Understanding Options

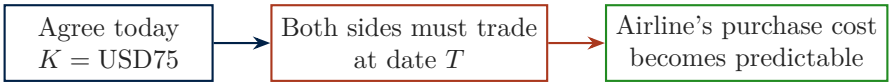
Rights, protection, and contingent decisions

Summer 2026

Computational Finance / FinTech Summer School

Day 6 | introductory derivatives

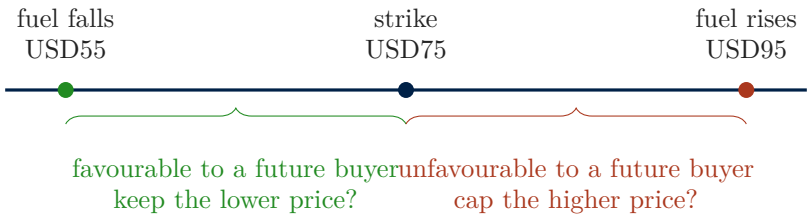
Day 5 fixed a price by accepting an obligation



- The forward protects the airline if fuel rises to USD95.
- The same obligation prevents the airline from benefiting if fuel falls to USD55.
- Day 6 asks whether protection can be purchased without surrendering that favourable state.

Obligation = a contractual duty to perform even when the market outcome later looks unattractive.

Price certainty and favourable-state participation are different objectives



> An option separates the decision to obtain protection from the later decision to use it.

Roadmap: six questions explain one-sided protection

1. **Choice:** why does a right differ from an obligation?
2. **Contract:** what exactly is bought and sold?
3. **Payoff:** how do the four basic positions behave at expiry?
4. **Profit:** where does the upfront premium enter?
5. **Combine:** how can options reshape an existing exposure?
6. **Judge:** what makes the right valuable, useful, and risky?

The sequence moves from economic purpose to contract, cash flow, portfolio, and judgement.

By the end, you should be able to read, build, and judge an option position

- **Read** the type, strike, expiry, style, premium, and multiplier of a quoted option.
- **Draw and calculate** the four basic option payoffs at expiration.
- **Separate** payoff, profit, break-even, and percentage return.
- **Combine** an option with a share or operating exposure to create a floor or cap.
- **Verify** a simple payoff equivalence using put–call parity.
- **Judge** whether the position serves a stated objective and identify writer risk.

Today stops before binomial valuation, Black–Scholes, Greeks, and implied volatility.

Two transparent cases will carry the lecture

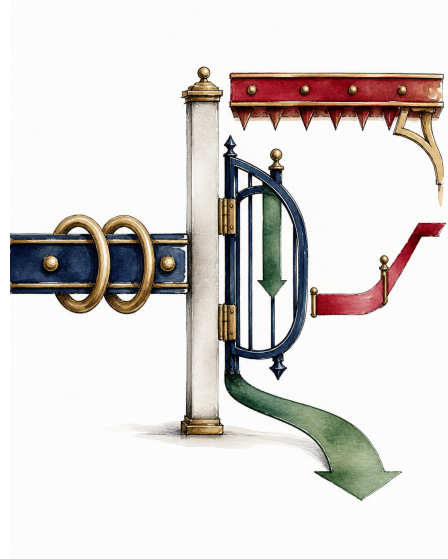
	Airline price cap	Shareholder floor
Exposure	Buy one fuel unit at T	Own one share worth USD100 today
Contract	Call, $K = 75$, premium $c_0 = 4$	Put, $K = 95$, premium $p_0 = 3$
States	$S_T \in \{55, 75, 95\}$	$S_T \in \{70, 95, 100, 130\}$
Question	Cap a high purchase price	Limit loss while retaining upside

S_T = underlying price at expiration; K = strike price; c_0 and p_0 = call and put premiums paid today. All case premiums and states are illustrative inputs.

Replace obligation with choice

What does the premium buy?

Begin with the policy problem. The payoff formula follows from the decision.



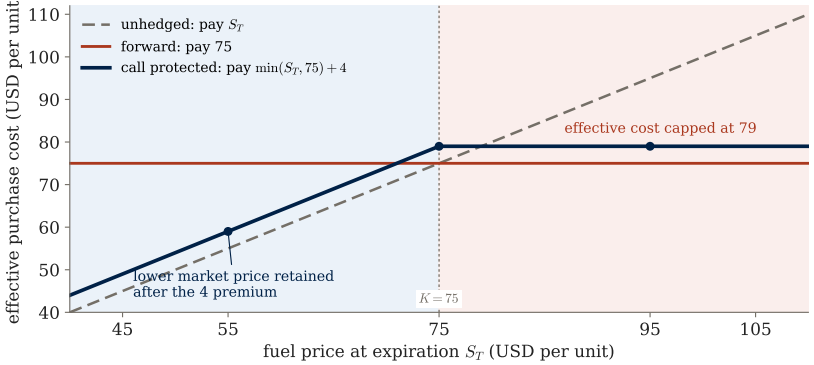
The forward fixes both the bad state and the good state

Fuel price S_T	Unhedged cost	Forward cost	What the forward does
USD55	USD55	USD75	gives up USD20 benefit
USD75	USD75	USD75	no difference
USD95	USD95	USD75	avoids USD20 increase

> The forward creates symmetry: a gain in the bad state is matched by a loss in the good state.

Costs are shown per fuel unit. The forward delivery price is USD75.

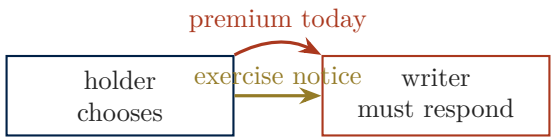
The call preserves lower prices and caps effective cost at USD79



With $K = 75$ and $c_0 = 4$, call-protected cost is $\min(S_T, 75) + 4$: lower prices still reduce cost, while higher prices cannot push it above USD79 per unit.

An option gives the holder a right and leaves the writer with a duty

> An **option** gives its **holder** a right to trade the underlying on specified terms. The **writer** must perform if the holder exercises that right.



Exercise = the holder invokes the contractual right. Writer = the party that sold the option and accepted the contingent obligation.

Calls and puts differ only in the direction of the holder's right

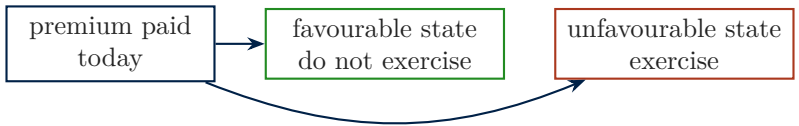
Contract	Holder's right	Natural protection example
Call	Buy the underlying at strike K	Future buyer worried about a price rise
Put	Sell the underlying at strike K	Owner or future seller worried about a price fall

- A right to buy is valuable when the market price rises above the strike.
- A right to sell is valuable when the market price falls below the strike.
- "Call" and "put" identify contract rights; "long" and "short" identify who owns or wrote them.

Strike price = the contractual price at which the underlying may be bought or sold.

The premium is the price of asymmetry

- If the future outcome is favourable, the holder can leave the option unused.
- If the outcome is unfavourable, the holder can require the writer to perform.
- The writer accepts a one-sided obligation and therefore receives compensation today.



Premium = the upfront price paid by the option holder and received by the option writer. It is distinct from the strike payment made only if exercise occurs.

The airline exercises the call only when market price exceeds the strike

S_T	Decision	Call payoff	Fuel acquisition
USD55	do not exercise	USD0	buy in market at USD55
USD75	indifferent	USD0	pay USD75 either way
USD95	exercise	USD20	buy at strike USD75

$$\text{call payoff at } T = \max(S_T - K, 0).$$

The maximum operator implements the holder's choice: use the positive exercise value or choose zero.

