()
entry_to_exit = 2 * 1_000 * (
    ledger["settlement_price"].iloc[-1]
    - ledger["settlement_price"].iloc[0]
)

assert daily_total == entry_to_exit
print(ledger[["day", "daily_profit_loss", "margin_call"]])

```

The assertion passes at USD2,000. The only margin call is USD4,000 on day 2.

← **Return to margin ledger**

Practical map

P5. Calculate effective cost under basis risk

```python
entry_futures = 570
closing_spot = 790
closing_futures = 585

# A long hedge gains when the futures price rises
futures_gain = closing_futures - entry_futures
effective_cost = closing_spot - futures_gain
closing_basis = closing_spot - closing_futures

print("Futures gain:", futures_gain)
print("Effective cost:", effective_cost)
print("Check F1 + basis:", entry_futures + closing_basis)

```

Expected output: futures gain 15; effective cost 775; the identity $F_1 + b_2 = 775$ confirms the calculation.

← **Return to basis example**

Practical map