Risk Management and Derivatives | Lecture 3

Hedging Strategies Using Futures

Group activity solutions

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Phase 1: Concept checks

- **CT1: (b).** The hedge fixed GoldHand's gold cost at the old high price. Then gold fell, and jewellery prices fell with it. GoldHand must still pay the old high cost while selling at the new low price, so its margin is squeezed. Sunmark did not hedge, so it buys cheaper gold and matches the lower prices. The hedge fixed GoldHand's absolute cost but left it worse off than its unhedged rival. This is the competitor trap.
- **CT2: (b).** A hedge removes a fraction ρ^2 of the variance, not ρ. Here $\rho = 0.7$, so $\rho^2 = 0.7 \times 0.7 = 0.49$, about 49 per cent. The leftover $1 - \rho^2 = 0.51$ is the basis risk the futures cannot track. A correlation of 0.7 sounds high, but it only buys about half the variance reduction. You square the correlation.
- **CT3: (b).** Diesel and crude do not move together perfectly. After the hedge, the firm still faces basis risk, the changing gap between the diesel price and the crude futures price. The hedge swapped a large price risk for a smaller basis risk. It did not make the cost certain.

Phase 2: Desk problem, fully worked

(a) Direction and the basis idea

Intuition. You must buy fuel later, so a price rise hurts you. You go *long* futures: if fuel rises, the futures gain offsets the higher cost. A hedge does not fix the spot price you pay; it fixes the *entry futures price* plus the *closing basis*.

Calculation.

(1) **Long.** You are a buyer, hurt by a rise, so you buy futures now.

(2) Closing basis $b_2 = S_2 - F_2 = 3.05 - 2.90 = \$0.15$.

(3) Effective cost $= S_2 - (F_2 - F_1) = 3.05 - (2.90 - 2.50) = 3.05 - 0.40 = \$2.65$.
Check: $F_1 + b_2 = 2.50 + 0.15 = \$2.65$. The two agree.

(4) Unhedged you pay \$3.05; hedged you pay \$2.65, so the hedge saved $3.05 - 2.65 = \$0.40$ per gallon. The only leftover uncertainty is the **basis** b_2, the gap between the diesel spot and the futures. You knew F_1 when you signed; you did not know b_2. The hedge swapped a large price risk for this smaller basis risk.

(b) The minimum-variance hedge ratio

Intuition. The volatility ratio σ_S/σ_F rescales the size: if the futures moves more than your fuel, each futures gallon does *more* than one gallon of work, so you need fewer of them. The correlation ρ then discounts the ratio, because diesel and heating oil do not move together perfectly.

Calculation.

$$h^* = \rho \frac{\sigma_S}{\sigma_F} = 0.9 \times \frac{0.04}{0.05} = 0.9 \times 0.8 = \mathbf{0.72}.$$

Read it: the futures is a bit more volatile ($\sigma_F > \sigma_S$), so the volatility ratio is 0.8, below 1. The high correlation 0.9 keeps h^* close to that ratio. So you hold about 0.72 futures gallons for each gallon of diesel.

(c) Number of contracts

Intuition. Scale the hedge ratio up from one gallon to your whole exposure, then divide by the size of one contract.

Calculation.

$$\frac{Q_A}{Q_F} = \frac{4{,}200{,}000}{42{,}000} = 100, \qquad N^* = h^* \frac{Q_A}{Q_F} = 0.72 \times 100 = \mathbf{72} \text{ contracts, long.}$$

You buy 72 heating-oil futures contracts.

(d) Effectiveness

Intuition. A hedge removes a fraction ρ^2 of the variance, *not* ρ. The square is what matters.

Calculation.

$$\rho^2 = 0.9 \times 0.9 = 0.81 = \mathbf{81}\%.$$

The hedge removes about 81 per cent of the variance. The leftover $1 - \rho^2 = 0.19$, about **19 per cent**, is the **basis risk**: the part of the diesel price the heating-oil futures cannot track.

(e) The equity hedge

Intuition. Beta is a hedge ratio. A portfolio with beta 1.2 moves 1.2 per cent for each 1 per cent of the market, so you need 1.2 contracts' worth of index for each contract's worth of portfolio to cancel the market move.

