Risk Management and Derivatives | Lecture 5

Pricing Forwards by Cost of Carry

In Class Group Activity

· SJTU Summer School 2026 · Fatih Kansoy

How this block runs. You work in your home group of four or five, textbook open. Roles rotate today: **Scribe** (writes the answer), **Reporter** (speaks at the microphone), **Sceptic** (must raise one objection), **Timekeeper**. Today's quotes are in US dollars ($), pounds sterling (£) and euros (€). All interest rates are continuously compounded unless stated.

- **Phase 1, Concept checks (~10 min).** For each question you vote on your own first. Then you talk for about 90 seconds with the people next to you, and vote again. Do not talk before the first vote.
- **Phase 2, Desk problem (~20 min).** One problem in parts. Your Reporter presents one part when the microphone reaches your group. Then we solve it together, part by part.
- **Phase 3, Case (~25 min).** You read one real story. The ending is hidden on purpose. You answer four questions and take a position. Then we reveal what happened.

A short word list for this block. The **spot price** S_0 is the price for delivery now. The **forward price** F_0 is the price agreed today for delivery at a future date T. **Cost of carry** is the net cost of holding the asset until T: the financing rate r, plus any storage cost u, minus any income yield q or convenience yield y. **Cash and carry** is the arbitrage trade of borrowing, buying the asset now, storing it, and delivering it into a forward. **Contango** means the forward sits above spot; **backwardation** means it sits below. **Convenience yield** is the value of holding the physical asset itself, which is high when the market is tight.

Phase 1 – Concept checks

~10 MIN

ConcepTest 1. Gold trades at a spot price of $2,000 per ounce. The one-year US dollar interest rate is 5 per cent (continuously compounded). A trader strongly believes gold will be worth $2,300 in one year. Ignoring storage, what is the correct one-year forward price of gold today?

(a) About $2,300, because that is the market's best forecast of the future spot price.

(b) About $2,103, because the forward price is the spot price carried forward at the interest rate.

(c) About $2,000, because forward and spot are always equal.

(d) It cannot be known, because nobody knows the future gold price.

ConcepTest 2. Crude oil spot is $80 per barrel. The six-month forward trades at $78 per barrel, which is below spot. What does this most likely tell you?

(a) The forward is mispriced and there is a free arbitrage profit.

(b) The market expects the oil price to fall over the next six months.

(c) Holders place a high value on having physical oil now (a high convenience yield), so the market is in backwardation.

(d) This is impossible; a forward can never be below spot.

ConcepTest 3. A non-dividend stock trades at \$100. The one-year rate is 4 per cent. A one-year forward is quoted at \$110. The fair forward is about \$104. What should an arbitrageur do?

(a) Nothing; the gap is only a theoretical idea, not a real trade.

(b) Buy the forward at \$110 and hope the stock rises.

(c) Short the forward at \$110, borrow \$100 and buy the stock now, then deliver in one year for a riskless gain of about \$6.

(d) Short the stock and buy the forward, locking a riskless loss.

Phase 2 – The desk problem

~20 MIN

The forward-pricing desk: five quotes before lunch

The setting. You are on the forward-pricing desk at a London bank this morning. Four clients want forward quotes before lunch, and a fifth screen is flashing a possible arbitrage. Work through them in order. All interest rates are continuously compounded.

(a) A non-income asset, fully scaffolded. Gold spot $S_0 = \$2{,}000$ per ounce. The six-month US dollar rate is $r = 5$ per cent. Gold pays no income; assume no storage cost for now. Quote the six-month forward price using $F_0 = S_0\, e^{rT}$.

(1) Pick the formula for an asset with no income: $F_0 = S_0\, e^{rT}$.

(2) Set $T = 0.5$, so $rT = 0.05 \times 0.5 = 0.025$.

(3) Find $e^{0.025} = 1.025315$.

(4) Multiply by S_0 to get the forward.

(b) A dividend-yield index, fully scaffolded. The S&P 500 index stands at $S_0 = 5{,}000$. The continuous dividend yield is $q = 1.5$ per cent. The nine-month dollar rate is $r = 5$ per cent. Quote the nine-month index forward using $F_0 = S_0\, e^{(r-q)T}$.

(1) An income-paying asset uses $F_0 = S_0\, e^{(r-q)T}$.

(2) Set $T = 0.75$, $(r - q) = 0.05 - 0.015 = 0.035$, so $(r - q)T = 0.02625$.

(3) Find $e^{0.02625} = 1.026598$.

(4) Multiply by S_0.

