Risk Management and Derivatives | Lecture 6

Interest Rate Futures

In Class Group Activity

· SJTU Summer School 2026 · Fatih Kansoy

How this block runs. You work in your home group of four or five, textbook open. Roles rotate today: **Scribe** (writes the answer), **Reporter** (speaks at the microphone), **Sceptic** (must raise one objection), **Timekeeper**. All money today is in US dollars ($), pounds sterling (£) and euros (€). Bond prices are in points per 100 face value unless stated.

- **Phase 1, Concept checks (~10 min).** For each question you vote on your own first. Then you talk for about 90 seconds with the people next to you, and vote again. Do not talk before the first vote.
- **Phase 2, Desk problem (~20 min).** One problem in parts. Your Reporter presents one part when the microphone reaches your group. Then we solve it together, part by part.
- **Phase 3, Case (~25 min).** You read one real story. The ending is hidden on purpose. You answer four questions and take a position. Then we reveal what happened.

A short word list for this block. The **quoted (clean) price** is the price you see on the screen. The **cash (dirty) price** is what you actually pay; it is the clean price plus the **accrued interest**, the slice of the next coupon the seller has already earned. A **day count** is the rule for counting days when you accrue interest; US Treasuries use actual/actual, money-market deals use actual/360. A **conversion factor** (CF) scales a deliverable bond to the futures contract's notional 6 per cent coupon. The **cheapest-to-deliver** (CTD) bond is the one the short can buy and deliver most cheaply. The **basis** is the cash price minus futures times CF. **Net carry** is coupon income minus the cost of financing the bond in repo.

Phase 1 – Concept checks

~10 MIN

ConcepTest 1. A US Treasury bond is quoted at 98-16 (that is 98 and 16/32, so 98.50 per 100 face). Coupons are paid on 15 May and 15 November. You buy 100,000 face for settlement on 15 July. Which best describes the cash you actually pay, before fees? Choose the best answer.

(a) Exactly 98,500, because that is the quoted price for 100,000 face.

(b) 98,500 *plus* the accrued interest since 15 May, because settlement is the dirty (cash) price.

(c) 98,500 *minus* accrued interest, because the seller has not yet earned the next coupon.

(d) The full face value of 100,000, because Treasuries settle at par.

ConcepTest 2. A CBOT Treasury bond future can be settled by delivering any one of several eligible bonds, each scaled by its own conversion factor. As yields move, the cheapest-to-deliver (CTD) bond can change. Who chooses which bond is delivered, and what does the futures price mainly track? Choose the best answer.

(a) The long chooses the bond, and the future tracks the highest-quality bond.

(b) The exchange chooses a fixed benchmark bond, and the future tracks that one bond for its whole life.

(c) The short chooses the bond, and the future tracks whichever bond is currently cheapest to deliver (the CTD).

(d) Both sides agree the bond at the start, and the future tracks that single bond.

ConcepTest 3. A three-month SOFR future is quoted as an index equal to 100 minus the annualised rate. You expect short rates to *rise*. To profit from that view with this future, you should: choose the best answer.

(a) Buy (go long) the future, because you are bullish on rates.

(b) Sell (go short) the future, because the index is 100 minus the rate, so the price falls when rates rise.

(c) Buy the future, because higher rates always lift futures prices.

(d) Do nothing, because rate futures do not respond to the level of rates.

Phase 2 – The desk problem

~20 MIN

A morning on the rates desk

The setting. You are on the rates desk at a London-based dealer. It is the morning of 15 July. You quote bonds, manage a Treasury-future delivery, run a short-rate hedge, size a duration hedge for the portfolio book, and price the basis trade your senior trader wants to put on. Work in points per 100 face unless told otherwise.

Givens (shared across the parts).

- **US Treasury "Bond X":** 4.00 per cent coupon, paid semiannually (15 May and 15 November), quoted 98-16 = 98.50. Day count actual/actual.
- **Money-market deposit:** 1,000,000 principal, 5.20 per cent rate, 90-day term, day count actual/360.
- **Treasury bond future:** settlement price $F = 119.00$ per 100 face; contract size 100,000 face; notional coupon 6 per cent.
- **Deliverable table for the future:**
  Bond A: quoted 132.90, conversion factor (CF) 1.1080
  Bond B: quoted 120.40, CF 1.0000
  Bond C: quoted 141.05, CF 1.1820
- **Three-month SOFR future:** index = 100 minus the rate; 1 basis point = \$25 per contract; \$2,500 per full index point.
- **Portfolio book:** $P = 24{,}000{,}000$; portfolio duration $D_P = 6.8$ years.
- **Repo rate for financing** = 4.80 per cent (actual/360); CTD coupon for carry = 5.50 per cent; accrued on the CTD now = 0.85 per 100.

(a) Day counts, accrued interest, dirty price. (Scaffolded.)