Appendix A1 · Notation Python P1 · Payoffs

The call buys a price cap, not a fixed purchase price

S_T	Unhedged	Forward	Call protected	Call decision
55	55	75	59	unused
75	75	75	79	unused
95	95	75	79	exercised

$$\text{call-protected purchase cost} = \min(S_T, K) + c_0.$$

Costs are USD per unit. The call strike is 75 and premium is 4. Premium is included in every state because it was paid at inception.

Python P4 · Airline policies

Choose the policy before seeing which state occurs

Policy	Downside if fuel rises	Benefit if fuel falls	Cash today
Unhedged	fully exposed	keeps all benefit	none
Forward	price fixed	gives up benefit	normally no premium
Call	price capped	keeps benefit after premium	premium paid

- The call is not automatically "better" than the forward; flexibility costs USD4.
- The right choice depends on budget tolerance, financing, competitive pass-through, and the value of lower-price participation.
- Ex post regret is not a risk-management criterion: the policy must be judged against the objective set ex ante.

Ex ante = before the uncertain price is known; ex post = after it is observed.

Read the contract before trading it

Which right, on what terms, for how many units?

A payoff diagram is useful only after the contract has been identified correctly.

Five fields define the economic right

1. **Underlying**: the share, commodity, currency, rate, or index referenced.
2. **Type**: call or put.
3. **Strike**: the contractual buy or sell price.
4. **Expiration**: the final date on which the right exists.
5. **Exercise style**: when the holder may exercise.

> Same underlying and same type do not imply the same contract: strike and expiration change the right materially.

Expiration = the date after which the option no longer exists. Exercise style is defined precisely later in this section.

A market quotation adds price, scale, and trading information

Field	What it tells the trader
Premium quote	price per underlying unit, commonly per share for equity options
Multiplier	number of underlying units represented by one contract
Bid / ask	prices available to sell / buy the option in the market
Volume	contracts traded during the stated period
Open interest	contracts still open after offsetting positions are netted

Bid = highest displayed buying price; ask = lowest displayed selling price. Open interest is not the number of contracts traded today.

Each field in an option quotation changes the contract or its scale

Underlying	Expiration	Type / strike	Bid / ask	Open interest
ACME	20 Dec 2026	Call / 95	2.80 / 3.00	4,250

- Buying at the USD3.00 ask acquires the right to buy ACME at USD95 until the stated expiration.
- The displayed USD3.00 is a per-share premium quote, not the total contract cash payment.
- Open interest records contracts still open; it does not measure today's trading volume.

ACME and the quotation are fictional case inputs, not a current recommendation or observed market quote. OI = open interest.

A USD3 quote means USD300 for one standard equity contract

$$\text{premium cash} = \text{quote} \times \text{contract multiplier} = 3.00 \times 100 = \text{USD300}.$$

- A standard U.S. equity option normally represents 100 shares.
- Corporate actions can create adjusted contracts with a different deliverable.
- Ten contracts at the same quote require USD3,000 before fees: $10 \times 3 \times 100$.

OCC, Equity Options Product Specifications, accessed August 2026.

Multiplier = units of the underlying represented by one option contract. Always check the actual contract specification.

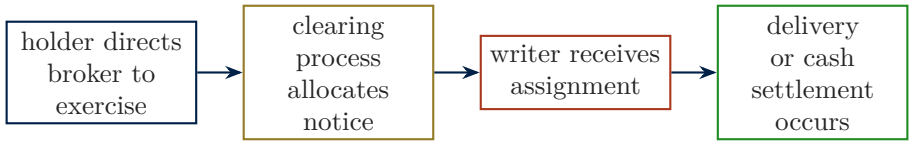
Appendix A5 · Multiplier Python P3 · Contract scale

Strike payment and option premium are different cash flows

- The USD300 premium is paid to acquire the option.
- The USD9,500 strike payment occurs only if the call is exercised and physically settled.
- Confusing them understates both cash needed on exercise and the investor's initial cost.

Physical settlement = exercise leads to delivery of the underlying asset against payment of the strike amount.

Exercise turns the holder's right into the writer's assignment



- The holder decides whether to exercise, subject to contract rules and broker cut-offs.
- An assigned writer must fulfil the obligation; receiving the premium did not remove that duty.

OIC, "Exercising Options" and "Understanding Assignment."

Assignment = notification that an option writer must fulfil the exercised contract. OCC = Options Clearing Corporation.

Appendix A12 · Exercise details

Rights and obligations determine all four basic positions

Position	Contractual position	Maximum terminal obligation
Long call	right to buy at K	may let option expire
Short call	duty to sell at K if assigned	potentially very large as S_T rises
Long put	right to sell at K	may let option expire
Short put	duty to buy at K if assigned	large if underlying falls toward zero

Long options own choice. Short options have sold that choice and accepted a contingent duty.

American and European describe exercise timing, not geography

Style	When holder may exercise	Common U.S. example
American	any permitted business day through expiry	standard equity options
European	only on the specified expiration date	many index options

- The naming convention does not identify where the contract trades.
- Early exercise is a right, not automatically an optimal action.
- Writers of American-style options face possible assignment before expiry.

OCC/OIC, Equity Options Product Specifications and Options Exercise FAQ.

Early exercise = exercise before the option's expiration date.

Appendix A12 · Style qualifications

Settlement determines what changes hands after exercise

Settlement	Call exercise	Put exercise
Physical	holder buys underlying at K	holder sells underlying at K
Cash	holder receives specified cash difference	holder receives specified cash difference

- Standard U.S. equity options are generally physically settled.
- Many index options settle in cash because the index itself cannot be delivered.
- The payoff formula may look similar, but operational cash and asset flows differ.

OCC, Equity Options Product Specifications; OIC, Equity vs. Index Options.

Cash settlement = the contract is discharged by a cash amount rather than delivery of the underlying asset.

Moneyness asks whether immediate exercise has positive value

Moneyness	Call	Put
In the money	$S > K$	$S < K$
At the money	$S \approx K$	$S \approx K$
Out of the money	$S < K$	$S > K$

Moneyness compares the current underlying price with the strike. It does not subtract the premium and therefore does not determine profit.

In the money = immediate exercise has positive intrinsic value. At the money is a market description, not an exact universal interval.

Appendix A11 · Moneyness audit

Contract checkpoint: three errors change the economics

1. Reading a USD3 premium quote as USD3 total ignores the 100-share multiplier.
2. Treating a call holder as obligated confuses the long position with the writer's duty.
3. Calling an option profitable because it is in the money ignores the premium paid.

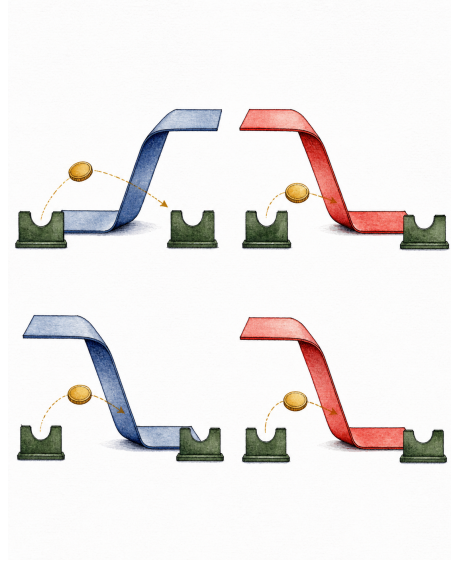
> Before calculating: identify the underlying, right, strike, expiry, style, settlement, quote unit, and multiplier.

The payoff diagram begins only after this contract-reading audit is complete.

