Risk Management and Derivatives | Lecture 4

Interest Rates

In Class Group Activity

· SJTU Summer School 2026 · Fatih Kansoy

How this block runs. You work in your home group of four or five, textbook open. Roles rotate today: **Scribe** (writes the answer), **Reporter** (speaks at the microphone), **Sceptic** (must raise one objection), **Timekeeper**. All money today is in US dollars ($) and pounds sterling (£). All rates are continuously compounded unless stated.

- **Phase 1, Concept checks (~10 min).** For each question you vote on your own first. Then you talk for about 90 seconds with the people next to you, and vote again. Do not talk before the first vote.
- **Phase 2, Desk problem (~20 min).** One problem in parts. Your Reporter presents one part when the microphone reaches your group. Then we solve it together, part by part.
- **Phase 3, Case (~25 min).** You read one real story. The ending is hidden on purpose. You answer four questions and take a position. Then we reveal what happened.

A short word list for this block. A **zero rate** $z(T)$ is the interest rate on money you lend now and get back, all at once, at time T. A **discount factor** is the value today of $1 paid at time T; with continuous compounding it is $e^{-z(T)T}$. **Bootstrapping** means building the rate for each maturity one step at a time, using the rates you already know for the earlier cash flows. A **forward rate** is a future borrowing or lending rate you can lock in today, found only from today's curve. **Duration** measures how much a bond's price moves when its yield moves; a longer duration means a larger price swing.

Phase 1 – Concept checks

~10 MIN

ConcepTest 1. You hold a 10-year US Treasury STRIPS, that is a zero-coupon US Treasury bond. It pays nothing until the end, then pays its face value once at year 10. Market interest rates then rise from 3 per cent to 5 per cent. A classmate says: "It is a US Treasury. The government always pays, so it cannot lose money." Is the classmate right? Choose the best answer.

(a) Yes. A US Treasury is safe, so its value cannot fall.

(b) No. The Treasury will not default, so you still get your face value at year 10. But its market price falls sharply when rates rise, because new bonds now pay more. A zero-coupon bond's duration equals its maturity, so this bond has a duration of 10. A 2 percentage point rise then cuts its market value by roughly 18 to 20 per cent. "Risk-free" means free of default risk, not free of interest-rate (price) risk.

(c) The price falls, but only if the government's credit rating is cut.

(d) The price is not affected as long as you hold the bond, because you still get the face value at the end.

ConcepTest 2. From today's zero curve you work out that the one-year interest rate, starting one year from now (the forward rate), is 4.3 per cent. A trader says: "So the market is predicting that the one-year rate will be 4.3 per cent next year." Is the trader right? Choose the best answer.

(a) Yes, the forward rate is the market's best forecast of the future rate.

(b) No. The 4.3 per cent is the breakeven rate you can lock in today by no-arbitrage. You get it by combining borrowing and lending across the curve. It need not equal anyone's forecast of next year's rate.

(c) Yes, but the forecast is shaded down a little to be safe.

(d) It is just an accounting number with no real meaning.

ConcepTest 3. For decades LIBOR was the world's main interest-rate benchmark. It was then replaced by SOFR. A classmate says: "LIBOR was scrapped because some bank traders were caught manipulating it." Is that the main reason? Choose the best answer.

(a) Yes, the rigging scandal was the whole reason.

(b) The manipulation was real and serious. But the deeper, structural reason is that after 2008 banks largely stopped lending to each other unsecured (without collateral). LIBOR asked a panel of banks "at what rate could you borrow unsecured?", but that market had mostly stopped trading, so even honest answers were guesses. SOFR is built from real overnight repo trades (short-term loans backed by US Treasuries), so there is nothing to estimate.

(c) No, LIBOR was replaced only because regulators wanted a government-run rate.

(d) No, LIBOR worked fine and was dropped for political reasons.

Phase 2 – The desk problem

~20 MIN

A bank builds the curve, then measures its risk

The setting. One problem in parts. All money is in US dollars ($). All rates are continuously compounded unless stated. You work on the rates desk of a US bank. First you build a small piece of the curve. Then you use it to measure, and then hedge, the bank's interest-rate risk.

(a) Compounding warm-up. A one-year deposit is quoted as 4 per cent with *semiannual* compounding (interest is added twice a year).

(1) Work out the effective annual rate, $(1 + 0.04/2)^2 - 1$.

(2) Convert the 4 per cent semiannual rate to a continuously compounded rate, using $R_c = 2\ln(1 + 0.04/2)$. Is the continuous quote larger or smaller than 4 per cent? Say why in one sentence.

(b) Bootstrap a zero rate. From short deposits you already know $z(0.5) = 3.0$ per cent and $z(1.0) = 3.4$ per cent. A 1.5-year US Treasury pays a 4 per cent coupon semiannually. Per $100 of face value it pays $2 at 0.5 years, $2 at 1.0 years, and $102 at 1.5 years. It trades at $100.40.

(1) Discount the first two coupons off the rates you already know: $2e^{-0.03\times0.5} + 2e^{-0.034\times1.0}$.

(2) Subtract that from the price to find the present value of the final $102 payment.

