Risk Management and Derivatives | Lecture 7

The Interest Rate Swap: Mechanics and Value

In Class Group Activity

· SJTU Summer School 2026 · Fatih Kansoy

How this block runs. You work in your home group of four or five, textbook open. Roles rotate today: **Scribe** (writes the answer), **Reporter** (speaks at the microphone), **Sceptic** (must raise one objection), **Timekeeper**. All money today is in US dollars ($), with one extension in pounds sterling (£). Payments are annual with annual compounding, to keep the arithmetic clean.

- **Phase 1, Concept checks (~10 min).** For each question you vote on your own first. Then you talk for about 60 seconds with the people next to you, and vote again. Do not talk before the first vote.
- **Phase 2, Desk problem (~20 min).** One problem in parts. Your Reporter presents one part when the microphone reaches your group. Then we solve it together, part by part.
- **Phase 3, Case (~25 min).** You read one real story. The ending is hidden on purpose. You answer four questions and take a position. Then we reveal what happened.

A short word list for this block. A **plain-vanilla interest-rate swap** is a contract where one side pays a **fixed** rate and receives a **floating** rate (here SOFR), on a **notional** that is only a scaling figure and is never exchanged. A **discount factor** $DF(t)$ is the value today of $1 paid at the end of year t. We value a swap as two bonds: a fixed-rate bond B_{fixed} and a floating-rate bond B_{float}. The **par swap rate** is the fixed rate that makes the swap worth zero at the start. **Comparative advantage** is the idea that two firms can both borrow more cheaply by swapping.

Phase 1 – Concept checks ~10 MIN

ConcepTest 1. GreenMill Foods enters a 3-year swap today as the **fixed-rate payer**: it will pay 4 per cent fixed each year and receive SOFR, on a notional of $100 million. The fixed rate was set at the current market par rate. At the moment of signing, what is the value of this swap to GreenMill? Choose the best answer.

(a) Positive, because GreenMill is betting that rates will rise and that bet has value.

(b) About zero, because the fixed rate was set so the present value of the fixed leg equals the present value of the floating leg.

(c) Negative, because GreenMill has to pay fixed, and paying is always a cost.

(d) It cannot be valued until the first SOFR fixing is known.

ConcepTest 2. AlphaCorp can borrow at 4.0 per cent fixed or SOFR + 0.30 per cent floating. BetaCo can borrow at 5.8 per cent fixed or SOFR + 1.20 per cent floating. A textbook swap lets both firms borrow more cheaply. The total saving the swap can share out is:

(a) 1.80 per cent, the difference in the two fixed rates.

(b) 0.90 per cent, the difference between the fixed-rate gap (1.80 per cent) and the floating-rate gap (0.90 per cent).

(c) Zero, because cheaper borrowing for both at once is impossible.

(d) 2.70 per cent, the two gaps added together.

ConcepTest 3. Two years ago two banks entered a 10-year fixed-for-floating swap at the then-par fixed rate. Since then market rates have risen sharply. No principal has ever changed hands. Which is true today? Choose the best answer.

(a) The swap is still worth zero to both sides, because no principal is exchanged.

(b) The swap is now an asset to one side and an equal liability to the other, so each side has counterparty credit exposure to the other.

(c) The swap is an asset to both sides at once.

(d) The swap is now worthless and should be torn up.

Phase 2 – The desk problem

~20 MIN

You are on the rates desk; price and risk-manage a swap

The setting. One problem in parts. You are on the rates desk at a London bank. A corporate client, GreenMill Foods, holds \$100 million of floating-rate debt linked to SOFR and wants to swap into fixed. You will price and risk-manage the swap. The notional is $N = \$100$ million throughout. Payments are annual with annual compounding.

Given (Table A): discount factors at the start. These are the prices today of \$1 paid at each future year-end.

year t	discount factor $DF(t)$
1	0.96
2	0.92
3	0.88

The contract. GreenMill pays 5 per cent fixed each year and receives SOFR, on \$100 million, for 3 years.

(a) Lay out the cash flows. Assume, for illustration only, that SOFR turns out to be 3.0 per cent in year 1, 3.5 per cent in year 2 and 4.0 per cent in year 3. Fill in the net cash flow each year to GreenMill (the fixed payer), where net = floating received minus fixed paid, on \$100 million.

(1) Year 1: receive $3.0\% \times 100$; pay $5\% \times 100$; net = ?

(2) Year 2: receive $3.5\% \times 100$; pay $5\% \times 100$; net = ?

(3) Year 3: receive $4.0\% \times 100$; pay $5\% \times 100$; net = ?

The floating leg is unknown in advance; you only assumed a path here to draw the picture. Pricing must not depend on a guessed path. That is why part (b) uses two bonds.

(b) Value the swap as two bonds. The fixed payer is long the floating bond and short the fixed bond, so to the fixed payer $V = B_{\text{float}} - B_{\text{fixed}}$. Use Table A.

(1) A floating-rate bond is worth its face value at a reset date, so $B_{\text{float}} = 100$. Write this down.

(2) Value the fixed bond: a 5 per cent bond on 100 pays a coupon of 5 each year, plus 100 principal back at year 3. Compute $B_{\text{fixed}} = 5\,(0.96 + 0.92 + 0.88) + 100\,(0.88)$.

(3) Work out $V = B_{\text{float}} - B_{\text{fixed}}$. Is the swap an asset or a liability to GreenMill at the start? Say why in one sentence.

