Finance and AI

When does an asset loss become a credit contraction?

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Day 1

Day 1 outline

1) Losses and the equity buffer
2) Why intermediary net worth matters
3) The incentive constraint and debt limit
4) Equilibrium amplification
5) Capital repair and policy

Bear Stearns poses a capital–liquidity puzzle

Bear Stearns was a major US investment bank and primary dealer that depended heavily on short-term secured wholesale funding.

Fiscal year-end 2007

$$
\begin{aligned}
A &= \$395.4\text{bn},\\
L &= \$383.6\text{bn},\\
N = A - L &= \$11.8\text{bn}.
\end{aligned}
$$

The reported balance sheet was highly leveraged.

Week of 10 March 2008

The holding-company liquidity pool fell from **$18.1bn** to **about $2bn** as counterparties withdrew customary secured funding. Bear reported capital above the standards then applied to it.

How can a firm report positive capital and still lose the ability to finance itself within days?

Bear Stearns 2007 Form 10-K; SEC liquidity chronology, 20 March 2008; Federal Reserve testimony, 3 April 2008.

A two percent asset loss is only the first step

The lecture asks what else must be true before firms lose access to credit.

1. **Loss absorption:** leverage must make the asset mark large relative to equity.
2. **Credibility:** lower equity must tighten credible outside funding.
3. **Equilibrium:** prices or alternative funders must not replace the lost capacity.
4. **Transmission:** loans must absorb the adjustment and borrowers must lack substitutes.

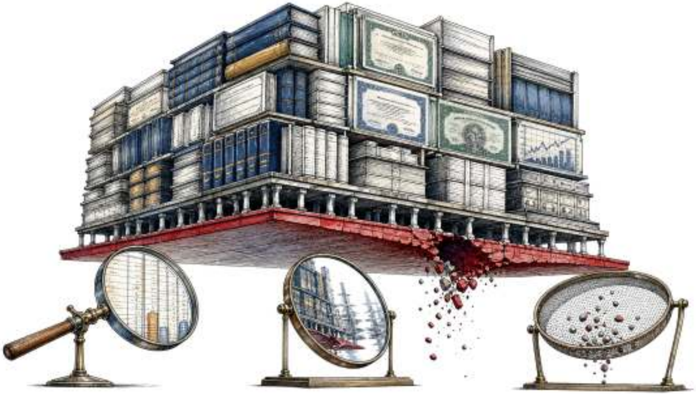
Each arrow needs different evidence. No capital ratio, funding statistic, or lending correlation establishes the entire chain.

Part I

Losses, leverage, and the equity buffer

How small can an asset loss be and still destroy a bank?

One bank, one loss, followed from the bond market to the capital ratio.



Bear Stearns reported $395bn of assets and $384bn of liabilities

The balance sheet contains one identity:

$$\underbrace{A}_{\text{assets}} = \underbrace{D}_{\text{promised claims}} + \underbrace{N}_{\text{equity}}.$$

For Bear Stearns at fiscal year-end 2007,

$$395.4 = 383.6 + 11.8 \qquad (\$\text{bn}).$$

The promised claims do not absorb the first loss. Equity is the residual that does.

Notation used throughout Day 1: A, economic value of assets; D, deposits and other promised claims; $N = A - D$, equity or net worth.

Bear Stearns 2007 Form 10-K, fiscal year ended 30 November 2007. Figures in $bn; values rounded to one decimal place.

This record supplies the scale for the loss arithmetic. It does not yet explain why Bear failed in March 2008.

Capital absorbs losses; cash settles payments

Two balance-sheet objects answer different questions:

Capital or equity N is the residual claim. It absorbs losses because promised creditors are paid first.

Cash and liquid assets sit inside A. They meet payments because they can be transferred or sold quickly.

A bank can therefore be well capitalised but short of cash, or liquid today but economically insolvent.

Capital asks who ultimately bears a loss. Liquidity asks whether a payment can be made now.

Bank of England (2013), *Bank capital and liquidity*.

$11.8bn is either a fortress or a rounding error

The number alone decides nothing. It has to be measured against the balance sheet it is protecting.

$$\kappa \equiv \frac{N}{A} = 2.98\% \qquad \ell \equiv \frac{A}{N} = 33.53$$

How far to the edge?

Just under three cents of cushion for every dollar of assets.

How fast do we get there?

Every dollar of equity is carrying $33.53 of assets.

One fact, two questions. $\kappa = 1/\ell$.

The claims do not fall when the assets do

Scale the opening balance sheet to $A = 100$. Then

$$\begin{aligned} \text{before the loss:} \qquad & A = 100, \quad D = 97.02, \quad N = 2.98, \\ \text{after a 2\% asset loss:} \qquad & A' = 98, \quad D' = 97.02, \quad N' = 0.98. \end{aligned}$$

The claims are fixed at the instant of marking, so

$$N' = A' - D = N - xA.$$

The two-unit fall in assets becomes a two-unit fall in equity. The loss is not shared proportionally across the balance sheet.

Here $x = 0.02$ is the proportional asset-value loss. This is a value shock, not a creditor withdrawal: D is therefore held fixed.

A one percent loss costs a third of the equity

$$\frac{-\Delta N}{N} = x\,\frac{A}{N} = x\,\ell$$

asset loss x	$x\ell$	equity left
1%	33.5%	two thirds
2%	**67.1%**	one third
2.98%	100%	**nothing**

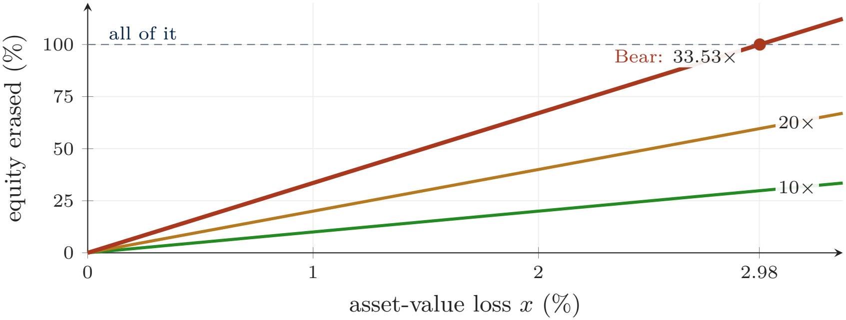
Equity is exhausted when $x\ell = 1$, that is when $x = 1/\ell = \kappa$.

Under fixed promised claims, κ **is the asset-value mark that exhausts book equity**. Failure can occur sooner through liquidity or later through support, recapitalisation, or forbearance.

Appendix A1

For a given asset shock, leverage determines the amplification slope

optional extension



The asset shock is held fixed on all three lines. Higher balance-sheet leverage makes the equity response steeper.

For intuition only, Bear's book-equity/assets ratio was 2.98%. Modern Basel III instead uses Tier 1 capital over a prescribed exposure measure and sets a 3% minimum; the two ratios are not directly comparable.

Basel Committee, leverage-ratio definition and minimum.

Appendix A1

Maturity alone is not a sufficient measure of rate risk

Now leave Bear Stearns and consider a stylised fixed-rate bond. You lend at 1% for ten years; market rates then rise and new bonds pay 4%. Your claim pays in full and on time, yet its market value falls.

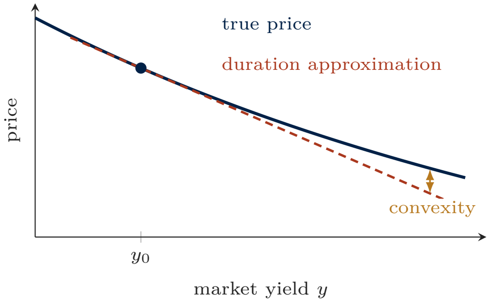
- The loss depends on **how long your money is out**.
- Final maturity misses the timing of interim coupons: a bond paying large coupons returns much of its present value early.
- What is needed is the *average* time you wait, each payment weighted by what it is worth today.

That average has a name.

Macaulay duration measures timing; modified duration measures sensitivity

$$\frac{\Delta P}{P} \approx -D_{\text{mod}}\,\Delta y$$

- D_{Mac} is the present-value-weighted payment time.
- $D_{\text{mod}} = D_{\text{Mac}}/(1+y)$ is the local price semi-elasticity.
- Here $D_{\text{Mac}} = 5$, $y = 3\%$, so $D_{\text{mod}} = 4.85$.



For this five-year zero-coupon claim, a 40bp rise gives $\Delta P/P \approx -4.85(.004) = -1.94\%$. The chart and approximation now refer to the same instrument. This is a worked bond calculation, not a historical estimate for Bear Stearns or a mark on a bank's entire asset book.

Assumptions: annual compounding, a small parallel yield shift, and no change in promised cash flows; 40 basis points equals .004.