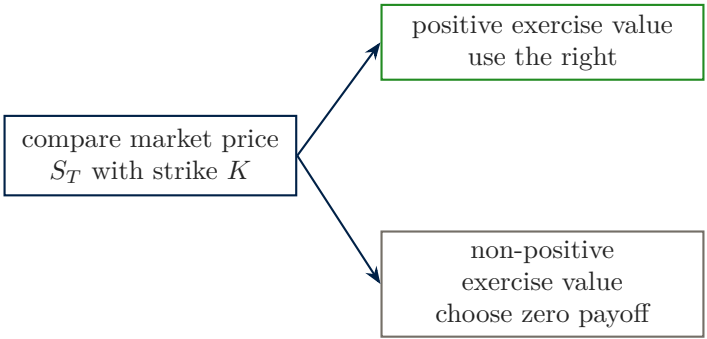
Build the four payoffs

What cash flow exists at expiration?

Derive each shape from the exercise decision; do not memorise four disconnected pictures.



Every expiration payoff begins with one comparison



- For a call, buying at K is valuable when the market price is higher.
- For a put, selling at K is valuable when the market price is lower.
- The writer's payoff is the negative of the holder's payoff for the same contract.

The long call keeps only the positive value of buying at the strike

$$C_T = \begin{cases} 0, & S_T \le K, \\ S_T - K, & S_T > K, \end{cases} \qquad = \max(S_T - K, 0).$$

- Below the strike, buying at K would be worse than buying in the market, so the holder does not exercise.
- Above the strike, exercise saves $S_T - K$ per unit.
- The payoff cannot be negative because exercise is a choice.

C_T = call payoff at expiration, before subtracting the premium paid at inception.

Appendix A1 · Exact notation

The state table makes the exercise rule visible

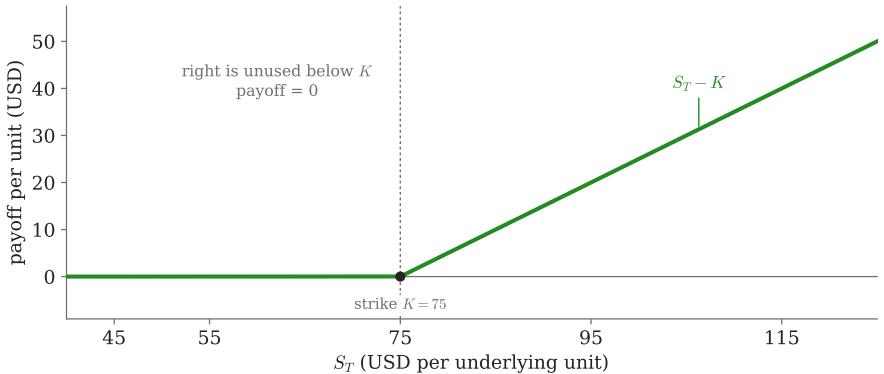
S_T	K	Exercise?	$\max(S_T - K, 0)$
55	75	no	0
75	75	indifferent	0
95	75	yes	20
115	75	yes	40

> Once the call is in the money, each additional USD1 in S_T adds USD1 to payoff per underlying unit.

Indifferent at $S_T = K$ means exercise value is zero; transaction and operational considerations can still matter in practice.

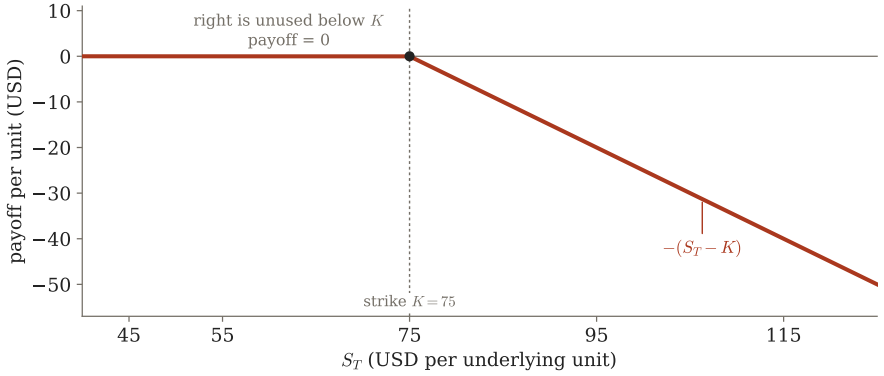
Appendix A2 · Call audit Python P1 · Payoffs

The long-call payoff is flat, then rises one for one



The kink is the point at which the exercise decision and payoff slope change. This is expiration payoff, not the option's market price before expiration.

The short call is the holder's payoff reflected through zero



Short-call payoff is $-\max(S_T - K, 0)$. The writer must sell at the strike if assigned; premium enters profit later.

Appendix A2 · Call audit

An uncovered short call has no finite terminal loss limit

$$\text{short-call payoff} = -\max(S_T - K, 0) \longrightarrow -\infty \quad \text{as } S_T \longrightarrow \infty.$$

- The premium received is finite; the obligation to deliver at K grows as the market price rises.
- "Uncovered" means the writer does not already own the underlying needed for delivery.
- Broker collateral and risk limits reduce counterparty exposure but do not change the payoff shape.

OCC, Characteristics and Risks of Standardized Options, June 2024.

Uncovered or naked call = short call without an offsetting long position in the underlying or another protective instrument.

The long put keeps only the positive value of selling at the strike

$$P_T = \begin{cases} K - S_T, & S_T < K, \\ 0, & S_T \geq K, \end{cases} \qquad = \max(K - S_T, 0).$$

- Below the strike, selling at K is better than selling in the market.
- Above the strike, the holder leaves the put unused.
- For a non-negative stock price, the largest per-share expiration payoff is K when $S_T = 0$.

P_T = put payoff at expiration, before subtracting the premium paid at inception.

Appendix A1 · Exact notation

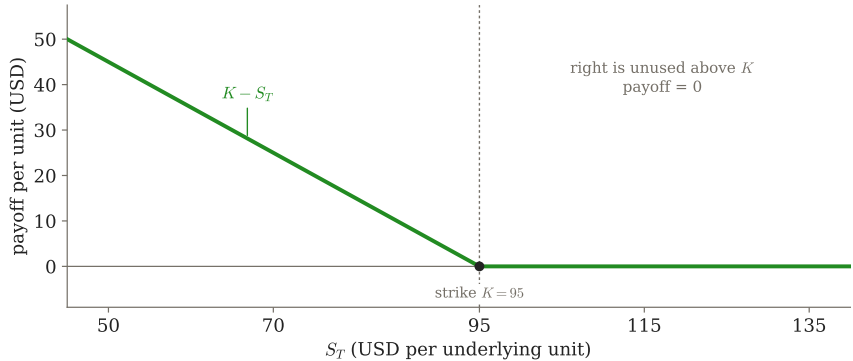
The put state table reverses the call's exercise region

S_T	K	Exercise?	$\max(K - S_T, 0)$
70	95	yes	25
95	95	indifferent	0
100	95	no	0
130	95	no	0

> The put is naturally matched to an owner or future seller harmed by a price fall.

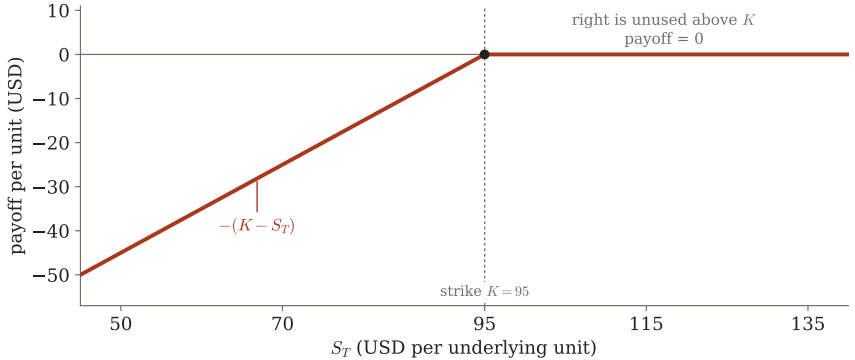
Appendix A3 · Put audit Python P1 · Payoffs

The long-put payoff falls to the strike, then becomes zero



A long put becomes a floor only when combined with an asset whose value falls with S_T; by itself it is a contingent cash payoff.

