Risk Management and Derivatives | Lecture 10

No-Arbitrage Bounds and Put-Call Parity

Group activity solutions

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Constants used throughout: $S = \$50$, $K = \$48$, $r = 6$ per cent, $T = 0.5$ yr; $e^{-rT} = e^{-0.03} = 0.970446$; $Ke^{-rT} = 48 \times 0.970446 = \$46.5814$, which we round to \$46.58.

Phase 1: Concept checks

- **CT1: (c).** With no dividend it is never best to exercise the American call early. Exercising now gives only the \$2 intrinsic value. The call is worth at least $S - Ke^{-rT} = \$3.42$, which you capture by selling it. Exercising throws away the interest you could earn on the \$48 strike, plus any remaining time value, so you either hold the call or sell it.
- **CT2: (c).** Parity is a model-free no-arbitrage identity. It comes from two portfolios with identical payoffs at T, not from any pricing model, so it holds whatever the price process. The only escape hatch is a friction that blocks the arbitrage, not a wrong model. (Option (d) over-claims, because frictions can in fact let parity look broken; that is exactly today's case.)
- **CT3: (c).** The bound $c \geq S - Ke^{-rT} = \$3.42$ is forced by no-arbitrage alone (buy the call, short the stock, invest Ke^{-rT}). A quote of \$1.80 is below the floor, so it lets you earn a riskless profit and cannot stand. You did not need volatility or any model to say this.

Phase 2: Desk problem, fully worked

(a) No-arbitrage bounds

Intuition. A call can never be worth more than the share itself, and never less than the share minus the discounted strike. A put can never be worth more than the present value of the strike, and never less than zero. These limits come from no-arbitrage alone, with no option model.

Calculation.

- Call lower bound $= \max(50 - 46.58, 0) = \$\mathbf{3.42}$. Call upper bound $= S = \$\mathbf{50.00}$.
- Put lower bound $= \max(46.58 - 50, 0) = \max(-3.42, 0) = \$\mathbf{0.00}$. Put upper bound (European) $= Ke^{-rT} = \$\mathbf{46.58}$.

The call lower bound \$3.42 is the call's discounted forward intrinsic value: the call is worth at least the stock today minus the present value of the strike, and never more than the share itself.

(b) Put-call parity check

Intuition. Parity says two portfolios with the same payoff at expiry must cost the same today. "Call plus the present value of the strike" and "put plus stock" both pay $\max(S_T, K)$ at T, so they must be priced equally. If they are not, one side is dear.

Calculation.

(1) Left side $= c + Ke^{-rT} = 5.70 + 46.58 = \$\mathbf{52.28}$.

(2) Right side $= p + S = 2.00 + 50.00 = \$\mathbf{52.00}$.

(3) Difference $= 52.28 - 52.00 = +\$\mathbf{0.28}$ per share (exactly \$0.2814). Parity *is* violated. The call side is dear; the put-plus-stock side is cheap.

Cross-check: the fair call implied by the put is $c^* = p + S - Ke^{-rT} = 2.00 + 50.00 - 46.58 = \$5.42$, and $5.70 - 5.42 = \$0.28$. Same number, so the work is consistent.

(c) Build the arbitrage

Intuition. Sell the dear side, buy the cheap side, and lock the gap in cash today. The combined holding pays a fixed \$48 at expiry whatever the stock does, so you fund it with a loan of exactly Ke^{-rT} and walk away with the mispricing in your pocket, risk free.

Calculation. Cash flow at $t = 0$ (the conversion):

trade	cash today
Sell (write) the call	+\$5.70
Buy the put	−\$2.00
Buy the stock	−\$50.00
Borrow Ke^{-rT}	+\$46.58
Net cash banked today	+\$**0.28**

At expiry T, the holding "long stock + long put − short call" is worth exactly $K = \$48$ whatever S_T is:

- If $S_T \geq 48$: the call you wrote is exercised against you, you deliver the stock and receive $K = 48$; the put expires worthless. Net $= 48$.
- If $S_T < 48$: you exercise your put and sell the stock for $K = 48$; the call expires. Net $= 48$.

You repay the loan of exactly $K = \$48$, so the terminal net is zero. The whole profit is the +**\$**0.28 **per share** banked today, riskless. Invested at r to expiry it grows to $0.2814 \times e^{0.03} = \$0.29$ per share. The clean answer is **about \$0.28 of riskless profit per share**.

Reversal note: had the call instead been under-priced, $c + Ke^{-rT} < p + S$, you would do the mirror trade (a reversal): buy the call, sell the put, short the stock, lend Ke^{-rT}.

(d) Early-exercise argument, American call, no dividend

Intuition. Exercising hands over the strike early and throws away the option's remaining value. Selling captures the full market price, which is at least the European floor. So you never exercise a non-dividend call early; you hold it or sell it.

Calculation.

- Exercise now gives intrinsic value $S - K = 50 - 48 = \$\mathbf{2.00}$.
- But the call (American or European, no dividend) is worth at least its European lower bound $S - Ke^{-rT} = \$\mathbf{3.42}$, realised by selling it in the market.
- Selling beats exercising by at least $3.42 - 2.00 = \$\mathbf{1.42}$ per share. That \$1.42 is exactly $K - Ke^{-rT} = 48 - 46.58$, the interest you forgo on the strike by paying it early, plus any remaining time value.