Appendix A2

An asset-side rate mark depends on exposure, duration, and hedging

optional extension

Let s be the share of bank assets exposed to the same yield move. To first order,

$$\frac{\Delta A}{A} \approx -sD_{\text{mod}}\Delta y.$$

For a stylised bank with $s = .30$, $D_{\text{mod}} = 5$, and $\Delta y = .004$,

$$\frac{\Delta A}{A} \approx -.30(5)(.004) = -0.6\%.$$

At $20\times$ leverage, that mark reduces equity by approximately

$$0.6\% \times 20 = 12\%.$$

This is an asset-side mark. For the whole balance sheet, $\Delta N = \Delta A - \Delta D$: liability duration and hedge payoffs can offset or reinforce the asset loss. A bank loss cannot be inferred from asset duration alone.

Appendix A2b

The loss is real before anyone reports it

1. **Economic value.** Under a stated valuation rule, rates or expected cash flows change the asset value immediately even if recognition is delayed.

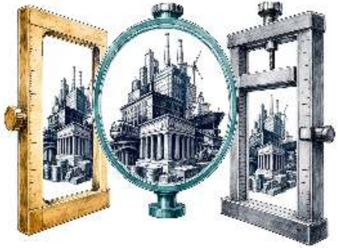
2. **Accounting recognition.** Fair value, impairment, provisions decide when it enters the books.

3. **Regulatory mapping.** Eligible-capital definitions decide whether the ratio moves at all.

Hold to maturity can defer recognition under specified accounting rules; it does not remove the economic-value loss, the opportunity cost of a below-market yield, or the risk of a forced sale.

Three different numbers are called capital

- **Book equity**: the accounting residual.
- **Market equity**: what the shares actually trade for.
- **Regulatory capital**: eligible instruments over risk-weighted assets or exposure.



They diverge when accounting recognition, market prices, and regulatory eligibility respond differently. The gap between them can itself be informative.

$\kappa = N/A$ is used here. It is not CET1/RWA or market-to-book.

Restoring an 8% target requires shrinkage, new equity, or bail-in

After the 2% mark, $A = 98$ and $N = 0.98$. Suppose an illustrative market target is $\bar{\kappa} = 8\%$; this is not the Basel leverage minimum.

shrink assets	**issue equity**	**bail in claims**
$\dfrac{0.98}{98 - s} = .08$	$\dfrac{0.98 + e}{98 + e} = .08$	$\dfrac{0.98 + b}{98} = .08$
$s = 85.75$	$e = 7.46$	$b = 6.86$
sell 87% of the balance sheet	raise eight times the equity left	break a promise

At 33 times leverage every private repair route is severe. That arithmetic helps explain public intervention in 2008, alongside funding runs, uncertainty, coordination failures, and systemic spillovers.

Appendix A3

Bear's capital ratios did not prevent a wholesale funding run

At the start of the week, Bear reported capital above prevailing regulatory standards. Its holding-company liquidity pool nevertheless changed as follows:

11 March, open
$18.1bn

11 March, close
$11.5bn

12 March
$12.4bn

14 March, close
about $2bn

Counterparties became unwilling to provide secured funding on customary terms, even against high-quality collateral. The confidence shock became a cash-timing constraint within days.

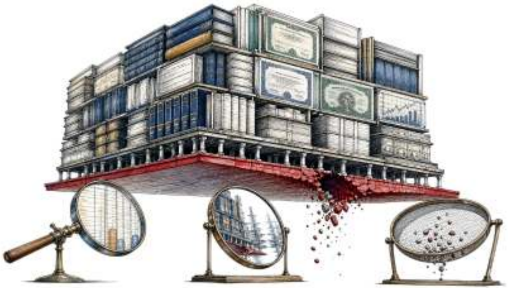
The 2007 balance sheet establishes leverage. The March chronology establishes a funding collapse. Neither record by itself proves that lower equity caused a contraction in bank lending.

The duration calculation is a teaching counterfactual, not an estimate of Bear's portfolio loss.
Bear Stearns 2007 Form 10-K; SEC liquidity chronology, 20 March 2008.

A capital problem does not stay inside the bank

- Selling 87% of total assets is a balance-sheet contraction. It becomes a credit contraction only if loans, rather than cash or securities, absorb the adjustment.

- Issuing equity needs an investor willing to hand over cash.



Accounting has told us who absorbs the loss. It cannot tell us why an outsider would fund a bank at all.

Why should an outsider continue to finance a highly leveraged intermediary after its own stake has fallen? Section II introduces the credibility problem.

We followed the value channel, $\Delta A = \Delta N$. Absent a loss on asset sales, a creditor leaving instead gives $\Delta A = \Delta D$ and leaves N untouched: that is liquidity, and it is Day 2.

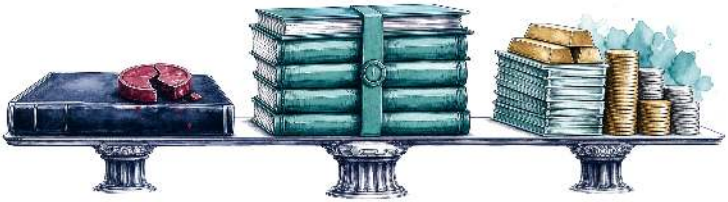
Part II

Why intermediary net worth matters

Why does intermediary net worth affect credible outside funding?

The institution that monitors borrowers must itself remain credible to funders.

Accounting does not determine lending



- New equity can replace the lost net worth.
- Securities or cash can adjust while the loan book is preserved.
- If funding remains available at unchanged terms, lower N need not change scale.

A balance-sheet loss becomes a lending contraction only after an economic constraint and an adjustment margin are specified.

The frictionless finance benchmark

$$\begin{tikzcd} \text{savers} \arrow[r, shift left, "\text{funding at the market price of risk}"] & \text{firms} \arrow[l, shift left, "\text{state-contingent repayment}"] \end{tikzcd}$$

No intermediary is required to create information or enforce the contract.

Direct finance is sufficient when project quality is observable, actions and outcomes are contractible, verification is costless, and promises are enforceable.

A financial contract must solve four frictions before scale is possible

Information Who can distinguish a productive borrower from a weak one?

Incentives Which action will be chosen after funding is committed?

Verification Can outsiders observe cash flows, collateral, and losses?

Enforcement Which repayment promises can actually be imposed?

Intermediation creates value when it solves these problems more cheaply than direct finance. It also creates a new contract between the intermediary and its own funders.

The model below isolates the incentive problem in that second contract.

Outside funding finances assets; it does not become net worth

Starting from

$$A = N + D,$$

N: intermediary equity/net worth; D: deposits plus other outside claims; A: assets financed.

From this point onward, D is the model's outside promised funding; it is not being set equal to every liability reported on Bear's 2007 balance sheet.

one additional unit of outside funding gives

$$\Delta A = 1, \qquad \Delta D = 1, \qquad \Delta N = 0.$$

Borrowing expands both sides of the balance sheet. It does not rebuild the first-loss stake unless an outside claim is converted into equity.

Agency separates the decision-maker from the risk-bearer

An **agency problem** arises when one party chooses an action while another bears part of its consequences. The **principal** supplies resources and delegates an action; the **agent** takes that action with information or incentives the principal cannot fully contract on.

Borrower–bank borrowers know more about project quality and effort; the bank screens and monitors.

Bank–saver the bank controls monitoring and risk after savers supply funding; savers need credible repayment.

Day 1 formalises the second conflict: who monitors the intermediary?

Delegated monitoring economises on information

optional extension

- The intermediary screens and monitors many borrowers.
- Outside funders hold simpler claims on the intermediary.
- One information producer can replace duplicated bilateral monitoring.

$$\begin{aligned} \text{Bilateral monitoring:} &\quad 100 \times 50 = 5{,}000 \text{ links,} \\ \text{Delegated monitoring:} &\quad 100 + 50 = 150 \text{ links.} \end{aligned}$$

Illustrative relationship count: 100 savers and 50 borrowers. It is not a monitoring-cost theorem.

Diamond (1984), delegated monitoring.

Net worth makes intermediation credible

The intermediary solves one information problem and creates another:

Who monitors the monitor?

- Own net worth absorbs the first loss.
- A larger equity stake raises the payoff from honouring outside claims relative to diversion.
- Outside funders can therefore credibly finance a larger balance sheet.

How much outside funding can a given level of N support?

Gertler and Kiyotaki (2010); Gertler and Karadi (2011).

Low net worth does not force the loan book to contract

After a loss, three adjustments can preserve existing loans:

1. New common equity supplies cash and rebuilds the loss-absorbing stake.
2. Cash or securities can fall while the stock of loans remains unchanged.
3. A borrower can replace this bank with another bank, a bond, or internal funds.

The accounting identity permits all three. A prediction for lending requires a behavioural constraint and a statement about the adjustment margin.

Prediction: if desired assets are below the funding ceiling, a fall in N changes capacity but not the chosen balance sheet.