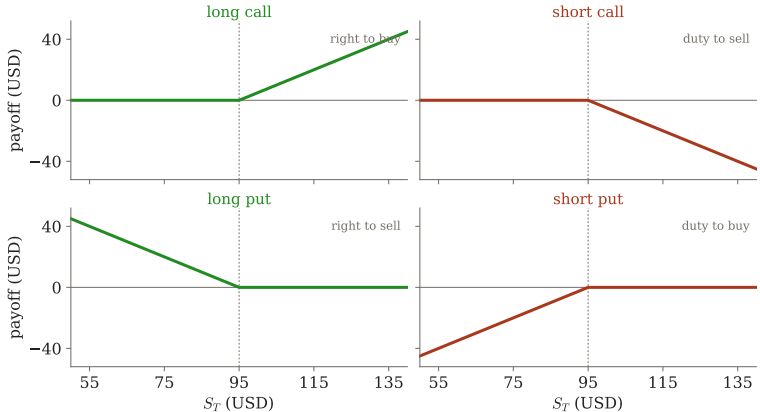
The short put accepts the duty to buy when price is below the strike



Short-put payoff is $-\max(K - S_T, 0)$. If assigned, a physically settled equity-put writer must pay K per share and receive the shares.

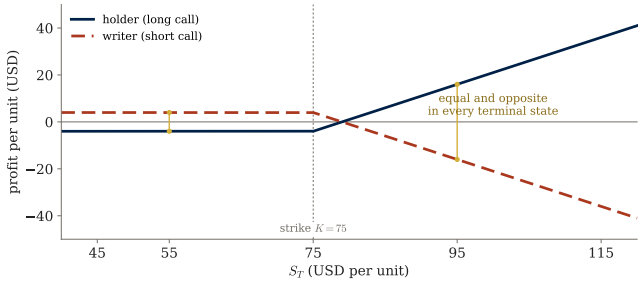
Appendix A3 · Put audit

One rights table and two mirror rules generate all four shapes



Green = holder payoff; crimson = writer payoff. All four panels use the same strike, terminal-price range, and payoff scale.

For the same contract, holder and writer profits are equal and opposite



- The holder's and writer's profits are equal and opposite for the matched option contract.
- Their complete business outcomes can differ because each may hold a different underlying exposure.

State-contingent = the cash flow depends on which future price state occurs.

Appendix A2 · Call audit Python P1 · Payoffs

Payoff checkpoint: infer the contract from the shape

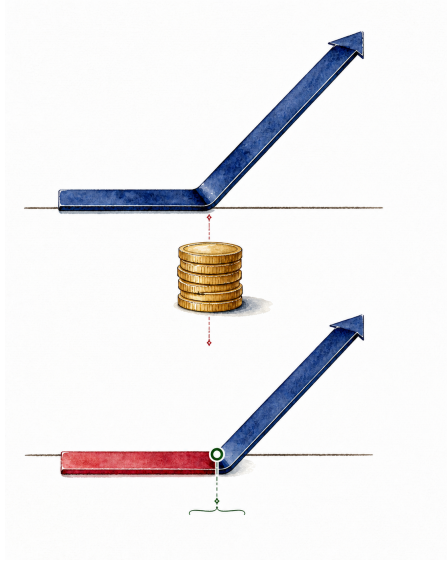
1. Flat at zero below K, rising above K: **long call**.
2. Positive below K, flat at zero above K: **long put**.
3. Mirror the holder through zero: the corresponding **short** position.
4. Match the instrument to exposure: future buyer and price ceiling $\rightarrow$ call; owner and price floor $\rightarrow$ put.

> Payoff tells us what the contract pays at expiration. It still does not tell us whether the trade made a profit.

Separate payoff, profit, and return

How does the premium move the result?

Keep the expiration right, upfront cost, and capital base as distinct quantities.



Payoff is received at expiration; profit also includes the premium

Quantity	Long call	Long put
Payoff at T	$\max(S_T - K, 0)$	$\max(K - S_T, 0)$
Profit at T	$\max(S_T - K, 0) - c_0$	$\max(K - S_T, 0) - p_0$
Maximum loss	premium c_0	premium p_0

- Exercise is based on positive payoff, not on whether the original trade has broken even.
- The premium shifts the holder's entire profit function downward.
- Financing and transaction costs can be added later; they are excluded from today's profit diagrams.

Profit here means expiration payoff minus inception premium, with interest on the premium ignored.

Appendix A4 · Break-even Python P2 · Profit curves

A long call breaks even above the strike

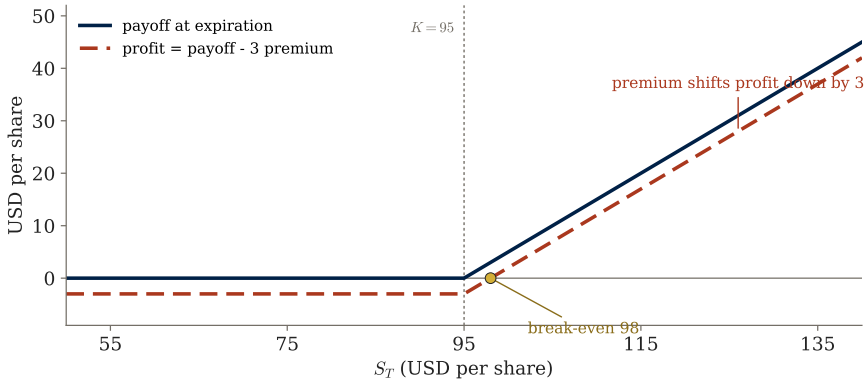
$$\begin{aligned} \max(S_T - K, 0) - c_0 &= 0, \\ S_T - K - c_0 &= 0 \quad \text{in the exercise region}, \\ S_T^{\mathrm{BE}} &= K + c_0. \end{aligned}$$

For $K = 95$ and $c_0 = 3$, the expiration break-even is USD98 per share.

Break-even = terminal underlying price at which the defined trade profit equals zero. It is not the strike and not an exercise threshold.

Appendix A4 · All break-evens

The premium lowers profit without changing exercise value



Both lines kink at the strike because the exercise decision is unchanged. Only the profit line crosses zero at the break-even.

One contract turns per-share profit into material cash outcomes

S_T	Payoff/share	Profit/share	Profit/contract	Exercise?
80	0	−3	−300	no
95	0	−3	−300	indifferent
98	3	0	0	yes
120	25	22	+2,200	yes

$$\text{contract profit} = 100\,[\max(S_T - 95, 0) - 3].$$

The standard 100-share multiplier is used. Fees and interest are ignored. “Indifferent” concerns exercise value at $S_T = K$, not the USD300 trade loss.

Python P3 · Contract scale

A long put breaks even below the strike

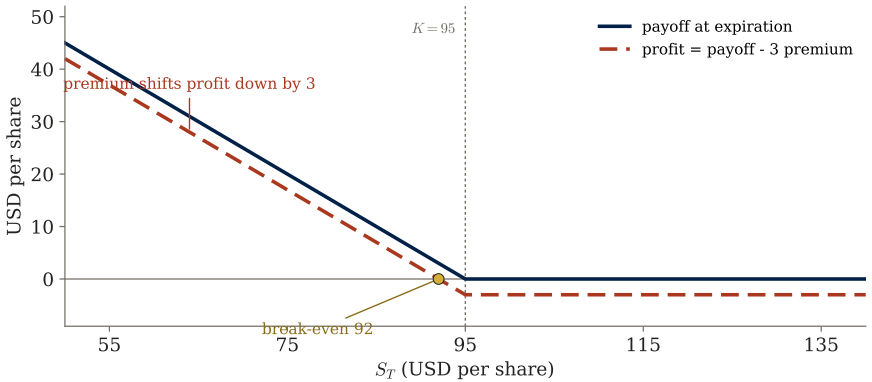
$$\begin{aligned} \max(K - S_T, 0) - p_0 &= 0, \\ K - S_T - p_0 &= 0 \quad \text{in the exercise region}, \\ S_T^{\mathrm{BE}} &= K - p_0. \end{aligned}$$

For $K = 95$ and $p_0 = 3$, the expiration break-even is USD92 per share.