With no dividend it is never optimal to exercise the American call early. So an American call on a non-dividend stock is worth the same as the European call.

(e) The dividend exception

Intuition. The only reason to ever exercise an American call early is to capture a dividend. You give up the interest you would save by waiting, in exchange for the cash dividend on the shares. This pays off only when the dividend is larger than that forgone interest.

Calculation.

(1) Critical dividend $= K(1-e^{-r\tau}) = 48\times(1-e^{-0.06\times 0.25}) = 48\times(1-0.985112) = 48\times 0.014888 =$ $\mathbf{0.7146}$. Here $D = \$2.00 > \$0.7146$, so the test says early exercise can be optimal.

(2) Show it with cash:

- *Exercise just before the ex-dividend date* (cum-dividend stock at \$50): you get $S_{\text{cum}} - K = 50 - 48 = \$\mathbf{2.00}$, and you also start to receive the \$2 dividend on the shares you now own.
- *Hold through the date instead*: the price drops by the dividend to about $50 - 2 = \$48$, and the call you keep is then worth at least $(50-2) - Ke^{-r\tau} = 48 - 48 \times 0.985112 = 48 - 47.285 = \$\mathbf{0.7146}$.

Exercising (\$2.00 of intrinsic, plus you own the dividend) beats the post-dividend call floor of \$0.71 by $2.00 - 0.71 = \$\mathbf{1.29}$ per share. So just before a large dividend, early exercise of the American call can be the right move.

The dividend is the only reason ever to exercise an American call early: you give up time value to capture a dividend that is larger than the interest you save by waiting.

Extension E1: American parity is a band

Intuition. The early-exercise feature of the American put turns the tight European equality into a band. The call and put need no longer line up exactly; their difference must only sit inside a range.

Calculation. For American options on a non-dividend stock, $S - K \le C - P \le S - Ke^{-rT}$. With our numbers: lower $= 50 - 48 = \$\mathbf{2.00}$, upper $= 50 - 46.58 = \$\mathbf{3.42}$. So $C - P$ must sit between \$2.00 and \$3.42, a band of width $K - Ke^{-rT} = \$\mathbf{1.42}$.

Extension E2: a discrete dividend lowers the call bound

Intuition. A dividend transfers value from the call holder to the share holder, so the call's floor drops by exactly the present value of the dividend.

Calculation. If a dividend $D = \$1.00$ is paid at $t = 3$ months (0.25 yr), the call lower bound falls to $c \ge S - De^{-rt} - Ke^{-rT} = 50 - 1 \times 0.985112 - 46.58 = \$\mathbf{2.43}$ (down from \$3.42). The drop is exactly the present value of the dividend, $1 \times e^{-0.015} = \$\mathbf{0.985}$.

The one idea behind every part: no-arbitrage alone pins option prices to exact bounds and to the parity identity $c + Ke^{-rT} = p + S$. A model is needed only to place the price inside those limits, not to set the limits.

Phase 3: Case, what happened

You were stopped just after the Palm IPO on 2 March 2000, with the Palm price soaring and 3Com about to be repriced. Here is what happened next.

On the first day of trading, **Palm closed at \$95.06** and **3Com closed at \$81.81**. Do the sum the way Lamont and Thaler did. Each 3Com share carried a claim on about 1.5 Palm shares, worth about $1.5 \times 95.06 =$ **\$143** at the close. But 3Com itself was trading at only about **\$82**. So the market was valuing "the rest of 3Com" at roughly **minus \$63 per share**, which across all the shares came to about **minus \$22 billion**. 3Com was a profitable company holding billions in cash, yet the market said its non-Palm business was worth less than nothing. That is impossible under no-arbitrage. There was, in the famous phrase, a **negative stub**.

You could get 150 Palm shares two ways: buy 150 Palm shares directly, or buy 100 3Com shares and end up with about 150 Palm shares *plus* a stake in 3Com's other businesses and cash. The second bundle is strictly more, yet it cost less. The market literally could not add and subtract.

So why did smart traders not arbitrage it away in minutes? Because the trade needed you to **short**

Palm, and to short you must borrow the shares. Only about 5 per cent of Palm was floating, demand to borrow was enormous, and the borrow was either impossible to find or astonishingly expensive. Short interest in Palm later reached about **147.6 per cent** of the float, meaning more than the entire floating supply had been sold short. This is exactly the **hard-to-borrow / short-sale constraint** you met in Lecture 9. The arbitrage was visible to everyone and executable by almost no one, so the mispricing survived for **weeks**, only closing as the spin-off date approached and the short-sale pressure eased. Owen Lamont and Richard Thaler wrote this up in "Can the Market Add and Subtract? Mispricing in Tech Stock Carve-Outs" (Journal of Political Economy, 2003).

The lesson, stated plainly. No-arbitrage bounds and put-call parity are iron laws *only* when the offsetting trade can actually be done. Remove the ability to short cheaply and the law stops binding. Bounds describe what prices must be when arbitrage is free; limits to arbitrage describe when they are not. This is exactly the corridor you met in the lecture: a "parity violation" can be the priced cost of a short-sale constraint, not a free lunch.

No-arbitrage gives exact bounds and the parity identity, but only when the arbitrage can be executed. Remove cheap shorting, as in Palm and 3Com, and a visible mispricing can survive for weeks.

End of solutions.