Net worth matters only through a binding credibility problem

Lower intermediary net worth reduces realised finance only when:

1. outside funders cannot contract on every action and payoff;
2. the intermediary needs outside claims to reach desired scale; and
3. the resulting credibility ceiling binds before another adjustment absorbs the loss.

$$\text{skin in the game} \quad \Longrightarrow \quad \text{credible promise} \quad \Longrightarrow \quad \text{funding capacity.}$$

The next section turns this intuition into an inequality.

How does a credibility problem become a numerical debt limit? Section III compares the banker's honest and diversion payoffs.

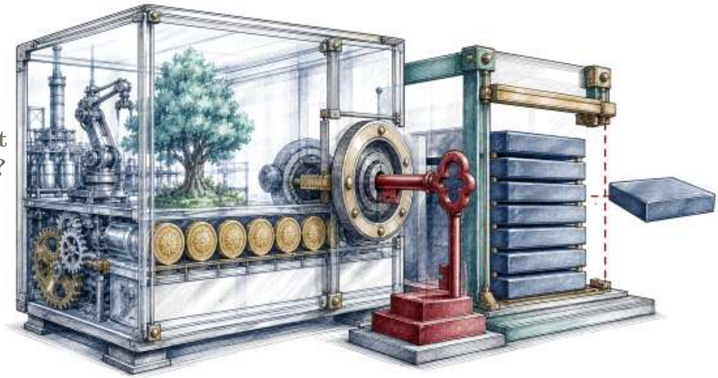
Four-link map: credibility is now the candidate second link; the model must still derive it and show when it binds.

Part III

The incentive constraint and the debt limit

How does an incentive constraint turn net worth into a debt limit?

First hold promised returns fixed; market clearing comes only after capacity is understood.



The model links savers, an intermediary, and productive assets

Savers provide outside funding D and are promised the gross payment $R_D D$.

Intermediary contributes equity/net worth N and finances assets $A = N + D$.

Productive assets represent loans or claims on firms and pay the gross amount $R_A A$.

R_A: gross asset return; R_D: gross promised funding return. For example, $R_A = 1.10$ means a 10% net asset return. The outside claim D may represent deposits, repo, bonds, or other promised funding.

A one-period teaching model isolates credible funding

Determined inside credible outside funding D, asset scale $A = N + D$, and whether the incentive constraint binds.

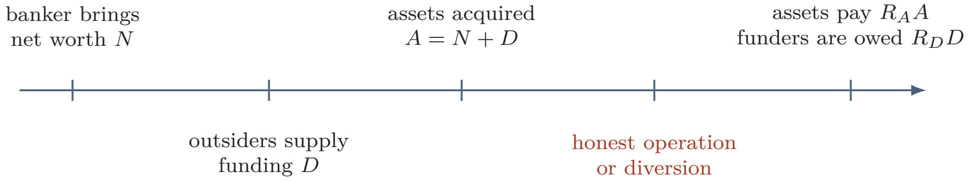
Held fixed first net worth N, asset return R_A, promised funding return R_D, and the diversion share λ.

Deliberately absent stochastic default, maturity, runs, regulation, market power, and endogenous asset prices.

This narrow contract is a feature: one mechanism is solved before other frictions are layered on. It is a teaching adaptation of the intermediary agency mechanism, not the full dynamic Gertler–Kiyotaki environment.

Gertler and Kiyotaki (2010); Gertler and Karadi (2011).

Diversion occurs after funding and before repayment



Funding arrives before the hidden action. Outsiders must therefore make repayment credible before they observe how the assets are operated.

The core model is deterministic and one-period. Market clearing is introduced only after the funding constraint is derived.

Stocks are chosen today; gross payments occur tomorrow

optional extension

$$\underbrace{N, D, A}_{\text{date-0 currency stocks}} \qquad \longrightarrow \qquad \underbrace{R_A A,\ R_D D}_{\text{date-1 currency payments}} .$$

- R_A, R_D, and λ are dimensionless; R_A and R_D are gross returns.
- $A = N + D$ is a date-0 financing identity; it is not a payoff equation.
- Diversion is a date-1 option, not a probability attached to date 0.

Keeping stocks and payments separate prevents the most common algebraic error in the incentive constraint.

The banker can divert a share of the asset payoff

After funding is committed, the banker can operate honestly or repudiate outside claims and retain

$$\lambda R_A A, \qquad 0 < \lambda < 1.$$

- λ is a reduced-form **diversion share**.
- It is not a default probability, fraud frequency, or loss given default.
- It is not directly observed; it summarises private benefits, weak monitoring, and limited pledgeability.
- Diversion is an off-equilibrium option that restricts credible promises.

With $R_A = 1.10$, $\lambda = .15$, and $A = 100$, diversion yields 16.5, leaving 93.5 as the payoff potentially pledgeable under honest operation.

Risk shifting, weak monitoring, and private benefits motivate agency more broadly; the model compresses them into one reduced-form parameter.

Diversion leaves only part of the payoff pledgeable

$$R_A A = \underbrace{(1-\lambda)R_A A}_{\text{pledgeable under honest operation}} + \underbrace{\lambda R_A A}_{\text{private diversion payoff}}.$$

Outside promises cannot exceed the pledgeable component while preserving the banker's incentive to operate honestly. If an asset produces 110 but 15 can be privately retained, outsiders cannot treat the full 110 as reliable backing for repayment.

Higher λ leaves less payoff that can credibly back outside funding.

Appendix A4

Honest and diversion payoffs

$$\Pi^{\text{hon}} = \underbrace{R_A(N+D)}_{\text{asset payoff}} - \underbrace{R_D D}_{\text{promised outside payment}},$$

$$\Pi^{\text{div}} = \underbrace{\lambda R_A(N+D)}_{\text{payoff privately retained after repudiation}}.$$

Π: the banker's end-of-period payoff under the indicated action.

Both choices operate the same asset base $A = N + D$. What differs is whether outside claims are honoured.

Appendix A4

A return spread creates a motive to expand the balance sheet

Using the honest payoff just defined,

$$\Pi^{\text{hon}} = R_A N + (R_A - R_D)D.$$

If $R_A > R_D$, an additional unit of outside funding raises honest payoff by

$$R_A - R_D > 0.$$

With constant returns and $R_A > R_D$, desired scale is unbounded unless some constraint intervenes. Credibility determines how much of that scale outsiders will actually finance.

Appendix A4

One more pound of outside funding consumes g units of incentive surplus

An extra unit of D changes both actions' payoffs:

$$\Delta\Pi^{\text{hon}} = R_A - R_D, \qquad \Delta\Pi^{\text{div}} = \lambda R_A.$$

Hence its effect on the surplus from remaining honest is

$$\begin{aligned}\Delta S &= (R_A - R_D) - \lambda R_A \\ &= -\underbrace{\big[R_D - (1-\lambda)R_A\big]}_{g}.\end{aligned}$$

When $g > 0$, each extra pound of outside finance uses scarce incentive surplus. Net worth supplies that surplus before any outsider is paid.

Appendix A4

Outside funding can erode the incentive to remain honest

Define the honest-operation surplus:

$$\begin{aligned} S(D) &\equiv \Pi^{\text{hon}} - \Pi^{\text{div}} \\ &= (1-\lambda)R_A N - \underbrace{\left[R_D - (1-\lambda)R_A\right]}_{g} D. \end{aligned}$$

The new object g is the **incentive funding gap**: the surplus used by one additional unit of outside finance. When $g > 0$,

$$\frac{\partial S}{\partial N} = (1-\lambda)R_A > 0, \qquad \frac{\partial S}{\partial D} = -g < 0.$$

Net worth supplies incentive surplus; outside funding uses it.

Appendix A4

Incentive compatibility limits funding

$$\boxed{R_A(N+D) - R_D D \;\geq\; \lambda R_A(N+D)} \tag{IC}$$

- If (IC) holds, honest intermediation gives the banker at least as much as diversion.
- If (IC) fails, that scale D cannot be financed on the stated terms (R_A, R_D).

The inequality restricts **credible scale**; it is not a probability of default.

Appendix A4

When funding capacity is finite

The sign of

$$g \equiv R_D - (1 - \lambda)R_A$$

determines whether this incentive constraint alone produces a finite limit.

$$\begin{aligned}
g > 0 \quad &\Longrightarrow \quad \text{finite outside-funding capacity,} \\
g = 0 \quad &\Longrightarrow \quad \text{no funding gap per extra unit of } D, \\
g < 0 \quad &\Longrightarrow \quad \text{(IC) alone supplies no finite cap.}
\end{aligned}$$

The debt-capacity formula is valid only in the first case.

Appendix A5

Debt and asset capacity

For $R_D > (1-\lambda)R_A$, holding R_A, R_D, and λ fixed:

$$D^{\max} = \frac{(1-\lambda)R_A}{R_D - (1-\lambda)R_A} N \equiv \phi N,$$

$$A^{\max} = N + D^{\max} = (1+\phi)N \equiv \psi N.$$

Here ϕ is the **outside-funding multiplier** and $\psi = 1 + \phi$ is the **total-asset capacity multiplier**. Both are defined only on the finite-capacity domain established on the preceding slide.