A put can be in the money, for example at USD94, while the trade still has a USD2 loss after its USD3 premium.

Appendix A4 · All break-evens

The long-put profit line sits one premium below its payoff



Above the strike, the put expires with zero payoff and the holder loses the premium.

The premium shifts the writer's profit upward

$$\pi_T^{\text{short call}} = c_0 - \max(S_T - K, 0),$$
$$\pi_T^{\text{short put}} = p_0 - \max(K - S_T, 0).$$

- Maximum profit is the premium received when the option expires unused.
- Break-even is $K + c_0$ for the short call and $K - p_0$ for the short put.
- Premium income is not free yield; it compensates for a state-contingent obligation.

π_T denotes trade profit at expiration under the same no-interest, no-fee convention used for long positions.

Appendix A4 · Writer break-evens

Compare stock and call on equal economic exposure first

Position	Initial cash	Profit if $S_T = 80$	Profit if $S_T = 120$
Buy 100 shares at 100	USD10,000	−2,000	+2,000
Buy one 95 call at 3	USD300	−300	+2,200

- Both positions reference 100 shares, but their payoff shapes and initial cash differ.
- The call sacrifices dividends and ownership rights and can expire worthless.
- A smaller premium does not make the same claim as owning the shares.

Economic exposure here means the same 100-share contract unit, not equal sensitivity before expiration.

Equal initial cash reveals leverage and concentration

USD9,900 invested in	Profit if $S_T = 80$	Profit if $S_T = 100$	Profit if $S_T = 120$
99 shares at 100	−1,980	0	+1,980
33 call contracts at 3	−9,900	+6,600	+72,600

- The call allocation controls 3,300 shares at expiration for the same initial cash.
- That leverage magnifies favourable percentage returns and concentrates the full initial outlay in contracts that can expire worthless.

The example assumes all positions are held to expiration and ignores interest, dividends, fees, and early sale. It demonstrates payoff leverage, not a recommendation.

Python P2 · Profit curves

Limited loss does not mean low risk

- A long option's dollar loss is limited to the premium, but that can be 100% of the amount invested.
- Direction alone is insufficient: the price move must be large enough and occur before expiration.
- Short maturity and far out-of-the-money strikes make complete premium loss more likely.
- Leveraged allocations can turn a modest market view into a concentrated portfolio outcome.

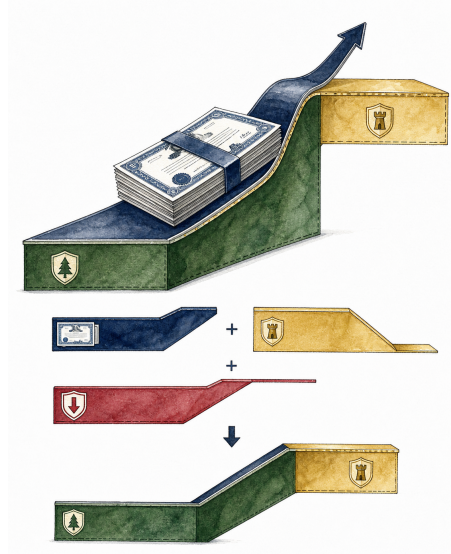
> Risk must be evaluated relative to capital committed, time available, and the economic objective, not only the maximum dollar loss per contract.

OCC, Characteristics and Risks of Standardized Options, June 2024.

Combine positions to reshape risk

Which boundary should the portfolio create?

Add payoffs state by state: the combination should follow from the exposure and objective.



The shareholder wants a floor without selling the upside

- The investor owns one share bought at USD100.
- A six-month event could reduce the share price, but immediate sale would also forfeit any subsequent recovery.
- A put with strike USD95 and premium USD3 creates a right to sell at USD95.

> Risk objective: preserve upside ownership while placing a known lower bound under the share's terminal sale value.

A boundary is a payoff limit such as a floor below which terminal value does not fall or a cap above which it does not rise.

Stock plus put creates the floor state by state

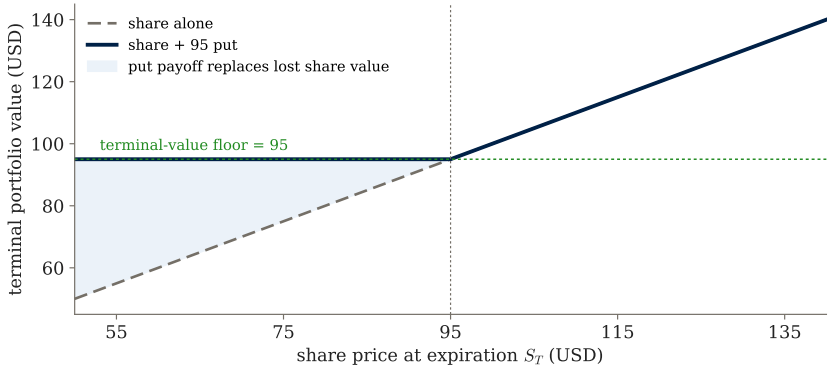
S_T	Stock value	Put payoff	Combined value	Put decision
70	70	25	95	exercise
95	95	0	95	indifferent
100	100	0	100	unused
130	130	0	130	unused

$$S_T + \max(K - S_T, 0) = \max(S_T, K).$$

Combined value excludes the USD3 premium paid at inception. The put changes terminal value only below the USD95 strike.

Appendix A6 · Protective put Python P5 · Portfolios

The protective put keeps the share's upside and replaces its downside



> A **protective put** is a long underlying position combined with a long put on that underlying.

Insurance cost lowers the floor in profit terms

$$\pi_T^{\text{protected}} = S_T + \max(95 - S_T, 0) - 100 - 3,$$
$$\text{maximum loss} = 100 + 3 - 95 = \text{USD8 per share.}$$

S_T	Stock profit	Protected profit	Difference
70	−30	−8	+22
100	0	−3	−3
130	+30	+27	−3

Maximum loss includes the share's USD5 fall from purchase price to strike plus the USD3 put premium.

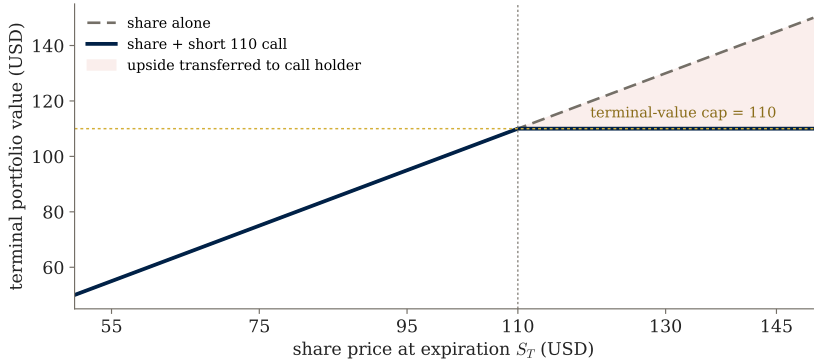
Appendix A6 · Maximum loss

An unused put can still have delivered the intended service

- If the share finishes at USD130, the put expires unused and costs USD3.
- The investor nevertheless retained the upside instead of selling the share at USD100.
- The premium purchased a six-month floor; it was not paid to guarantee that the put itself would be profitable.
- Evaluation should compare the realised portfolio with the pre-specified protection objective.

> Insurance is valuable because it changes outcomes in adverse states, not because an adverse state must occur.

Stock plus short call creates an upside cap



$S_T - \max(S_T - K, 0) = \min(S_T, K)$. **Covered call** = long underlying + short call.

