Risk Management and Derivatives | Lecture 6

Interest Rate Futures

Bond quotes, day counts, and the Treasury basis – group activity solutions

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Phase 1: Concept checks

- **CT1: (b).** Settlement is the dirty (cash) price, which is the quoted (clean) price plus the accrued interest since the last coupon. You always compensate the seller for the slice of the next coupon they have already earned, so accrued is *added*, not subtracted. The quoted price 98.50 is for comparison; the invoice is the dirty price.
- **CT2: (b)** is wrong; the correct answer is **(c).** The party who is *short* delivers, so the short picks the cheapest option for itself. The future therefore tracks the cheapest-to-deliver (CTD) bond, and that bond can change as yields move. The future does not track one fixed bond.
- **CT3: (b).** The index is 100 minus the rate, so the price moves *inversely* to the rate. A long future is long the price, so you win when rates fall. To bet on rising rates, you *short* the future.

Phase 2: Desk problem, fully worked

(a) Day counts, accrued interest, dirty price

Intuition. A bond's quoted price is a clean number for comparison, but the cash you hand over also pays the seller for the coupon they have earned since the last payment date. The day-count rule just tells you how to count those days. Treasuries divide by the real days in the period; money-market deals always divide by 360.

Calculation.

(1) Semiannual coupon per 100 $= 4.00/2 = 2.00$. Accrued $= 2.00 \times 61/184 = 2.00 \times 0.331522 =$ **0.6630** per 100.
Dirty price = clean + accrued $= 98.50 + 0.6630 =$ **99.1630** per 100.
For 100,000 face: cash paid = \$**99,163.0**.

(2) Money-market interest $= 1{,}000{,}000 \times 0.0520 \times 90/360 = 1{,}000{,}000 \times 0.0130 =$ \$**13,000.00**.

The two conventions share the same headline rate but accrue differently. Actual/actual divides by the real days in the period (184); actual/360 always divides by 360 whatever the real year, which slightly overstates the daily rate.

(b) Conversion factor, invoice price, cheapest-to-deliver

Intuition. The conversion factor tries to neutralise the coupon and maturity differences between the deliverable bonds, but it is built off a flat 6 per cent yield, so it is imperfect. One bond is always cheapest for the short to buy and deliver, and that is the CTD.

Calculation. Net cost of delivery = quoted price $- F\times$ CF.

- Bond A: $132.90 - 119.00 \times 1.1080 = 132.90 - 131.8520 = 1.0480$.
- Bond B: $120.40 - 119.00 \times 1.0000 = 120.40 - 119.0000 = 1.4000$.
- Bond C: $141.05 - 119.00 \times 1.1820 = 141.05 - 140.6580 =$ **0.3920** (smallest).

So the **CTD is Bond C**, with a net cost of 0.3920 per 100. The short will deliver Bond C.
Invoice price of the CTD $= F\times$ CF + accrued $= 119.00 \times 1.1820 + 0.85 = 140.6580 + 0.85 =$ **141.5080** per 100.

(c) Short-rate future, P&L and DV01

Intuition. Because the index is 100 minus the rate, a short position wins when rates rise (the index falls). This is exactly Concept Check 3 turned into a number: you shorted because you were bullish on rates, and the position made money as the index fell.

Calculation.

(1) Rates rise 25 bp, so the index falls 25 bp. P&L per contract = 25 bp × \$25 = \$625. The short wins because the index fell.
Total P&L = 20 × \$625 = \$**12,500** gain.

(2) DV01 of one contract = \$**25** per bp. Position DV01 = 20 × \$25 = \$**500** per bp.

(d) Duration-based bond-futures hedge

Intuition. The future's interest-rate sensitivity comes from the CTD, so you use the CTD's duration, not a generic number. You short enough contracts so that the futures' money duration matches the book's, then a yield move that hurts the bonds is met by a gain on the futures.

