

Risk Management and Derivatives | Lecture 5

Pricing Forwards by Cost of Carry

Group Activity solutions

· SJTU Summer School 2026

· Fatih Kansoy

Phase 1: Concept checks

- **CT1: (b).** The forward price is the spot price carried forward at the interest rate, $F_0 = S_0 e^{rT} = 2,000 e^{0.05 \times 1} = 2,000 \times 1.051271 = \$2,102.54$, about \$2,103. The trader's \$2,300 forecast plays no role at all. The forward is pinned by carry and arbitrage, not by anyone's opinion of the future price.
- **CT2: (c).** For a consumption commodity, $F_0 = S_0 e^{(r+u-y)T}$. When the convenience yield y is large enough to beat $r + u$, the forward sits below spot. Backwardation is normal in a tight physical market; it is not a pricing error and it is not a forecast of falling prices.
- **CT3: (c).** The fair forward is $100 e^{0.04} = \$104.08$. The quoted \$110 is too high, so sell the dear thing (the forward) and buy-and-carry the cheap thing (the stock, financed by borrowing). Deliver at \$110, repay \$104.08, and keep about \$5.92 per share with no price risk. This is exactly the trade that drags the quote back to fair value.

Phase 2: Desk problem, fully worked

(a) A non-income asset

Intuition. A forward is not a forecast. It is the spot price moved through time at the financing rate r . With no income and no storage, the only carry is the cost of funding the purchase, so the forward sits a little above spot (contango).

Calculation.

$$F_0 = S_0 e^{rT} = 2,000 e^{0.05 \times 0.5} = 2,000 e^{0.025} = 2,000 \times 1.025315 = \$2,050.63 \text{ per ounce.}$$

(b) A dividend-yield index

Intuition. Holding the index throws off dividends you do not get from the forward. So the dividend yield q works like a “negative interest rate” on the long side: it cuts the cost of carry. The forward still sits above spot here, but by less than financing alone would give.

Calculation.

$$F_0 = S_0 e^{(r-q)T} = 5,000 e^{(0.05-0.015) \times 0.75} = 5,000 e^{0.02625} = 5,000 \times 1.026598 = \$5,132.99 \text{ index points.}$$

(c) A currency forward

Intuition. A currency forward is covered interest parity. The foreign interest rate r_f plays the role of a dividend yield: holding the pound earns the sterling rate. Because sterling earns the higher rate here, the pound trades at a forward discount (the forward is below spot).

Calculation.

$$(r_d - r_f) = 0.045 - 0.0475 = -0.0025, \quad e^{-0.0025} = 0.997503, \\ F_0 = S_0 e^{(r_d - r_f)T} = 1.2700 \times 0.997503 = \$1.2668 \text{ dollars per pound.}$$

The pound is at a forward discount, because it earns the higher interest rate, so the forward gives up that extra yield.

(d) Cash-and-carry arbitrage

Intuition. The fair forward from part (a) is \$2,050.63. The offered \$2,100 is above fair, so the forward is too high. The rule is always the same: sell the dear thing and buy-and-carry the cheap thing. Here you sell the forward and buy the spot gold with borrowed money.

Calculation.

- (1) The offered \$2,100 is **above** the fair \$2,050.63, so the forward is too high.
- (2) The trade, today: short the forward at \$2,100 (agree to deliver in six months at \$2,100); borrow \$2,000 at 5 per cent for six months; buy one ounce of gold spot for \$2,000 and store it. In six months: deliver the ounce into the forward, receive \$2,100; repay the loan of $2,000 e^{0.025} = \$2,050.63$.
- (3) Profit per ounce = $2,100 - 2,050.63 = \$49.37$, riskless (no exposure to the gold price). Profit on 100 ounces = $49.37 \times 100 = \$4,936.98$.

This is the trade behind ConceptTest 3. Doing it pushes the forward quote down towards \$2,050.63, which is why the no-arbitrage price holds in practice.

(e) Storage cost and implied convenience yield

Intuition. Convenience yield is the value of holding the physical barrel itself: security of supply, the option to use it now. No one quotes y ; you back it out by inverting the cost-of-carry formula. A market price below spot means y is large.

Calculation. Rearrange $\ln(F_0/S_0) = (r + u - y)T$, so $y = r + u - \frac{\ln(F_0/S_0)}{T}$.

$$\begin{aligned}\ln(78/80) &= \ln(0.975) = -0.025318, & \frac{-0.025318}{0.5} &= -0.050636, \\ y &= 0.05 + 0.03 - (-0.050636) = 0.130636 \approx \mathbf{13.06\%} \text{ per year.}\end{aligned}$$

Cross-check. With no convenience yield ($y = 0$) the forward would be $80 e^{(0.05+0.03) \times 0.5} = 80 e^{0.04} = \83.27 , well above spot (contango). The market price of \$78 is below spot, so a large convenience yield of about 13 per cent is doing the work. The market is in **backwardation**, because the convenience yield beats the financing-and-storage cost. Hold this idea, that carry can push the forward away from spot, in mind for the case.