Appendix A7 · Covered call Python P5 · Portfolios

The covered call exchanges premium income for upside participation

- The call premium cushions a small price fall but does not create a hard downside floor.
- Above the strike, the share gain is offset by the short-call obligation.
- Assignment can require delivery of the shares at the strike even when the investor would prefer to keep them.
- The strategy fits only when the investor accepts the cap as part of the objective.

> Covered describes delivery capacity, not protection from a fall in the share price.

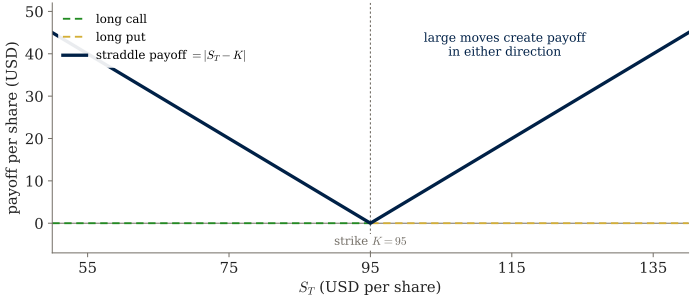
Opportunity cost = the gain forgone because the shares are delivered at the strike rather than sold at a higher market price.

Three positions answer three different risk objectives

Position	Downside	Upside	Cash at inception
Stock	falls with share	unlimited participation	pay share price
Stock + put	floor at strike before premium	retains participation	pay share + put premium
Stock − call	nearly stock-like below strike	capped at strike	receive call premium

- Protective put buys a boundary; covered call sells a boundary.
- The "best" combination cannot be selected without a stated exposure and objective.

A long straddle buys exposure to a large move in either direction



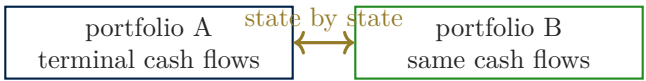
- One long call plus one long put with the same strike and expiration gives payoff $|S_T - K|$.
- The total premium creates two profit break-evens outside the strike.

Straddle = matched call and put position at the same strike and expiry. It is included to explain payoff addition, not as a trading recommendation.

Appendix A8 · Break-evens

Payoff engineering begins with terminal cash-flow equivalence

- Add the payoff of every component in each terminal price region.
- If two portfolios produce the same cash flow in every state, they are economically equivalent at expiration.
- In an arbitrage-free market, equivalent future cash flows must have the same value today after timing and contract terms are aligned.



Arbitrage = a self-financing trading opportunity with no loss in any state and a positive gain in at least one state.

Put–call parity links protection to a call plus safe cash

$$S_0 + p_0 = c_0 + PV(K)$$

- Left portfolio: one share plus one European put with strike K.
- Right portfolio: one European call with the same strike and expiry plus a safe asset that pays K at T.
- Both portfolios deliver $\max(S_T, K)$ at expiration.

$PV(K)$ = present value today of a default-free payment of K at expiration. The identity assumes matched European options on a non-dividend-paying underlying and aligned contract units.

Appendix A9 · Parity proof

Two terminal regions verify the parity payoff

Region	Share	Put	Call	Safe payoff
$S_T < K$	S_T	$K - S_T$	0	K
$S_T \geq K$	S_T	0	$S_T - K$	K
Left total	$\max(S_T, K)$		Right total $= \max(S_T, K)$	

> If the two matched portfolios had different prices today, buy the cheaper and sell the dearer; their equal terminal cash flows would cancel.

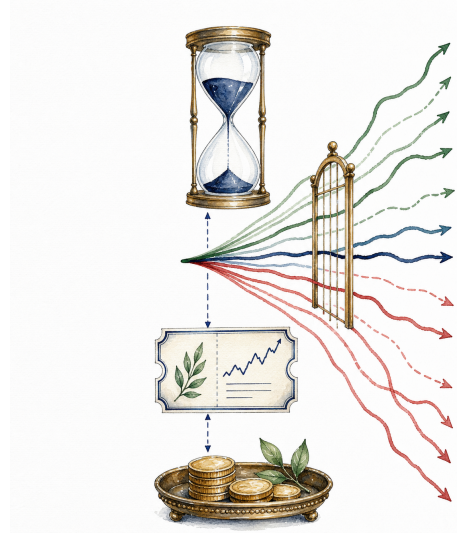
The argument is a no-arbitrage identity, not a model for the separate fair value of the call or put.

Appendix A10 · Arbitrage Python P6 · Verify parity

Judge value, use, and risk

What makes the right valuable, and when can it fail the objective?

Finish with disciplined qualitative reasoning, not a pricing formula.



Premium contains exercise value and remaining choice value

$$\text{option premium} = \text{intrinsic value} + \text{time value}.$$

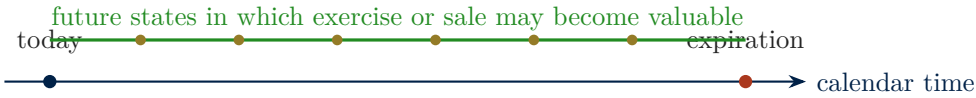
$S_0 = 100,\ K = 95$: immediate exercise value is $100 - 95 = 5$

- Call intrinsic value is $\max(S_0 - K, 0)$; put intrinsic value is $\max(K - S_0, 0)$.
- An out-of-the-money option can have zero intrinsic value and a positive premium because future prices remain uncertain.

S_0 = current underlying price. Time value is a residual definition, not "value caused only by calendar time."

Appendix A11 · Intrinsic value

More remaining time preserves more possible exercise opportunities

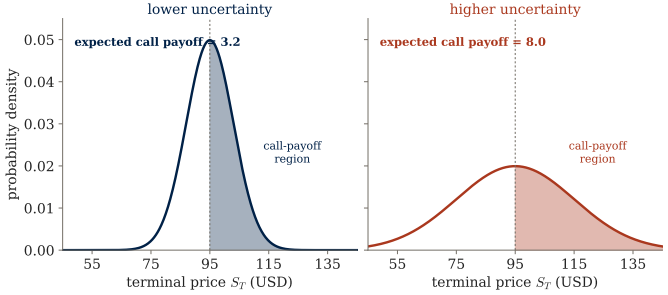


- With otherwise identical American-style options, a longer right contains all exercise dates available to the shorter right and additional dates.
- For European options, time effects interact with rates, dividends, and the distribution of future prices; the intuition needs qualification.

Holding all else fixed is essential: comparing two observed options with different strikes or market conditions does not isolate time.

Appendix A13 · Qualifications

More uncertainty can benefit both call and put holders



- Both distributions have mean USD95; only dispersion changes. The call uses the right tail and discards the left tail at zero payoff.
- Expected payoff rises from USD3.2 to USD8.0 in this normal-distribution illustration.

Convex = gains accelerate in favourable terminal regions while the holder's payoff is bounded below at zero. This is qualitative reasoning, not a numerical valuation.

Qualitative value drivers must specify call, put, and assumptions

Increase in	Call value	Put value	Intuition / qualification
Underlying price S_0	↑	↓	buy right improves; sell right weakens
Strike K	↓	↑	lower buy price / higher sell price
Uncertainty	↑	↑	favourable tail retained, zero exercise available
Time remaining	usually ↑	usually ↑	more possibilities; dividends and rates qualify European cases
Interest rate	usually ↑	usually ↓	changes present value of strike payment
Expected dividends	usually ↓	usually ↑	expected price drop around distribution date

Arrows are ceteris-paribus effects for otherwise matched vanilla options. “Usually” flags cases requiring contract-style and dividend qualifications.

Appendix A13 · Driver qualifications

Intrinsic value, moneyness, and profit answer different questions

Question	Quantity	Example: call $K = 95$, premium 8, $S_T = 100$
Is exercise valuable?	intrinsic payoff $\max(S_T - K, 0)$	yes; payoff is 5
Is it in the money?	compare S_T and K	yes; $100 > 95$
Did the trade profit?	payoff minus premium	no; profit is $5 - 8 = -3$

Exercise can be economically correct at expiration even when the original option trade has an overall loss.