Net worth is valuable because it supports credible outside claims. The result is a **capacity proposition**, not a statement that the bank necessarily chooses $A^{\max}$.

Appendix A6

Capacity rises only when credible repayment improves

For fixed returns and $g > 0$,

$$D^{\max} = \phi N, \qquad \phi = \frac{(1-\lambda)R_A}{R_D - (1-\lambda)R_A}.$$

$R_A \uparrow$ raises the payoff available under honest operation, so more outside repayment becomes pledgeable.

$R_D \uparrow$ makes each promised pound more expensive and widens the funding gap g.

$\lambda \uparrow$ raises the private benefit of diversion and reduces the pledgeable share.

These are fixed-return capacity effects. Equilibrium responses are smaller or different when the funding price and desired scale move.

Appendix A20

Capacity is a ceiling, not the chosen balance sheet

Let A^{des} denote the scale the bank would choose without the financing constraint; it is a maintained demand object, not derived in this model. Then

$$A^{*} = \min\{A^{\text{des}},\ A^{\max}\}.$$

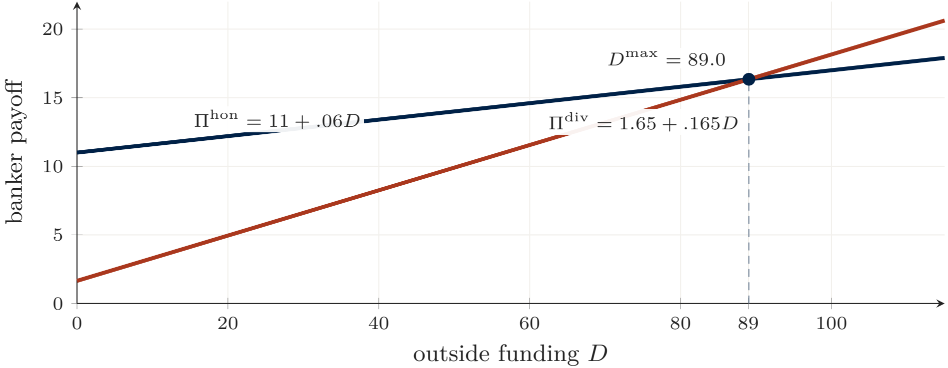
$$\begin{aligned} A^{\text{des}} < A^{\max} \quad &\Rightarrow \quad \text{constraint slack,} \\ A^{\text{des}} \geq A^{\max} \quad &\Rightarrow \quad \text{constraint binding.} \end{aligned}$$

A fall in net worth changes realised scale only after the capacity ceiling crosses desired scale.

Funding stops where honest and diversion payoffs cross

optional extension

Calibration: $R_A = 1.10$, $R_D = 1.04$, $\lambda = .15$, $N = 10$.



Left of the crossing, honesty dominates. At the crossing, the banker is indifferent and the incentive constraint binds.

Appendix A7

A fixed-return capacity calculation

optional extension

Let

$$R_A = 1.10, \qquad R_D = 1.04, \qquad \lambda = .15, \qquad N = 10.$$

$$(1-\lambda)R_A = .935, \qquad g = 1.04 - .935 = .105,$$

$$\phi = \frac{.935}{.105} = 8.90, \qquad D^{\max} = 89.0, \qquad A^{\max} = 99.0.$$

$$R_A \uparrow \Rightarrow \phi \uparrow, \qquad R_D \uparrow \Rightarrow \phi \downarrow, \qquad \lambda \uparrow \Rightarrow \phi \downarrow .$$

This is a **partial-equilibrium capacity calculation**: the returns and the agency parameter are held fixed.

Appendix A7

The model isolates credibility, not runs, prices, or default

Inside the model

- hidden action after funding;
- limited pledgeability;
- net-worth-dependent capacity;
- an incentive wedge when (IC) binds.

Requires additional structure

- stochastic default and recovery;
- funding runs and deposit insurance;
- regulation and market power;
- maturity and term structure;
- endogenous asset prices.

The section has answered its question: net worth becomes debt capacity because it raises honest-operation surplus relative to diversion. We have derived a ceiling at quoted returns, not an equilibrium quantity. For Bear Stearns, this is a **credibility lens**, not a causal account: the historical record also requires short-term funding, collateral, and a run.

At what quantity and return will outsiders actually fund the bank? Section IV adds funding supply and market clearing.

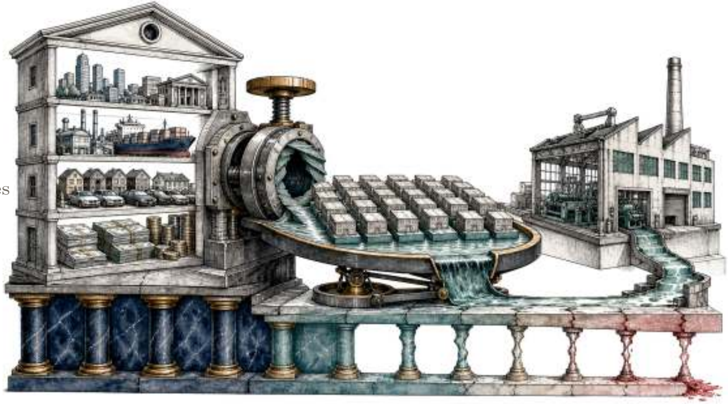
Four-link map: the credibility link is derived; equilibrium and transmission remain open.

Part IV

Equilibrium amplification

When does lower net worth reduce equilibrium lending and activity?

A debt limit matters only if market prices and real adjustment cannot offset it.



Equilibrium adds a funding price and an adjustment margin

The capacity result alone leaves three objects unresolved:

1. **Desired assets A^{des}:** productive opportunities, borrower demand, and risk appetite determine the unconstrained scale.
2. **Funding supply $R_D^s(D)$:** outsiders require a return for holding an increasing quantity of bank claims.
3. **Adjustment margin:** cash, securities, loans, prices, and borrower substitution determine how a smaller balance sheet reaches the economy.

Market clearing can determine D^*. It still cannot tell us, without more structure, which asset or borrower absorbs the change.

A funding-supply schedule closes the model

The fixed-return analysis gives a capacity boundary. Market equilibrium adds an inverse funding-supply schedule:

$$R_D = R_D^s(D), \qquad (R_D^s)'(D) > 0.$$

For the numerical example, use

$$R_D^s(D) = 1 + .00125D, \qquad R_A = 1.10, \qquad \lambda = .15.$$

Equilibrium outside funding satisfies

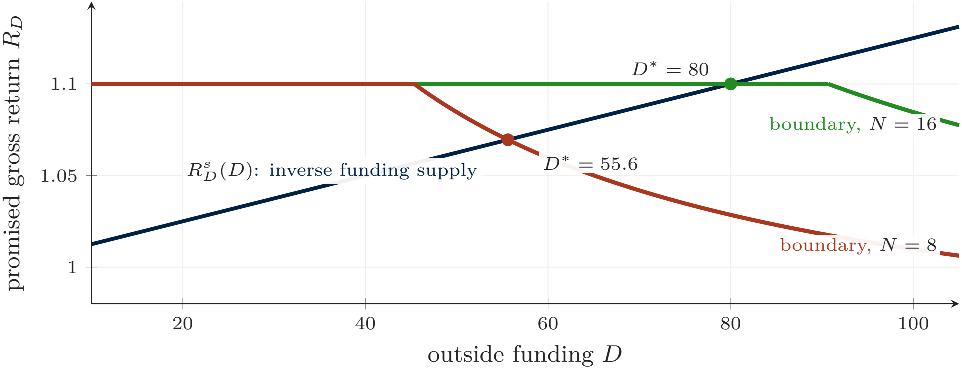
$$R_D^s(D^*) = R_D^d(D^*; N).$$

Superscript s: inverse funding supply; superscript d: the bank's incentive-compatible funding boundary, derived next.

The upward slope is a maintained reduced-form assumption about scarce risk-bearing and portfolio allocation; concavity alone does not imply it.

Appendix A16

Lower net worth shifts the funding boundary inward



$$R_D^d(D;N) = \min\left\{R_A,\ (1-\lambda)R_A\left(1+\frac{N}{D}\right)\right\}.$$

The same supply schedule meets a lower incentive-compatible boundary when intermediary net worth falls.

