Risk Management and Derivatives | Lecture 1

Introduction to Derivatives

Group activity solutions

· SJTU Summer School 2026 · Fatih Kansoy

Phase 1 – Concept checks

- **CT1: (b).** The \$9,360 is just today's price grown by the interest cost ($9{,}000 \times 1.04$). It tells you the cost of locking in copper today. It is not a guess about next year's price; nobody looked at copper supply or demand to get it.
- **CT2: (b).** The notional only sizes the interest payments. The £500 million principal is never exchanged, and at fair terms the swap is worth about zero on day one. The money truly at risk is how the rates move from here, which is much smaller than £500 million.
- **CT3: (b).** The payment at the end is a pure transfer (the €30 the bakery "loses" is exactly the €30 the farmer gains). But on the day they signed, both sides locked a price they could plan around. That certainty had value to each of them, so both could be better off.

Phase 2 – Desk problem, fully worked

(a) Alpine is long the forward

Alpine's worry is cocoa getting more expensive. A long forward locks the buying price, so when cocoa rises the forward gains and cancels the higher spot cost. The right side for a future buyer is always the long side.

Answer. Alpine should be **long**. Payoff per tonne $= S_T - K$, with $K = 3{,}200$.

state	S_T	payoff $S_T - K$
Fall	2,600	−600/t
Same	3,200	0/t
Jump	4,000	+800/t

The forward loses €600 when cocoa is cheap and gains €800 when cocoa is expensive. That is exactly the right way round for Alpine, which must buy cocoa anyway.

(b) Net cost locked at €3,200/t

Alpine pays the spot price for cocoa, and the forward pays Alpine back the difference from €3,200. When you add them, the spot price cancels and only €3,200 per tonne is left. The hedge does not aim for the lowest cost. It aims to make the cost the same in every state, so Alpine can plan.

Calculation. $Q = 10{,}000$. Net cost $= S_T \cdot Q - (S_T - K) \cdot Q = K \cdot Q$.

state	S_T	spot cost $S_T \cdot Q$	forward gain $(S_T - K)Q$	net cost
Fall	2,600	26,000,000	−6,000,000	**32,000,000**
Same	3,200	32,000,000	0	**32,000,000**
Jump	4,000	40,000,000	+8,000,000	**32,000,000**

Net $= K \cdot Q = 3{,}200 \times 10{,}000 =$ €32,000,000 in every state, which is €3,200 per tonne locked. In the fall state Alpine paid only €26,000,000 for spot cocoa but lost €6,000,000 on the forward, so

it still paid €32,000,000. The hedge does not let Alpine enjoy the cheap cocoa. You give up the chance of a lower cost in order to remove the risk of a higher cost.

(c) Counterparty credit risk at the jump

A hedge is only as good as the promise behind it. In the jump state the forward is worth a lot to Alpine, so the desk owes Alpine a lot. That is exactly the moment the desk is most tempted to walk away, and the moment Alpine can least afford it.

Calculation. Jump state, $S_T = €4{,}000$.

(1) Value of the forward to Alpine $= (S_T - K) \times Q = (4{,}000 - 3{,}200) \times 10{,}000 = €8{,}000{,}000$. The desk owes Alpine €8,000,000.

(2) **Replacement cost.** New cocoa forwards now cost €4,000 per tonne. Alpine's old contract was €3,200. To rebuild the same hedge Alpine must pay the gap of $4{,}000 - 3{,}200 = €800$ per tonne, over 10,000 tonnes: $800 \times 10{,}000 = €8{,}000{,}000$. The replacement cost equals the value Alpine lost.

(3) The desk is most tempted to default in the jump state, because that is when it owes the most. That is the worst moment for Alpine, because it is exactly when Alpine most needs the hedge to work. The promise is most valuable and most likely to be broken in the same state.

(d) No-arbitrage cash-and-carry

Two routes to owning cocoa in one year must cost the same. Route one: agree a forward today. Route two: borrow money, buy cocoa today, store it, repay the loan later. If a desk's forward price is different from route two, you can do one route cheaply and the other expensively, and pocket the gap with no risk. That gap is arbitrage and it should not exist.

Calculation.

(1) Fair forward $F_0 = S_0(1 + r) = 3{,}000 \times 1.04 = €3{,}120/\text{t}$.

(2) The second desk quotes €3,250, which is **above** the fair €3,120, so the forward is too expensive. When a forward is too expensive you sell it (agree to deliver at the high price) and carry the physical cocoa. This is the *cash-and-carry*.

(3) Per tonne:

step	today	in 1 year
Borrow €3,000 at 4%	+3,000	−3,120 (repay loan)
Buy 1 tonne of cocoa	−3,000	hold the tonne
Sell the forward at €3,250	0	+3,250 and deliver the tonne
total	**0**	+130

Profit $= 3{,}250 - 3{,}120 = €130/\text{t}$. You put in zero today and collect €130 in one year, with no risk. On 10,000 tonnes: $130 \times 10{,}000 = €1{,}300{,}000$.