Appendix A11 · Distinctions

Before expiration, selling can preserve value that exercise discards

- Exercising converts the option into its contractual underlying transaction.
- Selling the option transfers both its intrinsic value and any remaining time value to another buyer.
- Early exercise can therefore destroy time value, although dividends, rates, transaction costs, liquidity, and operational constraints can alter the decision.
- Distinguish “in the money” from “exercise immediately.”

> Decision sequence: estimate exercise value, observe sale value, include cash-flow consequences, then compare feasible alternatives.

This is decision logic, not a universal rule that early exercise never occurs.

Appendix A12 · Exercise qualifications

The writer must manage assignment, collateral, liquidity, and tail loss

- **Assignment risk**: an American-style writer can be required to perform before expiration.
- **Collateral risk**: adverse price moves can require additional margin or liquidation.
- **Liquidity risk**: closing the option may be costly when spreads widen or trading is thin.
- **Tail risk**: uncovered call loss can grow without a finite terminal bound; short-put loss can be large if the underlying collapses.
- **Operational risk**: contract multiplier, adjusted deliverable, expiry, and exercise instructions can produce unintended positions.

OCC, Characteristics and Risks of Standardized Options; OIC assignment guidance.

Tail risk = exposure to severe outcomes in relatively extreme parts of the price distribution.

Final checklist: design the boundary before choosing the option

1. State the underlying exposure and the price move that causes harm.
2. Choose the desired boundary: ceiling, floor, cap, or two-sided event exposure.
3. Read type, strike, expiry, style, settlement, quote, and multiplier.
4. Calculate payoff, premium-inclusive profit, break-even, and contract cash scale.
5. Add the option to the existing exposure state by state.
6. Check residual risk: premium, basis, timing, assignment, collateral, liquidity, and operational details.

> Options do not remove uncertainty. They purchase or sell a rule for reallocating cash flows after uncertainty is resolved.

References and source notes

- Kansoy, F., SJTU Derivatives Lecture 1, 2026: obligation-to-choice transition, premium-before-formula sequence, call/put payoff construction, and airline policy comparison.
- Brealey, Myers, Allen, and Edmans, *Principles of Corporate Finance*, Chapter 20, "Understanding Options": terminology, payoff/profit diagrams, protective put, covered call, combinations, and payoff equivalence.
- Berk and DeMarzo, *Corporate Finance*, Chapter 20, "Financial Options": contract mechanics, rights and obligations, option combinations, portfolio insurance, put–call parity, and qualitative value drivers.
- Options Clearing Corporation, *Characteristics and Risks of Standardized Options*, June 2024; theocc.com.
- OCC, "Equity Options Product Specifications"; OIC, "Exercising Options," "Options Assignment," "Basics," and "Equity vs. Index Options"; optionseducation.org.

All ACME quotations, option premiums, and terminal price states are disclosed illustrative inputs unless an official source is named.

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Technical appendix map

A1 Notation

A2 Call audit

A3 Put audit

A4 Break-even

A5 Contract scale

A6 Protective put

A7 Covered call

A8 Straddle

A9 Parity proof

A10 Parity arbitrage

A11 Value distinctions

A12 Exercise and settlement

A13 Value drivers

A14 Provenance

> The appendix holds derivations, qualifications, and audit trails that support the main narrative without interrupting it.

Python practical map

A1. Notation and exact expiration payoffs

Symbol	Meaning	Expiration payoff
S_T	underlying price at T	observed market outcome
K	strike price	fixed in the contract
C_T	long-call payoff	$\max(S_T - K, 0)$
P_T	long-put payoff	$\max(K - S_T, 0)$
c_0, p_0	premiums at inception	excluded from payoff; included in profit

$$\max(x, 0) = \begin{cases} 0, & x \leq 0, \\ x, & x > 0. \end{cases}$$

Subscripts 0 and T distinguish inception from expiration. The payoff convention is per underlying unit unless a multiplier is shown.

← **Return to call decision**

Appendix map

A2. Long-call and short-call state audit

S_T	K	Long call	Short call	Sum
55	75	0	0	0
75	75	0	0	0
95	75	+20	−20	0
115	75	+40	−40	0

$$\Pi_T^{LC} + \Pi_T^{SC} = \max(S_T - K, 0) - \max(S_T - K, 0) = 0.$$

LC = long call; SC = short call. Payoffs are USD per underlying unit and exclude premium.

← **Return to call table**

Appendix map

A3. Long-put and short-put state audit

S_T	K	Long put	Short put	Sum
70	95	+25	−25	0
95	95	0	0	0
100	95	0	0	0
130	95	0	0	0

$$\Pi_T^{LP} + \Pi_T^{SP} = \max(K - S_T, 0) - \max(K - S_T, 0) = 0.$$

LP = long put; SP = short put. For a non-negative share price, the short put's worst expiration payoff is $-K$ per share.

← **Return to put table**

Appendix map

A4. Profit and break-even for all four positions

Position	Profit at expiration	Break-even
Long call	$\max(S_T - K, 0) - c_0$	$K + c_0$
Short call	$c_0 - \max(S_T - K, 0)$	$K + c_0$
Long put	$\max(K - S_T, 0) - p_0$	$K - p_0$
Short put	$p_0 - \max(K - S_T, 0)$	$K - p_0$

- Buyer and writer share the same break-even because their profits are negatives of each other when the same premium is used.
- Break-even formulas assume hold-to-expiration profit and ignore financing and fees.

← **Return to call break-even**

Appendix map

A5. Premium quote, multiplier, and exercise cash

Cash calculation	One contract	Ten contracts
Premium at USD3 quote	$3 \times 100 = 300$	$3 \times 100 \times 10 = 3{,}000$
Call strike cash at USD95	$95 \times 100 = 9{,}500$	$95 \times 100 \times 10 = 95{,}000$
USD1 payoff per share	$1 \times 100 = 100$	$1 \times 100 \times 10 = 1{,}000$

- The premium is the price of the right; strike cash is the price of the underlying on exercise.
- Adjusted contracts can have non-standard deliverables after corporate actions.

OCC, Equity Options Product Specifications.

← **Return to multiplier**

Appendix map

A6. Protective-put algebra and maximum loss

$$
\begin{aligned}
V_T^{\mathrm{PP}} &= S_T + \max(K - S_T, 0) = \max(S_T, K), \\
\pi_T^{\mathrm{PP}} &= \max(S_T, K) - S_0 - p_0, \\
\pi_{\min}^{\mathrm{PP}} &= K - S_0 - p_0.
\end{aligned}
$$

At $S_0 = 100$, $K = 95$, and $p_0 = 3$, minimum profit is $95 - 100 - 3 = -8$ per share.

PP = protective put. The floor in terminal value is K; the floor in profit is lower because the share and put were purchased for $S_0 + p_0$.

← **Return to insured profit**

Appendix map

A7. Covered-call algebra and opportunity cost

$$V_T^{\mathrm{CC}} = S_T - \max(S_T - K, 0) = \min(S_T, K),$$
$$\pi_T^{\mathrm{CC}} = \min(S_T, K) - S_0 + c_0.$$

- For $S_T \leq K$, the call expires and the premium cushions the stock result.
- For $S_T > K$, portfolio value is capped at K before adding inception premium.
- Relative to uncalled stock, forgone terminal upside is $S_T - K$ above the strike.

CC = covered call. "Covered" means the share is available for delivery; it does not bound the share's downside.