Appendix A8 Appendix A10

Net worth determines when the agency wedge disappears

optional extension

Let $D^0 \equiv (R_D^s)^{-1}(R_A)$ be the quantity supplied when the promised funding return equals the

asset return. We call this the **competitive no-agency benchmark**: asset finance carries no agency wedge. It is feasible when

$$D^0 \leq \frac{1-\lambda}{\lambda} N.$$

Equivalently,

$$\boxed{N_{\text{crit}} = \frac{\lambda}{1-\lambda} D^0}.$$

For the stated calibration, $D^0 = 80$ and $N_{\text{crit}} = 14.12$.

$$N > N_{\text{crit}} \Rightarrow \text{no-agency benchmark and (IC) strictly slack,}$$
$$N = N_{\text{crit}} \Rightarrow \text{benchmark feasible and (IC) exactly binding,}$$
$$N < N_{\text{crit}} \Rightarrow \text{constrained equilibrium.}$$

Appendix A9

Lower net worth changes quantity and the funding return

Using $R_D^s(D) = 1 + .00125D$, $R_A = 1.10$, and $\lambda = .15$:

N	D^*	$A^* = N + D^*$	R_D^*
16	80.0	96.0	1.1000
8	55.6	63.6	1.0695

$$\Delta N = -8, \qquad \Delta D^* = -24.4, \qquad \Delta A^* = -32.4.$$

The fall in funding quantity lowers its promised return. In this calibration, the fixed-return multiplier overstates the local marginal response on the constrained segment; the displayed high-to-low comparison also crosses the kink.

Appendix A10 Appendix A12

The return spread is an agency wedge, not a profit margin

optional extension

On the constrained segment,

$$\frac{R_A - R_D^*}{R_A} = \lambda - (1-\lambda)\frac{N}{D^*}.$$

For $N = 8$, $D^* = 55.6$, and $\lambda = .15$:

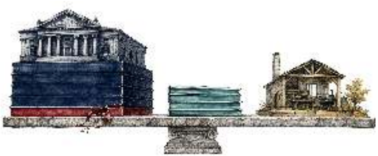
$$\frac{1.10 - 1.0695}{1.10} = 2.77\%.$$

The wedge measures the return not pledgeable to outside funders; it is not an observed net interest margin. The model has determined funding, not which assets shrink when funding falls.

An observed crisis spread also contains expected default loss, liquidity compensation, risk premia, and market power. It cannot be read directly as the model's agency wedge.

Appendix A11

A capacity loss reduces credit only if loans absorb the adjustment



The balance-sheet identity implies

$$A = C + S + L, \qquad \Delta A = \Delta C + \Delta S + \Delta L,$$

the same 20-unit contraction can be allocated differently:

$$(\Delta C, \Delta S, \Delta L) = (-5, -10, -5),$$
$$(\Delta C, \Delta S, \Delta L) = (0, 0, -20).$$

C: cash and reserves; S: securities; L: loans.

Only the second allocation makes the balance-sheet contraction a one-for-one loan contraction. Borrower substitution then determines the real effect.

The financial accelerator

optional extension

$$\begin{tikzcd} N_t \downarrow \arrow[r] & L_t \downarrow \arrow[r] & K_t, Y_t \downarrow \arrow[r] & \text{intermediary profits} \downarrow \arrow[r] & N_{t+1} \downarrow \arrow[llll, bend right, "\text{propagation}"'] \end{tikzcd}$$

t: current date; L_t: loans; K_t: borrower physical capital; Y_t: output.

This is a dynamic extension. A solved equilibrium also needs expectations, entry, survival, prices, and market clearing.

Bernanke, Gertler and Gilchrist (1999), financial accelerator.

Collateral prices and fire sales

optional extension

In a one-asset balance-sheet illustration, let the bank hold H units of collateral at price q, financed by outside claims D:

$$N = qH - D.$$

If $qH = 100$ and $D = 90$, then $N = 10$. A 5% fall in q gives

$$N' = 95 - 90 = 5.$$

The asset-price change halves net worth.

Forced sales lower q only when demand slopes downward, arbitrage capital is limited, or another price-setting mechanism is specified.

$$q \downarrow \Rightarrow N \downarrow \Rightarrow \underbrace{\text{capacity} \downarrow \Rightarrow \text{forced sales} \uparrow \Rightarrow q \downarrow}_{\text{when a balance-sheet constraint binds}}.$$

Kiyotaki and Moore (1997); Shleifer and Vishny (2011).

Appendix A17

Liquidity is a different constraint

optional extension

For collateral with market value V, the maximum secured funding F at haircut h is

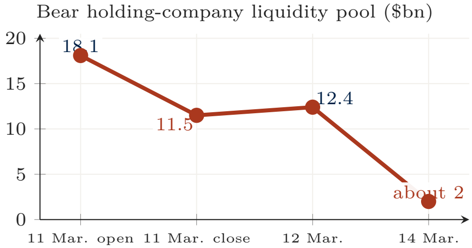
$$F = (1 - h)V.$$

For the illustrative value $V = 100$:

$$h : 2\% \to 10\% \quad \Longrightarrow \quad F : 98 \to 90.$$

The collateral need not default or fall in price; the lender has changed the advance rate. Bear's

counterparties then withdrew customary secured funding even against high-quality collateral.

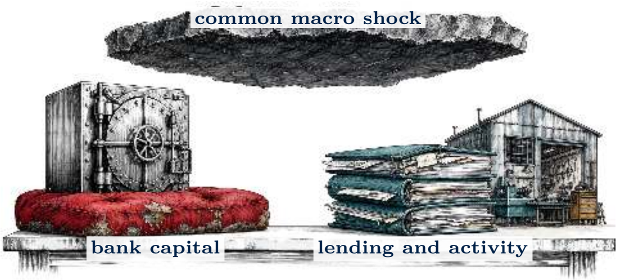


Bear's collapse therefore cannot be read from capital arithmetic alone. The one-period agency model does not generate this run; Day 2 develops that liquidity mechanism.

SEC, 20 March 2008 liquidity chronology.

Appendix A18

Capital and lending are not causally equivalent



$$\begin{aligned} \text{common macro shock} &\longrightarrow \{\text{asset losses},\ \text{loan demand}\},\\ \text{recognition and expected losses} &\longrightarrow \{\text{measured capital},\ \text{lending}\},\\ \text{policy and recapitalisation} &\longrightarrow \{\text{capital},\ \text{funding conditions}\}. \end{aligned}$$

A capital–lending correlation is compatible with several causal structures. Timing alone does not identify a bank-credit-supply effect. Policy must first identify which balance-sheet margin has failed.

Khwaja and Mian (2008), tracing bank liquidity shocks through matched borrowers.

A tighter capital constraint can reallocate credit across banks

Berrospide and Edge compare U.S. banks exposed to different post-stress-test capital-buffer requirements. A one-percentage-point larger buffer is associated with approximately

2.0 pp lower growth in utilised loans,
1.5 pp lower growth in commitments.

over the following four quarters. The estimated bank-level response does *not* translate one-for-one into firm-level contraction: exposed firms largely replaced the missing bank credit, so overall debt, investment, and employment changed little on average.

Capital requirements can shift which bank supplies a loan. The real effect depends on whether the borrower can substitute toward another lender or market.

The empirical design uses cross-bank variation in stress-capital buffers and borrower-level credit data; it is evidence about an average bank-credit-supply response, not a universal multiplier.
Berrospide and Edge (2019), Federal Reserve FEDS 2019-050.

Bank-equity losses predict contractions; panics amplify them

optional extension

Baron, Verner, and Xiong assemble bank-equity and panic histories for **46 countries from 1870 to 2016**. In their historical panel, a bank-equity decline of at least 30% predicts, after three years,

real GDP 3.4% lower and credit/GDP 5.7 percentage points lower.

Large bank-equity declines without creditor panics are also followed by substantial contractions: after three years, real GDP is about 2.7% lower and bank credit/GDP about 3.5 percentage points lower in the no-panic episodes.

This is broad historical predictive evidence, not a clean single-bank causal estimate. It shows that panic is an amplifier, not a necessary definition of banking-sector distress.

If distress can reach output through more than one margin, policy must diagnose the broken link before choosing the repair.

Baron, Verner, and Xiong (2021), *Quarterly Journal of Economics*, 136(1).

Part V

Capital repair and policy

How can capital be rebuilt without forcing an unnecessary credit contraction?

Capital, liquidity, and asset-price interventions repair different margins.

Banks may choose less capital than society prefers

Debt and deposits provide funding and liquidity services, but the private price of leverage need not include its full social cost.

1. Limited liability protects shareholders from losses beyond their stake.
2. Deposit insurance and expected support can reduce creditors' required compensation for downside risk.
3. Fire sales, disrupted payments, and credit contraction affect agents outside the bank.

For a common marginal-benefit schedule and increasing marginal costs, partial externalisation of losses can produce

$$MC_{\text{private}}(\ell) < MC_{\text{social}}(\ell) \qquad \Longrightarrow \qquad \ell_{\text{private}} > \ell_{\text{social}}.$$

ℓ: bank leverage; MC_{private}: marginal cost borne by the bank; MC_{social}: marginal cost including spillovers.
Hanson, Kashyap and Stein (2011), macroprudential rationale.