(4) **Independence of S_T.** You already own the tonne and you already promised to deliver it at €3,250, so the spot price never touches your cash flows.

in 1 year	$S_T = €2{,}000$	$S_T = €5{,}000$
Repay loan	−3,120	−3,120
Deliver tonne, receive forward price	+3,250	+3,250
profit	+130	+130

Same €130 either way. The profit comes only from the mispricing, €3,250 versus €3,120.

(e) Option versus forward, and leverage

The forward fixes the price both ways: Alpine is safe if cocoa rises, but gives up the gain if cocoa falls. The call is one-sided: Alpine is safe if cocoa rises, yet still enjoys cheap cocoa if it falls, because Alpine can simply not use the option. That extra freedom is not free; it costs the €100 premium.

Calculation.

(1) Breakeven = strike + premium = 3,200 + 100 = €3,300/t.

(2) Effective cost per tonne. Alpine always pays the €100 premium. If spot is above €3,200, Alpine uses the call and pays the €3,200 strike, so the all-in cost is 3,200 + 100 = €3,300. If spot is €3,200 or below, Alpine lets the call expire and buys at spot, so the all-in cost is spot +100.

spot in 1 year	unhedged	forward	call (incl. premium)
€2,600	2,600	3,200	2,700 (2,600 + 100)
€3,200	3,200	3,200	3,300 (3,200 + 100)
€4,000	4,000	3,200	3,300 (3,200 + 100)

(3) The forward and the call both cap the cost when cocoa is expensive. The difference shows in the fall state. The forward is stuck at €3,200, but the call lets Alpine buy at the cheap €2,700. The forward is better when Alpine just wants the lowest fixed cost and cocoa rises (€3,200 beats €3,300). The call is better when cocoa falls, because Alpine keeps most of the saving. The €100 is the price of keeping the upside.

(4) **Leverage**, budget €3,000,000.

- Buying physical cocoa at €3,000/t: 3,000,000 ÷ 3,000 = **1,000 tonnes**.
- Buying calls at €100/t: 3,000,000 ÷ 100 = **30,000 tonnes** of exposure. That is 30 times the exposure for the same cash.
- If cocoa jumps to €4,000:
  - Cocoa: profit = (4,000 − 3,000) × 1,000 = €1,000,000, a return of 1,000,000 ÷ 3,000,000 = +33.3%.
  - Calls: profit = (4,000 − 3,200 − 100) × 30,000 = €21,000,000, a return of 21,000,000 ÷ 3,000,000 = +700%.
- If cocoa ends at €3,200 or below, every call expires worthless and the whole €3,000,000 is lost. The cocoa buyer still owns cocoa worth about €3,200,000 and has lost little or nothing.

This is leverage. The calls magnify both the gain and the loss. It is the same idea, but the gains and losses are both much bigger.

Extension: reverse cash-and-carry at €3,050

This time the forward is too cheap, not too expensive. So you flip the trade: sell cocoa now while it is expensive in spot terms, park the cash at the bank, and buy the cocoa back cheaply through the forward. Again you lock both ends today, so the final spot price cannot touch you.

Calculation. Third quote €3,050 < fair €3,120, so the forward is too cheap. When a forward is too cheap you buy it (go long) and sell the physical short.

step	today	in 1 year
Short 1 tonne (sell borrowed cocoa)	+3,000	buy 1 tonne back to return it
Invest €3,000 at 4%	−3,000	+3,120
Buy the long forward at €3,050	0	−3,050, receive the tonne
total	**0**	+70

Profit = 3,120 − 3,050 = €70/t (on 10,000 t, €700,000). It is independent of S_T: you return the tonne using the tonne you bought through the forward, so the spot price never appears. At S_T = €2,000 or €5,000 you still collect +3,120 from the bank and pay −3,050 on the forward, for +70 either way.

Phase 3 – Case: what happened

The bank did not get its way quietly. On 24 January 2008 Société Générale went public. It said one of its traders, Jérôme Kerviel, had built enormous hidden positions in European stock-index futures. The bank had already started closing the position on 21 January 2008 and finished over about three days. It sold into a falling market. The bank reported a loss of **about €4.9 billion** from unwinding the trades. At the time this was one of the largest trading losses by a single trader in history.

The futures themselves were ordinary, safe, well-regulated contracts. They were exchange-traded, centrally cleared and margined every day. So why did clearing and margin not save the bank? Clearing and daily margin protect against one thing: a counterparty failing to pay. They do not stop a trader from taking a position that is too big. And they do not help when the position is hidden from your own risk managers. Two things beat the safeguards here:

- **The size was hidden.** The fake offsetting trades made the real exposure invisible inside the bank. Risk limits only work on the risk you can see.
- **The forced sale moved the market.** The position was so large that selling it pushed prices down further, so the act of closing the bet made the loss worse.

The instrument did not fail. The problem was the use: a desk built for tiny low-risk arbitrage was used for a giant one-way speculative bet, and the bet was hidden. The risk did not vanish into the clearing house. It sat on the bank's own balance sheet the whole time, waiting for the market to turn. Derivatives move risk from one party to another. They do not destroy it.

End of solutions.