← **Return to covered call**

Appendix map

A8. Long-straddle profit has two break-even prices

$$\pi_T^{\text{straddle}} = |S_T - K| - (c_0 + p_0),$$
$$S_T^{\text{BE,lower}} = K - (c_0 + p_0),$$
$$S_T^{\text{BE,upper}} = K + (c_0 + p_0).$$

- At the strike, both options have zero expiration payoff and the total premium is lost.
- Profit requires a move large enough to recover both premiums.
- The position expresses magnitude uncertainty, not a directional forecast by itself.

← **Return to straddle**

Appendix map

A9. Put–call parity proof by terminal regions

Region	S_T	Put	Call	Bond payoff
$S_T < K$	S_T	$K - S_T$	0	K
$S_T \geq K$	S_T	0	$S_T - K$	K

$$S_T + \max(K - S_T, 0) = \max(S_T - K, 0) + K = \max(S_T, K).$$

$$\therefore \quad S_0 + p_0 = c_0 + PV(K).$$

Matched European call and put, same underlying, strike, expiry, and multiplier; no dividends; frictionless no-arbitrage setting.

← **Return to parity**

Appendix map

A10. A parity gap implies a long-cheap, short-dear trade

$$\Delta = (S_0 + p_0) - (c_0 + PV(K)).$$

- If $\Delta > 0$, share-plus-put is dearer: sell it and buy call-plus-bond.
- If $\Delta < 0$, share-plus-put is cheaper: buy it and sell call-plus-bond.
- The terminal payoff difference is zero in every state, so the initial price gap is locked in under the assumptions.
- In practice, bid–ask spreads, short-sale constraints, dividends, exercise style, funding, and fees define a no-arbitrage band rather than exact screen equality.

Short-dear means sell the overpriced portfolio components; implementation requires the ability to borrow securities and trade all legs.

← **Return to parity table**

Appendix map

A11. Moneyness, intrinsic value, time value, and profit

Concept	Call calculation	What it answers
Moneyness	compare S with K	is exercise valuable?
Intrinsic value	$\max(S-K,0)$	how much exercise value exists now?
Time value	premium − intrinsic	how much premium exceeds exercise value?
Expiration profit	payoff − initial premium	did the position break even?

Example: $S_T = 100$, $K = 95$, $c_0 = 8$ gives intrinsic payoff 5, in-the-money status, and expiration profit -3.

← **Return to distinctions**

Appendix map

A12. Exercise-style and settlement qualifications

Feature	Basic rule	Qualification
American style	exercise through expiration	broker cut-offs and contract rules apply
European style	exercise on expiration date	“European” does not identify trading venue
Equity options	generally physical settlement	adjusted deliverables can follow corporate actions
Index options	often cash settlement	style and settlement vary by product
Assignment	writer must perform	allocation follows clearing and broker procedures

OCC/OIC product specifications, exercise, and assignment guidance.

Always verify the product specification and brokerage procedure; the table states common conventions rather than universal rules.

← **Return to exercise style**

Appendix map

A13. Qualitative value drivers and necessary qualifications

- **Underlying price:** call up, put down, holding other inputs fixed.
- **Strike:** call down, put up, because the contractual buy/sell term changes.
- **Uncertainty:** vanilla call and put values rise under standard no-arbitrage reasoning because their payoffs are convex.
- **Time:** a longer American option cannot be worth less than an otherwise identical shorter one; European comparisons can depend on rates and dividends.
- **Interest rates:** higher rates reduce the present cost of a future strike payment, usually helping calls and hurting puts.
- **Dividends:** expected distributions usually reduce the future ex-dividend stock price, hurting calls and helping puts; early-exercise incentives may change.

These are directional comparative statics, not a numerical pricing procedure. “All else fixed” includes the probability model and contract terms.

← **Return to driver table**

Appendix map

A14. Source and case-input provenance

Item	Status	Provenance
Airline 55 / 75 / 95 states	Illustrative input	continuity with Day 5 and SJTU exposure-first case
Call premium 4	Illustrative input	chosen to separate strike from break-even
Share 100, put 95 / 3	Illustrative input	protective-put arithmetic
ACME option quote	Fictional input	contract-reading exercise, not market data
100-share multiplier	Current convention	OCC equity-option specification
Exercise and assignment	Current convention	OCC/OIC official guidance
Payoff combinations	Textbook foundation	Brealey Ch. 20; Berk–DeMarzo Ch. 20

Machine-readable case inputs are stored in `data/cases/day06_option_states.csv` and the notebook metadata.

← **Return to sources**

Appendix map

Python practical map

P1 Call and put payoffs

P2 Payoff and profit

P3 Contract multiplier

P4 Airline policy costs

P5 Protective and covered

P6 Verify parity

- Code reproduces financial logic already derived in the lecture.
- The two notebooks use fixed case inputs and white-background figures.
- Each calculation carries units and a numerical check.

Technical appendix map

P1. Vectorise call and put payoffs

```python
import numpy as np

# Terminal share prices to evaluate
stock_prices = np.array([70, 95, 100, 130])
strike = 95

# maximum(..., 0) applies the holder's exercise choice
call_payoff = np.maximum(stock_prices - strike, 0)
put_payoff = np.maximum(strike - stock_prices, 0)

print(call_payoff)
print(put_payoff)

```

Expected call payoffs: 0, 0, 5, 35. Expected put payoffs: 25, 0, 0, 0.

← **Return to call table**

Practical map

P2. Keep payoff and profit in separate variables

```python
import numpy as np

stock_prices = np.arange(70, 131)
strike = 95
call_premium = 3

call_payoff = np.maximum(stock_prices - strike, 0)
call_profit = call_payoff - call_premium
break_even = strike + call_premium

print("break-even:", break_even)
print("profit at 120:", call_profit[stock_prices == 120][0])

```

Expected break-even: 98. Expected per-share profit at 120: 22.

← **Return to profit plot**

Practical map

P3. Scale per-share values to contracts

```python
contracts = 10
multiplier = 100
premium_quote = 3.00
strike = 95

# Premium is paid for every share represented by every contract
premium_cash = contracts * multiplier * premium_quote
exercise_cash = contracts * multiplier * strike

print("premium cash:", premium_cash)
print("strike cash if exercised:", exercise_cash)

```

Expected cash: USD3,000 premium and USD95,000 strike payment if all ten calls are exercised physically.

← **Return to multiplier**

Practical map

P4. Compare airline policy costs

```python
import numpy as np
import pandas as pd

fuel_prices = np.array([55, 75, 95])
strike = 75
call_premium = 4

policies = pd.DataFrame({"fuel_price": fuel_prices})
policies["unhedged"] = fuel_prices
policies["forward"] = strike
policies["call_protected"] = np.minimum(fuel_prices, strike) + call_premium

print(policies)

```

Expected call-protected costs: 59, 79, and 79 per fuel unit.

← **Return to airline comparison**

Practical map

P5. Add component payoffs into portfolios

```python
import numpy as np

stock_prices = np.array([70, 95, 100, 130])
put_strike = 95
call_strike = 110

put_payoff = np.maximum(put_strike - stock_prices, 0)
call_payoff = np.maximum(stock_prices - call_strike, 0)

protective_value = stock_prices + put_payoff
covered_value = stock_prices - call_payoff

print("protective:", protective_value)
print("covered:", covered_value)

```

Expected protective values: 95, 95, 100, 130. Expected covered values: 70, 95, 100, 110. Premiums enter profit in a separate step.

← **Return to portfolio comparison**

Practical map

P6. Verify put–call parity over a price grid

```python
import numpy as np

stock_prices = np.arange(50, 141)
strike = 95

put_payoff = np.maximum(strike - stock_prices, 0)
call_payoff = np.maximum(stock_prices - strike, 0)

left_portfolio = stock_prices + put_payoff
right_portfolio = call_payoff + strike

print("largest payoff gap:", np.max(np.abs(left_portfolio - right_portfolio)))

```

Expected largest gap: 0. Equality of terminal payoffs supports the present-value parity relation under matched assumptions.

← **Return to parity table**

Practical map