Risk weights and leverage ratios answer different questions

stylised asset	exposure	risk weight	risk-weighted assets
cash and reserves	20	0%	0
mortgages	50	50%	25
corporate loans	30	100%	30
total	100		55

Let $E_C = 6$ denote eligible common equity:

$$\underbrace{\frac{E_C}{A} = 6.0\%}_{\text{non-risk-weighted view}} \quad , \qquad \underbrace{\frac{E_C}{RWA} = 10.9\%}_{\text{risk-weighted view}} .$$

Here E_C/A is a simplified unweighted exposure ratio. A regulatory leverage ratio uses its prescribed exposure measure; the risk-weighted ratio responds to measured portfolio risk. The two constraints are complements, not substitutes.

Teaching weights, not the current Basel schedule. Basel Committee: the leverage ratio is a non-risk-based backstop to the risk-based capital framework.

Appendix A19

Capital regulation can move risk instead of removing it

optional extension

The resilience case for capital is strong, but implementation changes behaviour:

1. **Risk measurement:** a risk-weighted rule creates incentives to shift toward exposures with low measured weights.
2. **Procyclicality:** recognised losses and rising measured risk can tighten constraints during a downturn.
3. **Regulatory perimeter:** activity can migrate to less-regulated intermediaries while the underlying maturity or credit risk remains.

The leverage ratio limits model and weight error; risk-weighted ratios preserve risk sensitivity. Neither makes supervision or liquidity regulation redundant.

Basel Committee; Hanson, Kashyap, and Stein (2011).

A capital buffer is distance from the constraint

For an illustrative unweighted threshold $\kappa_{\min}$, define the buffer in units of equity:

$$B \equiv N - \kappa_{\min} A.$$

If $A = 100$, $N = 10$, and $\kappa_{\min} = 8\%$:

$$B = 10 - .08(100) = 2.$$

A two-unit asset loss, holding promised claims fixed initially, gives

$$A' = 98, \qquad N' = 8, \qquad B' = 8 - .08(98) = 0.16.$$

The bank remains above zero book equity, but almost all of its discretionary capital cushion has disappeared. A further marked loss of about 0.17 would exhaust the remaining buffer if promised claims were initially unchanged.

A regulatory minimum, a supervisory stress buffer, and a bank's management or market target are distinct thresholds; crossing any one has a different economic and legal meaning.

The illustrative 8% threshold is a teaching assumption, not the Basel leverage minim

Appendix A13 Appendix A14

A stress test asks whether capital survives a future scenario

The 2009 SCAP did not ask only whether banks met current minimum ratios. It applied the same adverse macroeconomic scenario to all major institutions.

1. project losses, revenues, reserves, and resources over the horizon;
2. calculate the resulting common-equity buffer under stress;
3. disclose the methodology and firm-level capital need;
4. require remediation by a deadline, with a public backstop available.

$$\text{capital need} = \max\{0,\ \text{required stressed buffer} - \text{projected stressed capital}\}.$$

The policy innovation combined loss recognition, comparable information, and a credible repair plan; it was not a declaration that every firm was currently insolvent.

Federal Reserve, SCAP design and results, May 2009.

The 2009 stress test converted losses into capital buffers

The Supervisory Capital Assessment Program examined the 19 largest US bank holding companies under a more adverse macroeconomic scenario.

19	10	$74.6bn
firms assessed	needed a larger or higher-quality buffer	required capital augmentation

By November 2009, the 10 firms had increased Tier 1 Common equity by more than $77bn through:

- $39bn of new common equity or other eligible securities;
- $23bn of preferred-to-common conversions;
- $9bn from business or portfolio sales, plus other actions.

Tier 1 Common was the stress test's common-equity measure; it is not the model's unweighted ratio N/A.

The exercise combined loss recognition, a forward scenario, a public capital standard, and a deadline for remedial action, reducing uncertainty about heterogeneous institutions.

Federal Reserve, Supervisory Capital Assessment Program, 9 November 2009.

The 2026 stress test still asks whether capital survives the path

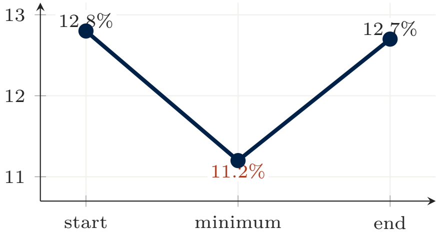
optional extension

The Federal Reserve's 2026 exercise applied a severely adverse, hypothetical nine-quarter scenario to 32 large banks. For the aggregate

common equity tier 1 ratio:

12.8% $\longrightarrow$ 11.2% minimum $\longrightarrow$ 12.7%.

The relevant object is the **minimum along the path**, not only the starting or ending ratio.



A stress test is a conditional capital calculation, not a forecast and not proof that short-term funding cannot run.

Board of Governors of the Federal Reserve System, *2026 Federal Reserve Stress Test Results*, 24 June 2026.

Capital repair changes cash, claims, and ownership differently

New common equity	cash and net worth rise; existing owners are diluted.
Creditor bail-in	a liability becomes equity; no new cash enters and the creditor bears the conversion.
Public-sector equity	cash and net worth rise; the state receives a claim and bears investment risk.

All three can raise measured capital. They do not allocate losses, control, or fiscal risk in the same way.

Appendix A15

Diagnosis determines the policy instrument

diagnosis	primary instrument	first-order effect
capital shortfall	common equity, conversion, or bail-in	raises loss-absorbing net worth N
funding run	liquidity facility or temporary guarantee for a solvent but illiquid institution	replaces withdrawals or stabilises funding terms
fire-sale market	market-liquidity operation or asset purchase	supplies cash or supports q; risk transfer depends on price

A fair-value liquidity operation and a subsidised risk transfer can share the same policy label but have different loss incidence. A liquidity facility cannot repair negative net worth.

Hanson, Kashyap and Stein (2011); Federal Reserve SCAP materials.

Bear Stearns and SCAP repaired different margins

Bear Stearns, March 2008

Observed secured funding vanished and the liquidity pool fell from $18.1bn to about $2bn.

Repair bridge funding through JPMorgan, then acquisition support. Maiden Lane acquired about $30bn of Bear mortgage-desk assets; the PDCF was a separate market-wide liquidity backstop.

Boundary liquidity support buys time; it does not make an economically insolvent portfolio solvent.

SCAP, 2009

Observed uncertainty about future losses and the quality of capital weakened confidence across major banks.

Repair one adverse scenario, comparable disclosure, a common-equity requirement, and a credible public backstop.

Boundary a capital exercise does not by itself prevent a short-term funding run.

Both episodes involved deleveraging pressure. The correct instrument followed from the failed margin, not from the word “crisis”.

Federal Reserve Bank of New York, Bear Stearns support; Federal Reserve, 2009 SCAP materials.

Capital policy changes incentives before the crisis

optional extension

Before distress Shareholders choose leverage, asset risk, and the size of their own loss-absorbing stake.

During distress Authorities may find intervention optimal because disorderly deleveraging imposes costs on borrowers, markets, and the payments system.

After intervention Expected future support can reduce market discipline unless regulation, supervision, pricing, and resolution rules make private risk-bearers absorb losses.

The ex-post case for support and the ex-ante case for discipline can both be valid. Capital policy is therefore partly a commitment problem.

The opening puzzle has a conditional answer

In the mechanism studied today, a small asset loss becomes a large economic contraction only when four statements hold together:

1. **Leverage:** the asset loss is large relative to initial equity.
2. **Credibility:** lower net worth tightens a binding incentive constraint.
3. **Equilibrium:** outside funding does not replace the lost capacity at unchanged terms.
4. **Transmission:** loans absorb the adjustment and borrowers cannot substitute toward other finance.

$$\text{asset loss} \not\equiv \text{credit contraction} \not\equiv \text{output loss}.$$

The agency mechanism supplies the credibility-to-capacity link. Portfolio adjustment, substitute finance, and real transmission require additional assumptions. Capital policy can weaken the mechanism, but also changes incentives.

Bear resolves the opening tension: its thin equity made losses dangerous, but the decisive March record is a wholesale funding collapse. Capital and liquidity are connected in crises without being the same mechanism.

The mechanism determines what should be measured

The four links turn a general crisis narrative into distinct empirical questions:

Loss absorption How large is the plausible asset mark relative to equity, after duration, hedges, and accounting treatment?

Credibility Does lower net worth change the quantity or terms of outside funding?

Equilibrium Do other funders or prices offset the original funding loss?

Transmission Which assets shrink, and can borrowers substitute toward another bank or market?

AI can organise documents, extract exposures, and estimate conditional risk. It becomes useful only after the economic target and decision are clear.

Prediction and causal identification are different tasks: a warning signal need not reveal the effect of a capital or liquidity intervention.

