Risk Management and Derivatives | Lecture 10

No-Arbitrage Bounds and Put-Call Parity

In Class Group Activity

· SJTU Summer School 2026 · Fatih Kansoy

How this block runs. You work in your home group of four or five, textbook open. Roles rotate today: **Scribe** (writes the answer), **Reporter** (speaks at the microphone), **Sceptic** (must raise one objection), **Timekeeper**. All money today is in US dollars ($). All rates are continuously compounded unless stated.

- **Phase 1, Concept checks (~10 min).** For each question you vote on your own first. Then you talk for about 90 seconds with the people next to you, and vote again. Do not talk before the first vote.
- **Phase 2, Desk problem (~20 min).** One problem in parts. Your Reporter presents one part when the microphone reaches your group. Then we solve it together, part by part.
- **Phase 3, Case (~25 min).** You read one real story. The ending is hidden on purpose. You answer four questions and take a position. Then we reveal what happened.

A short word list for this block. **No-arbitrage** means no free lunch: you cannot make a riskless profit from nothing. A **bound** is a price limit that any option must obey, found from S, K, r and T alone, with no option model. **Put-call parity** is the identity $c + Ke^{-rT} = p + S$ that links a European call c, a European put p, the stock S and the present value of the strike Ke^{-rT}. A **conversion** is the arbitrage trade you build when the call side is too dear: short the call, long the put, long the stock, borrow Ke^{-rT}. **Early exercise** means using your American option before expiry. **Limits to arbitrage** are the real frictions (such as not being able to borrow a stock to short it) that can let a mispricing survive even when everyone can see it.

Phase 1 – Concept checks ~10 MIN

ConcepTest 1. A US technology stock pays no dividends before your option expires. You hold an American call, strike $48, and the stock is at $50, so you are $2 in the money. You will not need the shares and you have no special view on the stock. What is the best action today? Choose the best answer.

(a) Exercise now to lock in the $2 gain before the stock can fall.

(b) Exercise now because the option is in the money, and in-the-money options should be exercised.

(c) Either hold the call or sell it in the market, because the call is worth at least $S - Ke^{-rT} =$ $3.42, which is more than the $2 you get by exercising.

(d) It does not matter, exercising and selling give the same cash today.

ConcepTest 2. A classmate says: "Put-call parity, $c + Ke^{-rT} = p + S$, only holds if the Black-Scholes assumptions are true, so in a crisis it can break." Is this right? Choose the best answer.

(a) Yes, parity is a Black-Scholes formula, so it needs lognormal prices and constant volatility.

(b) Yes, parity is a forecast of where prices should go, so it can be wrong.

(c) No, parity is a model-free no-arbitrage identity for European options on the same stock and date, so it holds whatever the price process, as long as the trade can be done.

(d) No, parity always holds exactly in every real market with no exceptions.

ConcepTest 3. A European call has strike \$48, the stock is at \$50, $r = 6$ per cent and $T = 6$ months, so $Ke^{-rT} = \$46.58$. Someone quotes the call at \$1.80. Without any option model, what can you say? Choose the best answer.

(a) Fine, the call is out of the money on a discounted basis, so \$1.80 is plausible.

(b) Fine, a call can never be worth more than its \$2 intrinsic value, and $\$1.80 < \$2$.

(c) The quote is too low: the call must be worth at least $S - Ke^{-rT} = \$3.42$, so \$1.80 is an arbitrage.

(d) You cannot say anything without the volatility.

Phase 2 – The desk problem

~20 MIN

The equity-derivatives desk: bounds, parity and the arbitrage

The setting. You are on the equity-derivatives desk at a London bank. A US large-cap stock pays **no** dividend before your options expire, except in part (e), where a dividend is announced. Quotes are per share. All money is in US dollars (\$) and all rates are continuously compounded.

Given (parts (a) to (d)). $S = \$50$, $K = \$48$, $r = 6$ per cent per year, $T = 6$ months (0.5 years). You may use the pre-computed constants $e^{-rT} = e^{-0.03} = 0.970446$ and $Ke^{-rT} = 48 \times 0.970446 = \$46.58$.

(a) No-arbitrage bounds. Compute the largest and smallest possible price for the European call and the European put. Use this skeleton:

- Call: lower $= \max(S - Ke^{-rT}, 0)$; upper $= S$.
- Put: lower $= \max(Ke^{-rT} - S, 0)$; upper $= Ke^{-rT}$.

State each of the four numbers, then say in one sentence what the call lower bound means.