Calculation.

(1) Number of contracts:

$$N = \beta \frac{V_P}{V_F} = 1.2 \times \frac{50{,}000{,}000}{50{,}000} = 1.2 \times 1{,}000 = \mathbf{1{,}200} \text{ contracts, short.}$$

You short the index, because you fear a fall.

(2) **Stress test, 10 per cent fall** (index 5,000 to 4,500, a move of 500 points):

- Portfolio loss $\approx \beta \times 10\% \times V_P = 1.2 \times 0.10 \times 50{,}000{,}000 =$ £6,000,000.
- Short-futures gain $= 1{,}200 \times$ £10 $\times 500 =$ £6,000,000.

The two nearly cancel: the loss on the shares is met by the gain on the short futures. The market risk is removed.

(3) **Target beta of 0.6:**

$$N = (\beta - \beta_{\text{target}}) \frac{V_P}{V_F} = (1.2 - 0.6) \times 1{,}000 = 0.6 \times 1{,}000 = \mathbf{600} \text{ contracts, short.}$$

You short only 600, so you keep half the market exposure. You still hold the stock-picking (residual) part of the portfolio; you have only trimmed the market part.

Extension: funding the hedge

Intuition. A futures hedge is a cash commitment, not a free insurance policy. When the hedge moves against the futures leg, you pay cash now, even if the business is doing fine.

Calculation. Keep the full short of 1,200 contracts. The market *rises* 10 per cent (index 5,000 to 5,500, a move of 500 points).

(1) Short-futures loss $= 1{,}200 \times$ £10 $\times 500 =$ £6,000,000. This is paid **in cash now**, as variation margin out of your account.

(2) Your shares are up about £6,000,000, but that gain is **on paper**: you have not sold the shares, so no cash has come in.

(3) So the hedge demands £6,000,000 of cash in a week when the business is doing fine. This is the Lecture 2 lesson: margin converts a price risk into a **cash (liquidity) need**. Size the hedge so the worst-week margin call cannot exceed your funding line.

The one idea behind both halves: a futures hedge does not remove risk. It swaps a large price risk for a smaller basis risk (Part One) and a funding need (the extension). The diesel desk and the London fund are two faces of the same lesson.

Phase 3: Case, what happened

You were stopped in the summer of 2008 with oil near its peak and Southwest's hedges hugely valuable. Here is what happened next.

Oil did not climb to $200. It collapsed. From above about $147 per barrel in July 2008, crude fell to about $33 per barrel by December 2008, as the global financial crisis cut demand.

The price dropped far below the locked-in levels. So Southwest had to record large accounting charges (mark-to-market) on the part of its hedge book that covered future periods. These charges did not use cash. The third-quarter 2008 charge was about $247 million. Southwest reported its first quarterly loss in more than 17 years.

A point to keep clear. In that same third quarter the hedges still produced large cash gains on the fuel Southwest actually burned (cash settlement gains of about $448 million, up from about $189 million a year earlier). The headline loss came from the accounting mark-to-market on future-period hedges, not from the hedges failing on current fuel. This is the lecture point that a hedge moving against you can show an accounting loss that is hard to explain.

The competitor trap, in reverse. For a while, Southwest's locked-in fuel cost was now above the spot price its unhedged rivals were paying. When oil crashed, the unhedged rivals bought cheap fuel, while Southwest was still tied to higher hedged prices. The hedge fixed Southwest's absolute cost but flipped its position relative to rivals. This is exactly Concept Check 1, seen from the other side. When the input price falls, the hedger can end up worse off than the unhedged rival.

The hindsight lesson. Over the full cycle, the disciplined programme had still saved billions of dollars. The 2008 accounting loss did not make years of sound hedging wrong. The right question was never "did the price go the way the hedge bet?" The right question was "when we signed, did this hedge remove a risk we could not afford to carry?" By that test the programme was sound. You judge a hedge by the risk it removed when you signed, not by the price that later happened.

Both lessons land here at once: the competitor trap can flip on you when prices fall, and a hedge must be judged by the risk it removed, not by hindsight on the price.

End of solutions.