Technical appendix: derivation map

1. A1–A3: losses, duration, and capital-ratio repair
2. A4–A7: incentives, finite capacity, and the numerical solution
3. A8–A12: funding equilibrium, threshold, and wedge
4. A13–A15: buffers, debt overhang, and recapitalisation
5. A16: funding supply as a maintained assumption
6. A17: collateral-price loss
7. A18: haircut and secured funding
8. A19: risk-weighted assets and capital ratios
9. A20: fixed-return capacity comparative statics

Each central algebraic result is worked from its starting equation. Institutional claims and maintained assumptions are labelled separately.

Back to lecture

A1 – Leverage maps an asset mark into equity loss

Let initial outside claims be $D = A - N$. Mark a fraction x from asset value while holding those promised claims fixed at that instant:

$$\begin{aligned} A' &= A - xA, \\ N' &= A' - D = (A - xA) - (A - N) = N - xA, \\ \Delta N &= N' - N = -xA, \\ \frac{-\Delta N}{N} &= x\frac{A}{N} = x\ell. \end{aligned}$$

Equity is exhausted when $N' = 0$:

$$0 = N - xA \quad \Longrightarrow \quad x_{\text{wipeout}} = \frac{N}{A} = \frac{1}{\ell}.$$

For Bear Stearns, $\ell = 33.53$, hence $x_{\text{wipeout}} = 2.98\%$.

Back to lecture

A2 – The duration approximation

For promised cash flows CF_t, price is

$$P(y) = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t}.$$

Differentiate with respect to the yield:

$$\frac{dP}{dy} = -\sum_{t=1}^{T} \frac{tCF_t}{(1+y)^{t+1}}.$$

Define modified duration

$$D_{\text{mod}} \equiv -\frac{1}{P}\frac{dP}{dy}.$$

A first-order Taylor approximation around the current yield gives

$$\Delta P \approx \frac{dP}{dy}\Delta y \quad \Longrightarrow \quad \frac{\Delta P}{P} \approx -D_{\text{mod}}\Delta y.$$

For a five-year zero-coupon claim at $y = .03$, Macaulay duration is 5 and

$$D_{\text{mod}} = \frac{5}{1.03} = 4.854.$$

With $\Delta y = 40\text{bp} = .004$, the local price change is therefore $-4.854(.004) = -1.94\%$.

This first-order calculation assumes fixed contractual cash flows, annual compounding, and a small parallel shift in the relevant yield curve. Convexity and changes in credit or liquidity spreads are omitted.

Back to lecture

A2b – From asset duration to the bank's net-worth exposure

Let asset and liability values be A and L, with modified durations D_A and D_L. For a small common yield change Δy,

$$\Delta A \approx -D_A A\,\Delta y, \qquad \Delta L \approx -D_L L\,\Delta y.$$

Because net worth is $N = A - L$,

$$\boxed{\Delta N \approx (-D_A A + D_L L)\Delta y + \Delta H},$$

where ΔH is the value change of the bank's interest-rate hedges. Equivalently, dividing by assets,

$$\frac{\Delta N}{A} \approx -\left(D_A - \frac{L}{A} D_L\right)\Delta y + \frac{\Delta H}{A}.$$

Thus a long asset duration alone does not determine the equity loss: liability duration, leverage, basis risk, and hedges determine the net exposure.

A common parallel shift is still a teaching approximation. Different curves, option-adjusted durations, and non-rate marks require separate shocks.

Back to lecture

A3 – Three capital-ratio repair calculations

Bear scaled to $A = 100$ gives $N = 2.98$. After a 2% mark, $(A_1, N_1) = (98, 0.98)$. Target $\bar{\kappa} = .08$.

$$\text{Asset reduction:} \quad \frac{N_1}{A_1 - s} = \bar{\kappa} \quad \Longrightarrow \quad s = A_1 - \frac{N_1}{\bar{\kappa}} = 98 - 12.25 = 85.75.$$

$$\text{New cash equity:} \quad \frac{N_1 + e}{A_1 + e} = \bar{\kappa} \quad \Longrightarrow \quad e = \frac{\bar{\kappa} A_1 - N_1}{1 - \bar{\kappa}} = \frac{6.86}{0.92} = 7.46.$$

$$\text{Liability bail-in:} \quad \frac{N_1 + b}{A_1} = \bar{\kappa} \quad \Longrightarrow \quad b = \bar{\kappa} A_1 - N_1 = 6.86.$$

Asset reduction uses proceeds to repay claims, equity issuance adds cash to both sides, and bail-in converts an existing claim into equity. At $\ell = 33.53$ each route is large relative to the balance sheet: the same target ratio is reached by selling 87% of the assets, by raising eight times the remaining equity, or by writing down 7% of the claims.

Back to lecture

A4 – Rearranging the incentive constraint

Start from honest and diversion payoffs:

$$R_A(N+D) - R_D D \geq \lambda R_A(N+D).$$

Move diversion payoff to the left:

$$(1-\lambda)R_A(N+D) \geq R_D D.$$

Expand the left side and move the outside-funding term:

$$\begin{aligned}(1-\lambda)R_A N + (1-\lambda)R_A D &\geq R_D D,\\ (1-\lambda)R_A N &\geq \left[R_D - (1-\lambda)R_A\right]D.\end{aligned}$$

The left side is incentive surplus supplied by inside net worth. The bracketed term is the funding gap created by one additional unit of outside finance.

Back to lecture

A5 – Why capacity is finite only in one case

Define the per-unit funding gap

$$g \equiv R_D - (1-\lambda)R_A.$$

Then the incentive constraint is

$$(1-\lambda)R_A N \geq gD.$$

$$\begin{aligned} g > 0: &\quad D \leq \frac{(1-\lambda)R_A}{g}N && \text{finite capacity,} \\ g = 0: &\quad (1-\lambda)R_A N \geq 0 && \text{no cap from this constraint,} \\ g < 0: &\quad gD < 0 \text{ for } D > 0 && \text{no finite cap from this constraint.} \end{aligned}$$

The familiar debt-limit formula must not be used when $g \leq 0$; additional frictions would be needed to determine scale.

Back to lecture

A6 – Deriving debt and asset capacity

For $g > 0$, divide the incentive constraint by the positive funding gap:

$$(1-\lambda)R_A N \geq gD,$$

$$D \leq \frac{(1-\lambda)R_A}{g} N,$$

$$D^{\max} = \frac{(1-\lambda)R_A}{R_D - (1-\lambda)R_A} N \equiv \phi N.$$

Use $A = N + D$:

$$A^{\max} = N + D^{\max} = (1+\phi)N \equiv \psi N.$$

At fixed (R_A, R_D, λ), one additional unit of net worth supports ϕ additional units of outside funding and ψ units of total assets. This is capacity, not necessarily chosen scale.

Back to lecture

A7 – Numerical capacity solution and payoff check

Let $R_A = 1.10$, $R_D = 1.04$, $\lambda = .15$, and $N = 10$:

$$\begin{aligned}
(1-\lambda)R_A &= .85(1.10) = .935, \\
g &= 1.04 - .935 = .105, \\
\phi &= .935/.105 = 8.9048, \\
D^{\max} &= 8.9048(10) = 89.048, \\
A^{\max} &= 99.048.
\end{aligned}$$

At the boundary, up to rounding,

$$\begin{aligned}
\Pi^{\text{hon}} &= 1.10(99.048) - 1.04(89.048) = 16.343, \\
\Pi^{\text{div}} &= .15(1.10)(99.048) = 16.343.
\end{aligned}$$

The payoff equality verifies that the reported capacity is the crossing point.

Back to lecture

A8 – The bank funding-demand boundary

When the constraint binds,

$$(1-\lambda)R_A(N+D) = R_D D.$$

Divide by $D > 0$:

$$R_D = (1-\lambda)R_A\left(1+\frac{N}{D}\right).$$

Maintain the competitive benchmark assumption that the bank does not promise more than the asset return. Its boundary is

$$R_D^d(D) = \min\left\{R_A, (1-\lambda)R_A\left(1+\frac{N}{D}\right)\right\}.$$

At the kink, set the two expressions equal and cancel $R_A > 0$:

$$1 = (1-\lambda)\left(1+\frac{N}{D_k}\right) \quad \Longrightarrow \quad D_k = \frac{1-\lambda}{\lambda}N.$$

Back to lecture

A9 – Deriving the no-agency-wedge threshold

Let $D^0 = (R_D^s)^{-1}(R_A)$ be funding supplied when the promised return equals the asset return. The no-agency benchmark is feasible if it lies on the flat segment:

$$D^0 \leq D_k = \frac{1-\lambda}{\lambda} N.$$

Multiply by $\lambda/(1-\lambda) > 0$:

$$\frac{\lambda}{1-\lambda} D^0 \leq N.$$

Therefore

$$N_{\text{crit}} \equiv \frac{\lambda}{1-\lambda} D^0.$$

$$\begin{aligned} N > N_{\text{crit}} &: \text{ no-agency benchmark, constraint slack,} \\ N = N_{\text{crit}} &: \text{ benchmark, constraint exactly binding,} \\ N < N_{\text{crit}} &: \text{ market clearing occurs on the constrained segment.} \end{aligned}$$

Back to lecture

A10 – Solving the high- and low-net-worth equilibria

Use $R_D^s(D) = 1 + .00125D$, $R_A = 1.10$, and $\lambda = .15$. No-agency-benchmark funding solves

$$1 + .00125D^0 = 1.10 \quad \Longrightarrow \quad D^0 = 80,$$

so $N_{\text{crit}} = (.15/.85)80 = 14.12$. For $N = 16 > N_{\text{crit}}$, $D^* = 80$ and $R_D^* = 1.10$. For $N = 8 < N_{\text{crit}}$, solve market clearing on the binding segment:

$$1 + .00125D = .935\left(1 + \frac{8}{D}\right),$$

$$.00125D^2 + .065D - 7.48 = 0,$$

$$D^* = \frac{-.065 + \sqrt{.065^2 + 4(.00125)(7.48)}}{.0025} = 55.6,$$

$$R_D^* = 1 + .00125(55.6) = 1.0695.$$

The negative quadratic root is economically inadmissible.