Calculation.

$$V_F = \frac{119.00}{100} \times 100{,}000 = \$119{,}000 \text{ per contract},$$
$$N^* = \frac{P \times D_P}{V_F \times D_F} = \frac{24{,}000{,}000 \times 6.8}{119{,}000 \times 8.2} = \frac{163{,}200{,}000}{975{,}800} = \mathbf{167.25}.$$

So you **short about 167 contracts** to hedge a long bond book against rising yields. Notice that the hedge ratio uses the CTD's duration (8.2), because the future moves with the CTD.

(e) The basis and the cash-and-carry trade

Intuition. The basis trade buys the cheap cash bond, shorts the future, and finances the bond in repo. The trader earns the basis as it narrows toward fair value by delivery, but pays the carry to hold the position. In a high-repo environment the carry can be *negative*, so the trade only pays if the basis tightens enough to cover the financing drag.

Calculation.

(1) Basis = cash price $- F\times$ CF $= 141.05 - 119.00 \times 1.1820 = 141.05 - 140.6580 =$ **0.3920** per 100. This is the same as the CTD net cost in part (b), by construction.

(2) Coupon income (90 days) $= 100 \times 0.0550 \times 90/365 =$ **1.3562** per 100.
Repo cost (90 days) $= 141.90 \times 0.0480 \times 90/360 =$ **1.7028** per 100.
Net carry $= 1.3562 - 1.7028 =$ **−0.3466** per 100. This is *negative* carry: financing costs more than the coupon earns.

(3) The trader is betting the basis will narrow toward fair value (cash converges to futures times CF by delivery), earning the basis change while paying the negative carry. The positive basis (0.3920) roughly equals minus the net carry (0.3466) plus a small residual, which is the delivery option the short owns.

Extension: the short's delivery option

Intuition. The short owns optionality: the right to deliver the cheapest bond and to choose the delivery day within the month. That option has value, so the cash bond trades at a small premium over pure carry, which shows up as a positive basis.

Calculation. The residual is $0.3920 - 0.3466 = 0.0454$ per 100. In 32nds, $0.0454 \times 32 \approx$ **1.5/32**. A typical delivery-option value is a few 32nds (for example $6/32 = 0.1875$ per 100). The buyer of the future is short that option and is compensated through a lower futures price.

The one idea behind every part: a bond future does not track one fixed bond. The short owns a delivery option, so the future tracks the cheapest-to-deliver bond, its price is driven by the cost of carry, and the basis is net carry plus the value of that option.

Phase 3: Case, what happened

You were stopped at the start of 1998, with LTCM running on 25-to-1 leverage and calling its convergence trades "sure to converge". The fund is **Long-Term Capital Management (LTCM)**. Here is what happened next.

On **17 August 1998**, Russia devalued the rouble and declared a moratorium on its government debt. Instead of converging, the spreads LTCM had bet on *diverged*. In the panic, investors fled to the most liquid bonds, exactly the ones LTCM was short, and abandoned the cheap, illiquid ones it was long. Every convergence trade moved against the fund at once, and they were correlated in a way the models had not priced.

Leverage did the rest. The fund lost about **4.6 billion dollars** in under four months. By late September its equity had collapsed from roughly 4.7 billion to a few hundred million, and the leverage ratio exploded. Because LTCM sat at the centre of a 1.25 trillion dollar web of derivatives with the world's biggest banks, regulators feared its forced liquidation would drag the whole system down.

On **23 September 1998**, the Federal Reserve Bank of New York brought 14 financial institutions into a room and organised a recapitalisation of about **3.6 billion dollars**. The banks put in the money, not the Fed itself, and they wound the book down slowly. The "sure thing" had nearly broken the financial system.

The lesson, stated plainly. The prices did eventually converge, mostly. But LTCM was gone before that day arrived; it could not survive the path. Being right about the final price is worthless if you are forced to close at the worst point. The danger was never "will it converge". The danger was "can you fund the position and stay liquid long enough to get there". Parts (d) and (e) are exactly LTCM's world: a duration and basis position that looks low-risk on paper, financed in repo, where the real risk lives in funding and liquidity, not in the model's fair value.

Convergence is about the destination; ruin happens on the path. A leveraged relative-value trade is only as safe as your ability to fund and hold it, because funding and liquidity risk can force you out before the prices ever converge.

End of solutions.