(3) Solve $102\,e^{-1.5\,z(1.5)} =$ (that value) for $z(1.5)$. State $z(1.5)$ as a percentage.

(c) The forward rate. Using $z(1.0) = 3.4$ per cent and your $z(1.5)$ from part (b), work out the six-month rate that starts in one year:

$$f(1, 1.5) = \frac{z(1.5) \times 1.5 - z(1.0) \times 1.0}{1.5 - 1.0}.$$

Is the forward above or below the zero rates? Say why in one sentence.

(d) Interest-rate risk, measured. A regional bank holds a bond portfolio worth \$5,000,000,000 (\$5 billion) with duration $D_P = 6$ years. The central bank raises rates and the whole curve rises by 2 percentage points ($\Delta y = +0.02$).

(1) Work out the loss, using $\Delta B \approx -P \times D_P \times \Delta y$.

(2) What is that loss as a percentage of the portfolio?

(3) If the bank's equity (its capital buffer) is smaller than this loss, what has happened to the bank once these bonds are valued at market prices?

(e) The duration hedge. The bank hedges with 10-year US Treasury note futures. Each contract is \$100,000 of face value, and the cheapest-to-deliver bond has duration $D_F = 7.5$.

(1) How many contracts do you trade, and long or short? Use $N = \dfrac{P \times D_P}{F \times D_F}$.

(2) **Stress test.** Rates rise by 50 basis points ($\Delta y = +0.005$). Work out the loss on the portfolio and the gain on the short futures. Do they cancel?

Extension (for fast groups): convexity. Take a 10-year zero-coupon US Treasury, \$100 face value, yield 4 per cent. Its price is $B = 100\,e^{-0.04\times 10}$, and its duration is 10 (a zero's duration equals its maturity). Rates now rise by 1 percentage point ($\Delta y = +0.01$).

(1) Work out the straight-line (duration) estimate of the price change, $\Delta B \approx -B \times D \times \Delta y$.

(2) Work out the exact new price, $B' = 100\,e^{-0.05\times 10}$, and the exact change.

(3) Compare the two. Did the bond fall more or less than the straight line said? The gap is **convexity**. Does it help or hurt the bondholder?

Phase 3 – The case

~25 MIN

Silicon Valley Bank, March 2023 (United States)

How to run this. Read the story with your group. The ending is hidden on purpose. Your group must take a position on the four questions and hold your decision. Do not search the internet. Use only what you know from the lecture. I will reveal what happened only after every group has answered. All money is in US dollars (\$).

The story. You are on the risk team at a US bank in early 2023. Look back at how the bank got here.

During 2020 and 2021, deposits grew very fast. The bank served technology start-ups, and those start-ups had raised large sums from investors. They parked their cash at the bank.

A bank must do something with deposits. This bank bought a large pile of US government bonds: Treasury bonds and government-backed mortgage bonds. These bonds are about as safe as any asset on earth. The US government has never failed to pay them, so the default risk was close to zero.

But the bank bought *long-dated* bonds, with many years left to run. Long-dated bonds have high duration. Duration measures how much a bond's price moves when interest rates move. The longer the duration, the larger the price swing.

Through 2022 and early 2023, the US central bank raised interest rates very fast. Rates went from near zero to about 4.5 to 4.75 per cent in about one year. When market rates rise, the price of

existing bonds falls. New bonds now pay more, so older low-rate bonds are worth less.

The bank's bonds were still perfectly safe from default. Every coupon would still be paid. But their *market price* had fallen by a large amount: about $15 to 17 billion of losses on paper. That figure was close to the whole equity (the capital buffer) of the bank.

This is how the danger stayed hidden. The bank held most of these bonds in a "held to maturity" bucket. Under the accounting rules, bonds in that bucket are shown at their original cost, not at today's lower market price. So the loss was real, but it did not show up in the headline numbers. It stayed hidden, as long as the bank never had to sell.

Then deposits started to leave. Start-up clients were spending their cash quickly and not raising new money, so they drew down their accounts. To pay them, the bank needed cash. To get cash, it had to sell bonds. And selling meant turning the hidden paper loss into a real, reported loss.

Stop on about 8 March 2023. On that day the bank announces three things at once:

- It has sold about $21 billion of bonds.
- This sale crystallised a loss of about $1.8 billion.
- It will try to raise about $2.25 billion of new capital to repair the balance sheet.

Most of the bank's depositors are start-ups. Their balances sit far above the $250,000 insured limit, so most of their money is *not* protected if the bank fails. They talk to each other constantly, and many share the same investors.

The news is now public. Your group must decide what happens next.

The story stops here.

Four discussion questions (answer all four in your group; take a position):

1. **Safe bonds, large loss.** The bonds were US government bonds that will never default. So why did they lose so much value?
2. **Two different risks.** What is the difference between default risk and interest-rate risk? Which one hurt this bank?
3. **The hidden loss.** The "held to maturity" accounting let the problem grow without showing in the headline numbers. Why did selling the bonds change everything?
4. **Should they have hedged?** Should the bank have hedged its duration earlier, for example with bond futures or swaps? What would that protection have cost, and why might the bank have skipped it?

End of activity handout. Solutions are distributed after the block.