Risk Management and Derivatives | Lecture 4

Interest Rates

Group activity solutions

· SJTU Summer School 2026 · Fatih Kansoy

Phase 1: Concept checks

- **CT1: (b).** A US Treasury has no default risk, so you still get your face value at year 10. But its market price still falls when rates rise, because new bonds now pay more. A zero-coupon bond's duration equals its maturity, so this 10-year bond has a duration of 10. A 2 percentage point rise then cuts its market value by roughly 18 to 20 per cent. "Risk-free" means free of default risk, not free of interest-rate (price) risk.
- **CT2: (b).** The 4.3 per cent is a breakeven, not a forecast. You build it only from today's zero rates, by combining borrowing and lending across the curve. No view about next year goes in. So it need not equal anyone's forecast.
- **CT3: (b).** The rigging was real and serious. But the deeper reason is structural: after 2008 banks largely stopped lending to each other unsecured (without collateral). That market mostly stopped trading, so even honest LIBOR answers were guesses. SOFR fixes this because it is built from real overnight repo trades (short-term loans backed by US Treasuries).

Phase 2: Desk problem, fully worked

(a) Compounding warm-up

Intuition. The same growth can be quoted at different compounding frequencies. The more often you compound, the smaller the quoted rate needs to be for the same end result. So the continuous quote is the smallest of all for the same growth.

Calculation.

(1) Effective annual rate $= (1 + 0.04/2)^2 - 1 = 1.02^2 - 1 = 0.0404 = \mathbf{4.04}\%$.

(2) Continuous rate $= 2\ln(1 + 0.04/2) = 2\ln(1.02) = 2 \times 0.019803 = 0.039605 = \mathbf{3.96}\%$. The continuous quote (3.96 per cent) is *smaller* than the 4 per cent semiannual quote, because compounding more often does more of the work, so the rate itself can be lower for the same growth.

(b) Bootstrap a zero rate

Intuition. A bond is a bundle of dated cash flows. Discount the early coupons off rates you already know. Whatever value is left must come from the final payment, and that one equation gives the last unknown zero rate.

Calculation.

(1) Discount the first two coupons:

$$2e^{-0.03\times0.5} + 2e^{-0.034\times1.0} = 2(0.98511) + 2(0.96657) = 1.97022 + 1.93314 = 3.90337.$$

(2) Present value of the final \$102 payment:

$$100.40 - 3.90337 = 96.49663.$$

(3) Solve for the last zero rate:

$$102\, e^{-1.5\, z(1.5)} = 96.49663 \;\Rightarrow\; e^{-1.5\, z(1.5)} = 0.946045,$$

$$z(1.5) = -\frac{1}{1.5}\ln(0.946045) = \mathbf{3.70}\%.$$

(c) The forward rate

Intuition. A forward rate is a breakeven you can lock in today. It comes only from today's zero curve, not from any forecast. On a rising curve the longer money costs more, so the forward sits above the zero rates.

Calculation.

$$\begin{aligned} f(1, 1.5) &= \frac{z(1.5)\times 1.5 - z(1.0)\times 1.0}{1.5 - 1.0} = \frac{0.037\times 1.5 - 0.034\times 1.0}{0.5} \\ &= \frac{0.0555 - 0.034}{0.5} = \frac{0.0215}{0.5} = \mathbf{4.30}\%. \end{aligned}$$

The forward 4.30 per cent sits above both $z(1.0) = 3.4$ per cent and $z(1.5) = 3.7$ per cent, because the curve is rising.

(d) Interest-rate risk, measured

Intuition. Duration links a yield move to a price move. For a small yield change, the loss is about price times duration times the yield change. A long duration turns a small rate move into a large loss.

Calculation.

(1) Loss $\approx -P \times D_P \times \Delta y = -5{,}000{,}000{,}000 \times 6 \times 0.02 = -\$\mathbf{600{,}000{,}000}$ (\$600 million).

(2) As a share of the portfolio: $600/5{,}000 = 0.12 = \mathbf{12}\%$.