Back to lecture

A11 – Deriving the intermediation wedge

On the constrained segment,

$$R_D = (1-\lambda)R_A\left(1+\frac{N}{D}\right).$$

Subtract this expression from R_A and divide by $R_A > 0$:

$$\begin{aligned}\frac{R_A - R_D}{R_A} &= 1-(1-\lambda)\left(1+\frac{N}{D}\right),\\ &= 1-(1-\lambda)-(1-\lambda)\frac{N}{D},\\ &= \lambda-(1-\lambda)\frac{N}{D}.\end{aligned}$$

At $N = 8$, $D = 55.6$, and $\lambda = .15$:

$$.15 - .85\left(\frac{8}{55.6}\right) = .0277 = 2.77\%.$$

This is an incentive wedge in the model, not an observed net interest margin.

Back to lecture

A12 – The equilibrium net-worth derivative

Write constrained market clearing as

$$F(D, N) = R_D^s(D) - (1-\lambda)R_A\left(1 + \frac{N}{D}\right) = 0.$$

Implicit differentiation gives

$$F_D \frac{dD}{dN} + F_N = 0,$$

where

$$F_D = (R_D^s)'(D) + \frac{(1-\lambda)R_A N}{D^2}, \qquad F_N = -\frac{(1-\lambda)R_A}{D}.$$

Hence, after using the binding condition,

$$\frac{dD}{dN} = \frac{(1-\lambda)R_A}{D(R_D^s)'(D) + R_D^s(D) - (1-\lambda)R_A}.$$

At the low-N equilibrium this equals 4.58, so $dA/dN = 1 + dD/dN = 5.58$, below the fixed-return multiplier 9.90.

Back to lecture

A13 – Capital-buffer arithmetic

Let the equity buffer above a fixed minimum be

$$B = N - \kappa_{\min} A.$$

Its exact finite change is

$$\Delta B = \Delta N - \kappa_{\min}\Delta A - A\Delta\kappa_{\min} - \Delta\kappa_{\min}\Delta A.$$

For a fixed minimum and an immediate asset mark m,

$$\Delta A = -m, \qquad \Delta N = -m, \qquad \Delta\kappa_{\min} = 0.$$

Substitute:

$$\Delta B = -m + \kappa_{\min} m = -(1 - \kappa_{\min})m.$$

With $\kappa_{\min} = 8\%$ and $m = 2$, the buffer falls by 1.84, because the denominator falls along with equity.

Back to lecture

A14 – Debt-overhang solution

Assume one-period, risk-neutral, same-date valuation. Existing assets pay 80 or 120, each with probability 1/2; debt has face value 100. Existing equity is

$$E_0 = \tfrac{1}{2}\max(80 - 100, 0) + \tfrac{1}{2}\max(120 - 100, 0) = 10.$$

A project costs 10 and pays 15 in either state, so

$$NPV = 15 - 10 = 5 > 0.$$

After investment, equity values are $\max(95 - 100, 0) = 0$ and $135 - 100 = 35$:

$$E_1 = \tfrac{1}{2}(0) + \tfrac{1}{2}(35) = 17.5.$$

Existing owners pay 10 but gain only $E_1 - E_0 = 7.5$. They reject the positive-NPV project because the remaining expected gain accrues to creditors in the low state.

Stylised debt-overhang example after Myers (1977).

Back to lecture

A15 – Two recapitalisation multipliers

For fixed promised returns,

$$A^{\max} = \psi N \quad \Longrightarrow \quad \Delta A^{\max} = \psi \Delta N.$$

With $\psi = 9.90$ and $\Delta N = 2$:

$$\Delta A^{\max} = 9.90(2) = 19.8.$$

When funding supply is endogenous, use the local equilibrium response derived in A12:

$$\Delta A^* \approx \frac{dA}{dN} \Delta N = 5.58(2) = 11.2.$$

Both exceed the two-unit injection because net worth supports additional outside funding. The equilibrium response is smaller because the funding price moves with quantity; neither calculation says that all additional assets are loans.

Back to lecture

A16 – Household funding supply is a maintained assumption

A representative two-period household chooses saving d_h:

$$\max_{0 \le d_h \le W} u(W - d_h) + \beta u(R_D d_h).$$

For an interior solution,

$$-u'(W - d_h) + \beta R_D u'(R_D d_h) = 0,$$

or

$$u'(W - d_h) = \beta R_D u'(R_D d_h).$$

The return changes both the reward to saving and the resources needed to reach future consumption. Concavity alone therefore does not guarantee that saving rises with R_D. With CRRA coefficient γ,

$$\operatorname{sign}\left(\frac{dd_h}{dR_D}\right) = \operatorname{sign}(1 - \gamma).$$

With M identical households, aggregate funding is $D = M d_h$. The upward inverse schedule in the core remains a reduced-form assumption, not a theorem from concavity alone.

Back to lecture

A17 – Collateral prices and intermediary equity

Let H be fixed collateral holdings, q the price per unit, and D promised outside claims:

$$N = qH - D.$$

Its change is

$$\Delta N = H\Delta q + q\Delta H - \Delta D.$$

Holding holdings and promised claims fixed at the mark gives

$$\Delta N = H\Delta q.$$

If $q_0 H = 100$ and $D = 90$, then $N_0 = 10$. A 5% price fall implies

$$q_1 H = .95(100) = 95, \qquad N_1 = 95 - 90 = 5.$$

The collateral price falls by 5%; equity falls by 50%.

Back to lecture

A18 – Haircuts and secured funding capacity

For collateral value V and haircut h, maximum secured funding is

$$F = (1 - h)V.$$

Holding collateral value fixed,

$$\frac{\partial F}{\partial h} = -V < 0.$$

With $V = 100$:

$$\begin{aligned} h_0 = .02 \quad &\Longrightarrow \quad F_0 = .98(100) = 98, \\ h_1 = .10 \quad &\Longrightarrow \quad F_1 = .90(100) = 90, \\ \Delta F = F_1 - F_0 &= -8. \end{aligned}$$

The haircut shock removes eight units of funding without requiring a change in the collateral's price or expected payoff.

Back to lecture

A19 – Risk-weighted and unweighted capital ratios

Risk-weighted assets are the weighted sum of exposures:

$$RWA = \sum_i w_i A_i.$$

For cash $(20, w = 0)$, mortgages $(50, w = .5)$, and corporate loans $(30, w = 1)$:

$$RWA = 0(20) + .5(50) + 1(30) = 55.$$

Total exposure is

$$A = 20 + 50 + 30 = 100.$$

With eligible common equity $E_C = 6$:

$$\frac{E_C}{A} = 6.0\%, \qquad \frac{E_C}{RWA} = \frac{6}{55} = 10.9\%.$$

Different denominators produce different constraints from the same numerator.

Back to lecture

A20 – Comparative statics of fixed-return capacity

Let $b \equiv (1-\lambda)R_A$ and $g \equiv R_D - b > 0$. Then

$$\phi = \frac{b}{g} = \frac{(1-\lambda)R_A}{R_D - (1-\lambda)R_A}.$$

Differentiate while holding the other arguments fixed:

$$\frac{\partial \phi}{\partial R_A} = \frac{(1-\lambda)R_D}{g^2} > 0,$$

$$\frac{\partial \phi}{\partial R_D} = -\frac{(1-\lambda)R_A}{g^2} < 0,$$

$$\frac{\partial \phi}{\partial \lambda} = -\frac{R_A R_D}{g^2} < 0.$$

Since $A^{\max} = (1+\phi)N = (R_D/g)N$, these derivatives describe a fixed-return capacity boundary. They are not equilibrium derivatives when R_D, desired assets, or asset prices respond to the shock.

Back to lecture

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Appendix map

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Appendix map

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