(c) A currency forward, formula only. GBP/USD spot is $S_0 = 1.2700$ dollars per pound. The one-year US dollar rate is $r_d = 4.5$ per cent and the one-year sterling rate is $r_f = 4.75$ per cent. Treat the US dollar as the domestic currency. Using $F_0 = S_0\, e^{(r_d - r_f)T}$, quote the one-year forward. Is the pound at a forward premium or discount? Say why in one sentence.

(d) Cash-and-carry arbitrage, no scaffold. Back to gold. Spot is still \$2,000 per ounce, the six-month rate is 5 per cent, and there is no storage cost. A counterparty offers to BUY gold from you six months forward at a forward price of \$2,100 per ounce.

(1) Is this forward too high or too low relative to fair value?

(2) Write down the trade that locks a riskless profit.

(3) Compute the profit per ounce, and the profit on a 100-ounce deal.

(e) Storage cost and implied convenience yield, no scaffold. Crude oil spot is $S_0 = \$80$ per barrel. The six-month dollar rate is $r = 5$ per cent. Storage costs run at $u = 3$ per cent per

year (paid like a continuous rate). The six-month oil forward is quoted at $F_0 = \$78$ per barrel. Oil is a consumption asset, so use $F_0 = S_0\, e^{(r+u-y)T}$. Back out the implied convenience yield y. Then say in one sentence whether the market is in contango or backwardation, and why.

Extension (for fast groups): valuing an existing forward. One month ago you went LONG a gold forward at a delivery price $K = \$1{,}950$ per ounce. Conditions today match part (a): spot \$2,000, six-month rate 5 per cent, the current six-month forward $F_0 = \$2{,}050.63$. What is the value today of your existing long forward contract? Use $f = (F_0 - K)\, e^{-rT}$. As a check, also compute it the direct way for a non-income asset, $f = S_0 - K\, e^{-rT}$, and confirm the two agree.

Phase 3 – The case

~25 MIN

London gold and New York gold futures, March 2020 (United Kingdom, United States and Switzerland)

How to run this. Read the story with your group. The ending is hidden on purpose. Your group must take a position on the four questions and hold your decision. Do not search the internet. Use only what you know from the lecture. I will reveal what happened only after every group has answered. All prices are in US dollars ($) per troy ounce.

The story. Gold trades in two big places at once. **London** is the home of the physical spot market: real bars sit in vaults, and the standard "good delivery" bar weighs about 400 ounces. **New York** is the home of the main gold future, the COMEX contract, which is settled by physical delivery in a New York vault, but in a different unit: its bars are 100 ounces or one kilo.

In normal times these two prices are tied together by arbitrage. If the New York future ever rose much above the London spot price, a trader would do a cash-and-carry: buy a bar in London, ship it across, and deliver it into the New York future for a riskless turn. That trade keeps the gap between the future and spot tiny. The market even quotes the gap directly, as the "exchange for physical", or EFP. For years the EFP was just a couple of dollars an ounce, the small cost of moving and financing the metal.

Then came March 2020. Covid hit the West. Two things happened at once. First, passenger flights were grounded across Europe and the United States: from 14 March the US restricted flights from Europe, and arrivals at London Heathrow fell sharply. This matters because gold normally rides in the cargo holds of passenger planes. With the planes on the ground, there was suddenly no easy way to fly metal from London to New York. Second, on 23 March the three big Swiss refineries in Ticino (Valcambi, Argor-Heraeus and PAMP) shut down under lockdown. These are the refineries that recast a London 400-ounce bar into the 100-ounce and kilo bars that New York will accept. With them closed, you could not even change the bars into the right shape.

So traders who were short the New York future, and who had planned to deliver London metal against it, looked at their screens and realised something. The arbitrage that is supposed to tie the two prices together could not be carried out. You could not move the gold, and you could not reshape it.

The story stops here.

Four discussion questions (answer all four in your group; take a position):

1. **Recall.** The link between the New York future and the London spot price is held by one specific trade. Name that trade, and write down the cost-of-carry relationship it is meant to enforce.
2. **Mechanism.** Step through that arbitrage trade one action at a time (buy where, ship

where, reshape how, deliver into what). At which exact step does the grounded-flights, shut-refineries situation of March 2020 stop you?

3. **Prediction.** If the arbitrage cannot be carried out, the New York future is no longer pinned to London spot. Which way do you expect the future to move relative to spot, and roughly how far could the gap go: a couple of dollars, or much more?

4. **Judgement.** Was this a "broken market" or the cost-of-carry relationship working exactly as written, once the real cost of moving the metal exploded? Defend your view.

End of activity handout. Solutions are distributed after the block.