(3) If the bank's equity is smaller than \$600 million, the bank is **insolvent** once these bonds are valued at market prices: the losses have eaten the whole capital buffer. This is the lesson of the day, measured.

(e) The duration hedge

Intuition. To hedge a bond portfolio with bond futures, match the money durations. You short enough futures so that the futures' price sensitivity equals the portfolio's. Then a yield move that hurts one side helps the other.

Calculation.

(1) Number of contracts:

$$N = \frac{P \times D_P}{F \times D_F} = \frac{5{,}000{,}000{,}000 \times 6}{100{,}000 \times 7.5} = \frac{30{,}000{,}000{,}000}{750{,}000} = \mathbf{40{,}000} \text{ contracts, short.}$$

You short the futures, because you lose on the bonds when rates rise.

(2) **Stress test, $+50$ basis points** ($\Delta y = +0.005$):

- Portfolio loss $= 5{,}000{,}000{,}000 \times 6 \times 0.005 = \$150{,}000{,}000$.
- Short-futures gain $= 40{,}000 \times 100{,}000 \times 7.5 \times 0.005 = 40{,}000 \times 3{,}750 = \$150{,}000{,}000$.

The two nearly cancel: the loss on the bonds is met by the gain on the short futures. The interest-rate risk is removed.

Extension: convexity

Intuition. The duration line is only a first guess. The true price-yield curve bends upward, so it always lies above the straight line. The bond falls a little less than the line says (and rises a little more). That gap always helps the bondholder.

Calculation. A 10-year zero-coupon US Treasury, \$100 face value, yield 4 per cent, duration 10.

(1) Price now: $B = 100\,e^{-0.04\times 10} = 100\,e^{-0.4} = \$67.03$. Straight-line estimate of the change: $\Delta B \approx -67.03 \times 10 \times 0.01 = -\$\mathbf{6.70}$.

(2) Exact new price: $B' = 100\,e^{-0.05\times 10} = 100\,e^{-0.5} = \$60.65$. Exact change: $60.65 - 67.03 = -\$\mathbf{6.38}$.

(3) The bond fell about \$0.32 *less* than the straight line said ($6.70 - 6.38 = 0.32$). That gap is **convexity**, and it always helps the bondholder: the bond falls a little less when rates rise, and rises a little more when rates fall.

The one idea behind every part: every promise has a date, and every date has a price (the zero curve). Interest-rate risk is the curve changing shape under a schedule of dated promises. "Safe from default" is not the same as "safe from interest-rate moves".

Phase 3: Case, what happened

You were stopped on about 8 March 2023, just after the bank announced the \$21 billion bond sale, the \$1.8 billion loss and the plan to raise \$2.25 billion. The bank is **Silicon Valley Bank (SVB)**. Here is what happened next.

The announcement frightened depositors. Venture investors told their start-ups to pull their money out at once. On **9 March 2023**, customers tried to withdraw about \$42 billion in a single day. This was the largest bank run on record.

The capital raise collapsed. The bank could not fund the outflow. On **10 March 2023**, California regulators closed the bank and appointed the FDIC as receiver. It was the second-largest US bank failure at that time, and the largest since 2008.

On **12 March 2023**, US regulators announced that all depositors, including those above the \$250,000 limit, would get full access to their money, to stop a wider panic.

The lesson, stated plainly. No government bond defaulted. Every bond would have paid in full at maturity. The bank failed purely because of **interest-rate risk (duration)** plus a deposit run. "Safe from default" is not the same as "safe from interest-rate moves". Rising rates cut the market value of the long-duration bonds. The "held to maturity" label hid that loss until a forced sale revealed it. Once the loss was revealed, uninsured depositors ran. This is Concept Check 1 and the lesson of the day, made real.

A note on the hedge (question 4). SVB could have cut its duration with interest-rate swaps or bond futures, the same duration hedge you built in part (e). It largely did not. Hedging has a cost and lowers reported income while rates are low, so the bank chose the income and kept the risk. That choice is exactly what failed.

A bond that is safe from default is not safe from interest-rate moves. A bond's price always moves opposite to its yield, and duration measures how much. That is the risk that brought down a bank holding only government bonds.

End of solutions.