(1) For Bond X settling on 15 July, compute the accrued interest on actual/actual and the dirty price. The days from 15 May to 15 July are 61; the days in the coupon period from 15 May to 15 November are 184. The semiannual coupon per 100 is $4.00/2 = 2.00$. Accrued = (semiannual coupon) × (days accrued / days in period). State the dirty price per 100 and the cash paid on 100,000 face.

(2) For the money-market deposit, compute the interest on actual/360, using $1{,}000{,}000 \times 0.0520 \times 90/360$.

(b) Conversion factor, invoice price, cheapest-to-deliver. (Scaffolded.)

(1) For each deliverable bond, compute the net cost of delivery = quoted price $- F \times$ CF. The CTD is the bond with the *smallest* net cost (cheapest for the short to buy and deliver). Identify the CTD.

(2) Compute the invoice price of the CTD, using invoice = $F \times$ CF + accrued, with accrued on the CTD = 0.85.

(c) Short-rate future, P&L and DV01. (Partly scaffolded.) You short 20 three-month SOFR futures because you expect rates to rise. Rates then rise by 25 basis points. Remember: 1 basis point = \$25 per contract, and a short gains when the index falls.

(1) Compute the profit or loss.

(2) State the DV01 (the dollar value of a one-basis-point move) of one contract and of the whole position.

(d) Duration-based bond-futures hedge. (Unsupported.) You want to hedge the 24,000,000 book ($D_P = 6.8$) with the Treasury bond future ($F = 119.00$, size 100,000). The CTD (Bond C) has duration $D_F = 8.2$ years. Using

$$N^* = \frac{P \times D_P}{V_F \times D_F},$$

where V_F is the dollar value of one futures contract, find the number of contracts and the direction (long or short).

(e) The basis and the cash-and-carry trade. (Unsupported.) Your senior trader wants the classic basis trade on the CTD (Bond C): buy the cash bond, short the future, finance the purchase in repo for 90 days.

(1) Compute the basis = cash price $- F \times$ CF.

(2) Compute the net carry over 90 days = coupon income − repo cost. Use coupon = 5.50 per cent on 100 face with 90/365, and repo = 4.80 per cent (actual/360) on the dirty purchase price of 141.90 (which is $141.05 + 0.85$ accrued).

(3) Say in one line what the trader is really betting on.

Extension (for fast groups): the short's delivery option. The basis you found in part (e) is about 0.3920 per 100. The simple "minus net carry" figure is about 0.3466 per 100. The basis exceeds it by about 0.0454 per 100. Express that residual in 32nds, and explain in one or two lines what it pays for.

Phase 3 – The case

~25 MIN

Long-Term Capital Management, 1998 (United States)

How to run this. Read the story with your group. The ending is hidden on purpose. Your group must take a position on the four questions and hold your decision. Do not search the internet. Use only what you know from the lecture. I will reveal what happened only after every group has answered. All money is in US dollars (\$).

The story. In 1994 John Meriwether, formerly the head of bond trading at Salomon Brothers, founded a hedge fund called Long-Term Capital Management. On the board sat Robert C. Merton

and Myron Scholes, who in 1997 shared the Nobel Prize in Economics for option-pricing theory. The fund hired the sharpest quantitative minds on Wall Street.

LTCM ran relative-value, or "convergence", trades. It looked for two assets that should trade at almost the same price but did not quite: on-the-run versus off-the-run US Treasuries, swap spreads, and the bond basis you just computed. Each gap was tiny, a few 32nds, a few basis points, so to make real money the fund used enormous leverage. At the start of 1998 it had equity of about 4.7 billion dollars, had borrowed over 124.5 billion dollars, and held assets of around 129 billion dollars, a debt-to-equity ratio over 25 to 1. On top of that sat off-balance-sheet derivatives with a notional value of roughly 1.25 trillion dollars, much of it interest-rate swaps.

The first three years were spectacular: returns after fees of about 21 per cent, 43 per cent and 41 per cent. The trades, the partners argued, were "sure to converge". The mispricings could only close, given time. The models said the risk was small.

The story stops here.

Four discussion questions (answer all four in your group; take a position):

1. **Tiny edge, huge leverage.** A convergence trade is built from positions like the basis trade you just priced: long the cheap asset, short the dear one. Why does a tiny mispricing force a fund to use huge leverage to earn a meaningful return?
2. **Two distinct risks.** The partners called the trades "low-risk" because the two prices "must" converge. Name two distinct risks that could still hurt them *before* convergence.
3. **Leverage in reverse.** With 25-to-1 leverage, what does a 2 per cent adverse move in the value of the assets do to the equity? Work it out roughly.
4. **Make a prediction.** It is August 1998 and Russia has just defaulted on its rouble debt. Investors are dumping anything risky and crowding into the most liquid US Treasuries. For a fund that is short the liquid, expensive bonds and long the illiquid, cheap ones, which way does the spread move, and what happens to a 25-to-1 book? Write down your prediction before the reveal.

End of activity handout. Solutions are distributed after the block.