Extension: valuing an existing forward

Intuition. At inception the forward had zero value, because K was set equal to the then forward price. As the market forward F_0 moves above your locked-in K , your long position gains value. The gain is the difference $F_0 - K$, discounted back to today.

Calculation.

$$f = (F_0 - K) e^{-rT} = (2,050.63 - 1,950) e^{-0.05 \times 0.5} = 100.63 \times e^{-0.025} = 100.63 \times 0.975310 = \mathbf{\$98.15}.$$

Independent check (non-income asset): $f = S_0 - K e^{-rT} = 2,000 - 1,950 e^{-0.025} = 2,000 - 1,950 \times 0.975310 = 2,000 - 1,901.85 = \mathbf{\$98.15}$. The two routes agree. This previews how a futures position is marked to market each day: the contract's value moves as the forward price moves away from the price you locked in.

The one idea behind every part: a forward price is the spot price carried forward at the net cost of holding the asset, $F_0 = S_0 e^{(r+u-q-y)T}$. Financing r and storage u push it up; income q and convenience y pull it down. Arbitrage, the cash-and-carry trade, is what forces the price to hold.

Phase 3: Case, what happened

You were stopped in late March 2020, with the cash-and-carry arbitrage between London spot gold and the New York gold future jammed: you could not fly the metal across, and you could not reshape a 400-ounce London bar into the 100-ounce or kilo bars New York would take. Here is what happened next.

With the arbitrage trade switched off, the New York future was no longer pinned to London spot. It pulled **above** spot, and the gap (the EFP) blew out. The normal EFP is about **\$1.50** per ounce. In late March 2020 it widened to about **\$60** per ounce on 25 March, and intraday quotes for the New York premium reached as high as roughly **\$70** an ounce. A spread that is normally smaller than a tip on a coffee became larger than a day's normal move in the gold price itself.

Why the gap? The arbitrage that closes the gap (buy a London bar, ship it to New York, deliver it into the future) is what holds the future near spot. In March 2020 you could not run it. The metal could not move (grounded passenger flights, which carry gold in their cargo holds), and it could not be reshaped (the Swiss refineries that recast London 400-ounce bars into New York 100-ounce and kilo bars had shut). The banks that were short the New York future, expecting to deliver London metal, were trapped: they could not deliver, so they had to buy their shorts back, which pushed the future even further above spot. Several banks took heavy losses on the EFP.

In cost-of-carry language, the real cost of carry between London and New York exploded. The formula $F_0 = S_0 e^{(r+u-y)T}$ never says the gold future must equal London spot. It says the future equals spot plus the cost of carrying the metal to the delivery point. Normally that carry is tiny, a couple of dollars, so the two prices sit almost on top of each other. When moving and reshaping the metal became impossible, the effective carry cost jumped from a couple of dollars to many tens of dollars, and the future detached from spot exactly as the formula allows. The link did not break because the maths was wrong. It loosened because the arbitrage that enforces the maths could not be executed.

How it was fixed. On 24 March 2020 the CME announced a new gold future, “Gold (Enhanced Delivery)” (symbol **4GC**), with a first expiry in April 2020. The new contract lets you deliver a London 400-ounce bar (as well as 100-ounce and kilo bars), using a mechanism called Accumulated Certificates of Exchange, so a single 400-ounce London bar can satisfy four 100-ounce contracts. By accepting the London bar directly, it removed the reshaping step and helped relink the two markets.

The verified facts. Normal EFP about \$1.50 per ounce; peak about \$60 per ounce on 25 March 2020, with the New York premium reaching roughly \$70 intraday. US restrictions on passenger flights from Europe began 14 March 2020; gold normally travels in passenger-plane cargo holds. The three Swiss refineries in Ticino (Valcambi, Argor-Heraeus and PAMP) suspended production on 23 March 2020 for at least a week. London good-delivery bars are about 400 ounces; the main COMEX contract delivers 100-ounce or kilo bars. The CME announced the Gold (Enhanced Delivery) contract (4GC), with 100-ounce, 400-ounce and 1-kilo delivery, on 24 March 2020, first expiry April 2020. Sources: CME Group press release, 24 March 2020; CME Group, Gold (Enhanced Delivery) contract specifications and Accumulated Certificates of Exchange; FXEmpire / Voima, “What caused the New York vs. London gold price spread”, 2020; Bloomberg, “Spread Blowout That Jolted Gold”, July 2020. Figures stated as “about”.

The wide EFP was not a broken market. It was the cost-of-carry relationship holding exactly as written: the future equals spot plus the cost of carrying the metal to delivery, and when that carry could not be paid (no flights, no refining) the future detached from spot. Arbitrage pins the price only when the arbitrage can actually be done.

End of solutions.