(b) Put-call parity check. The market now quotes the call at $c = \$5.70$ and the put at $p = \$2.00$. Is parity violated, and by how much? Use the identity

$$c + Ke^{-rT} = p + S.$$

(1) Work out the left side (call plus the present value of the strike).

(2) Work out the right side (put plus stock).

(3) State the difference per share, and say which side is dear.

(c) Build the arbitrage. The call is overpriced relative to put plus stock, so sell the dear side and buy the cheap side. This trade is a **conversion**: short the call, long the put, long the stock, and borrow the present value of the strike.

(1) Write the four cash flows at $t = 0$ and add them up. How much do you bank today?

(2) Check the position at expiry T. Show that "long stock + long put − short call" is worth exactly $K = \$48$ whatever the final stock price S_T, so the loan is repaid and the terminal net is zero.

(3) State the riskless profit per share.

(d) Early-exercise argument, American call, no dividend. Show that selling the American call beats exercising it. Compare the intrinsic value you get by exercising now with the European lower bound you can realise by selling. By how much does selling win, and what does that gap equal?

(e) The dividend exception. Now the stock will pay a cash dividend of $D = \$2.00$ and goes ex-dividend in a moment. After the ex-dividend date there are $\tau = 3$ months (0.25 years) left to expiry. The strike is still $K = \$48$ and $r = 6$ per cent.

(1) Use the rule of thumb that early exercise just before an ex-dividend date can be optimal only if the dividend is large enough, that is if $D > K(1 - e^{-r\tau})$. Work out the critical dividend $K(1 - e^{-r\tau})$ and compare it with $D = \$2.00$.

(2) Take the cum-dividend stock price as \$50. If you exercise just before the ex-dividend date you get $S_{\text{cum}} - K$ and you start to receive the dividend. If instead you hold through the date, the price falls by about the dividend and the call you keep is worth at least $(S - D) - Ke^{-r\tau}$. Compute both and say which wins.

Extension (for fast groups, pick either).

E1. American parity is a band, not an equality. For American options on a non-dividend stock, parity loosens to $S - K \leq C - P \leq S - Ke^{-rT}$. State the two ends of the band with our numbers, and the width $K - Ke^{-rT}$.

E2. A discrete dividend lowers the call bound. If a dividend $D = \$1.00$ is paid at $t = 3$ months (0.25 yr), the call lower bound falls to $c \geq S - De^{-rt} - Ke^{-rT}$. Compute the new bound, and say what the drop from \$3.42 equals.

Phase 3 – The case ~25 MIN

Palm and 3Com, March 2000 (United States): the negative stub

How to run this. Read the story with your group. The ending is hidden on purpose. Your group must take a position on the four questions and hold your decision. Do not search the internet. Use only what you know from the lecture. I will reveal what happened only after every group has answered. All money is in US dollars (\$).

The story. In early 2000, near the very top of the dot-com boom, the US networking company **3Com** owned a fast-growing subsidiary, **Palm**, the maker of the Palm Pilot handheld organiser. 3Com decided to separate them in two steps. On **2 March 2000**, 3Com floated about **5 per cent** of Palm to the public in an initial public offering (IPO). It kept the other **roughly 95 per cent** and announced that, later in the year, it would hand those remaining Palm shares to its own 3Com shareholders. Under the plan, each 3Com share would entitle the holder to **about 1.5 Palm shares**.

Palm's IPO was priced at **\$38 a share**. When it started trading, demand was wild: the price spiked to about **\$165** during the day before settling. Now think about the simple arithmetic a first-year student could do. If you own one 3Com share, you own a slice of 3Com's profitable, cash-rich networking business *and* a claim on about 1.5 Palm shares. So one 3Com share should be worth at least 1.5 times the Palm price, plus the value of the rest of 3Com.

The story stops here.

Four discussion questions (answer all four in your group; take a position):

1. **The no-arbitrage relationship.** Write in words the lowest price one 3Com share should trade at, given the Palm price and the 1.5 ratio. Why can the "rest of 3Com" never be a negative number?
2. **Do the sum.** Suppose after day one Palm closes at about $95 and 3Com closes at about $82. Compute the implied value of "the rest of 3Com" (the **stub**) per 3Com share. What do you find?
3. **Design the arbitrage.** A negative stub means the market is pricing a profitable business at less than zero. Design the textbook arbitrage that should erase it.
4. **Why might it fail?** The arbitrage looks riskless. Name the real-world reason it might *not* work, and link it to something from Lecture 9. (Hint: to short Palm you must first borrow Palm shares. What if almost no one will lend them, or the borrow is extremely expensive?)

End of activity handout. Solutions are distributed after the block.