(c) Find the par swap rate. The par swap rate c^* is the fixed rate that makes the swap worth zero at the start:

$$c^* = \frac{1 - DF(3)}{DF(1) + DF(2) + DF(3)}.$$

(1) Numerator: $1 - 0.88$.

(2) Denominator: $0.96 + 0.92 + 0.88$.

(3) Divide, and state c^* as a percentage to two decimals. If GreenMill had agreed to pay c^* instead of 5 per cent, what would the swap be worth at the start? Link this back to Concept check 1.

(d) The curve shifts up; re-value the swap. One year has *not* passed; instead, immediately after signing, market rates jump. Use this new discount-factor table (Table B).

year t	new $DF(t)$
1	0.94
2	0.88
3	0.82

Re-value the same 5 per cent swap to GreenMill, and say who now holds an asset.

(1) New $B_{\text{fixed}} = 5\,(0.94 + 0.88 + 0.82) + 100\,(0.82)$.

(2) $B_{\text{float}} = 100$ (still at a reset).

(3) Work out $V = B_{\text{float}} - B_{\text{fixed}}$ to GreenMill. Who now holds the asset, and who holds the liability? How much did the value swing? Tie this back to Concept check 3.

(e) Comparative advantage with an intermediary bank. Two firms can borrow as follows.

firm	fixed	floating
AlphaCorp (strong credit)	4.0%	SOFR + 0.30%
BetaCo (weaker credit)	5.8%	SOFR + 1.20%

AlphaCorp wants floating debt; BetaCo wants fixed debt. A bank stands in the middle and keeps 0.10 per cent for itself. Construct the swap, show each firm's all-in cost, and split the remaining gain equally.

(1) Total gain available = (fixed gap) minus (floating gap). Compute it.

(2) The bank keeps 0.10 per cent. How much is left to split, and how much goes to each firm?

(3) Who has the comparative advantage in fixed, and who in floating? So who borrows which, before they swap?

(4) Write the all-in cost for each firm and check that the savings reconcile.

(5) **The critique.** In one or two sentences, say why this 0.90 per cent is *not* a pure free lunch.

Extension (for fast groups): a fixed GBP for fixed USD currency swap. A UK firm wants USD funding; a US firm wants GBP funding. The spot rate is 1.25 USD per GBP. They exchange principals at the start: £80 million against $100 million. The UK firm pays 5 per cent on the USD leg and receives 3 per cent on the GBP leg, for 3 years, then re-exchanges principal.

(1) Work out the annual USD interest, $5\% \times 100$.

(2) Work out the annual GBP interest, $3\% \times 80$.

(3) Unlike the single-currency swap, principal *is* exchanged here. Name the two things that now drive the swap's value, and say why the "no principal" intuition from Concept check 3 is specific to single-currency swaps.

Phase 3 – The case

~25 MIN

"Synthetically fixed": Jefferson County, Alabama (United States)

How to run this. Read the story with your group. The ending is hidden on purpose. Your group must take a position on the four questions and hold your decision. Do not search the internet. Use only what you know from the lecture. I will reveal what happened only after every group has answered. All money is in US dollars ($).

The story. In the 1990s a US federal court ordered Jefferson County, Alabama (the county that contains the city of Birmingham) to rebuild its old, leaking sewer system to meet clean-water rules. The county borrowed heavily to pay for it.

To keep the headline borrowing cost low, the county did not issue ordinary fixed-rate municipal bonds. Instead it used variable-rate and "auction-rate" debt, where the interest rate is reset every few weeks at auction, and then it layered large interest-rate **swaps** on top. The idea was to "synthetically" fix the cost: the county would pay floating to its bondholders, and in the swaps it would pay a fixed rate to banks (notably JPMorgan) and receive a floating rate back, so the floating payments would, in theory, cancel out and leave the county paying a low fixed rate.

By the mid-2000s the county had run up over $3 billion of sewer debt structured this way, and the swaps had a notional far larger than that. On paper, the county looked hedged.

The story stops here.

Four discussion questions (answer all four in your group; take a position):

1. **Mechanics.** Draw the county's position. On the bond side it pays floating to investors. In the swap it pays fixed to the bank and receives floating from the bank. If the floating leg the county *receives* exactly matched the floating leg it *pays*, what would its net cost be, and would it be hedged?

2. **Find the gap.** The floating rate the county *paid* its bondholders was an auction-rate, variable municipal rate. The floating rate it *received* in the swap was a different market index. These two floating rates usually move together, but not always. What is the name of the risk created when the two floating legs do not move together? What would happen to the county's net cost if its own borrowing rate jumped while the swap's received rate did not?

3. **Stress it.** Now imagine a market-wide credit panic. The specialist insurers who guaranteed the county's bonds get downgraded, and the auction market for this kind of debt simply stops working, so no buyers turn up at the auctions. Under the auction rules, a failed auction sends the interest rate to a high penalty level. What happens to (a) the rate the county must pay its bondholders and (b) the value of the swaps the county holds, if overall rates are *falling* at the same time?

4. **Exit.** The county now wants out of the swaps. To cancel a swap that has moved against you, you must pay the other side its value, the termination payment. If rates have fallen,

which side is the swap an asset to, and who must write the cheque to tear it up? What does this do to a county already struggling to pay its bondholders?

End of activity handout. Solutions are distributed after the block.