Risk Management and Derivatives | Lecture 7

The Interest Rate Swap: Mechanics and Value

Group activity solutions

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Phase 1: Concept checks

- **CT1: (b).** A par swap is priced so neither side pays the other to enter. The fixed rate is set so the present value of the fixed leg equals the present value of the floating leg, so the swap starts at about zero. A view that rates will rise is a reason to *want* the swap, but the market price already reflects the whole forward curve. You are not paid for a view; you take on the view at fair value.
- **CT2: (b).** The shareable gain is the credit-quality spread differential: the fixed gap minus the floating gap, $1.80 - 0.90 = 0.90$ per cent. Part of that 0.90 per cent is not free. It exists because the weaker firm pays a wider spread in fixed than in floating, and after the swap the weaker borrower still rolls floating debt and bears rollover and default risk.
- **CT3: (b).** Principal is only a scaling figure (the notional); value comes from the interest legs. Once rates move, the fixed-rate payer and receiver no longer have equal-valued legs, so the swap is worth $+X$ to one and $-X$ to the other. The in-the-money side now worries the other side might default, which is exactly the credit exposure that central clearing was built to manage.

Phase 2: Desk problem, fully worked

(a) Lay out the cash flows

Intuition. The fixed payer pays the same 5 every year and receives whatever SOFR turns out to be. Drawing an assumed path shows the shape, but the picture cannot be the price, because the floating leg is unknown in advance. That is why part (b) values the swap off the curve, not off a guess.

Calculation. On \$100 million, net = floating received minus fixed paid:

- Year 1: receive $3.0\% \times 100 = 3.00$; pay $5\% \times 100 = 5.00$; net $= -\$\mathbf{2.00}$ million.
- Year 2: receive $3.5\% \times 100 = 3.50$; pay 5.00; net $= -\$\mathbf{1.50}$ million.
- Year 3: receive $4.0\% \times 100 = 4.00$; pay 5.00; net $= -\$\mathbf{1.00}$ million.

These net figures depend on the assumed SOFR path, so they are an illustration, not a valuation.

(b) Value the swap as two bonds

Intuition. The fixed payer is long a floating bond and short a fixed bond. A floating bond is worth its face value at a reset, because each coupon is set fresh to the market. So the only real work is the fixed bond, and the swap value is the difference.

Calculation.

(1) $B_{\text{float}} = \mathbf{100}$ (worth face value at a reset date).

(2) $B_{\text{fixed}} = 5\,(0.96 + 0.92 + 0.88) + 100\,(0.88) = 5 \times 2.76 + 88 = 13.80 + 88 = \mathbf{101.80}$.

(3) $V = B_{\text{float}} - B_{\text{fixed}} = 100 - 101.80 = -\$\mathbf{1.80}$ million. The swap is a small *liability* to GreenMill at the start, because it agreed to pay 5 per cent, which is above the fair market rate.

(c) Find the par swap rate

Intuition. The par swap rate is the fixed rate that makes the two legs worth the same, so the swap is worth zero at the start. It is built only from today's discount factors, exactly the curve from Lecture 6.

Calculation.

$$c^* = \frac{1 - DF(3)}{DF(1) + DF(2) + DF(3)} = \frac{1 - 0.88}{0.96 + 0.92 + 0.88}$$
$$= \frac{0.12}{2.76} = 0.043478\ldots = \mathbf{4.35}\% \text{ (to two decimals).}$$

If GreenMill had agreed to pay 4.35 per cent instead of 5 per cent, the swap would be worth zero at the start. It is the off-market 5 per cent, not any directional bet, that makes part (b) non-zero. This is the answer to Concept check 1, made concrete.

(d) The curve shifts up; re-value the swap

Intuition. Nothing was paid and no principal moved, but the curve changed. The fixed bond is now discounted harder, so it is worth less, and the gap to the floating bond opens up. The swap that started slightly out of the money is now firmly in the money for GreenMill.

Calculation.

(1) New sum of discount factors $= 0.94 + 0.88 + 0.82 = 2.64$. $B_{\text{fixed}} = 5 \times 2.64 + 100 \times 0.82 = 13.20 + 82 = \mathbf{95.20}$.

(2) $B_{\text{float}} = \mathbf{100}$ (still at a reset).

(3) $V = B_{\text{float}} - B_{\text{fixed}} = 100 - 95.20 = +\$\mathbf{4.80}$ million to GreenMill.

The swap is now an **asset** to GreenMill worth \$4.80 million, and an equal **liability** of $-\$4.80$ million to the bank on the other side. The value swung by $4.80 - (-1.80) = \mathbf{6.60}$ million on a parallel rate rise. No principal moved, yet the swap now has value, and the in-the-money side (GreenMill) now carries counterparty credit exposure to the bank. This is Concept check 3 in numbers. (Optional cross-check: the new par rate is $(1 - 0.82)/2.64 = 0.18/2.64 = 6.82$ per cent, well above the 5 per cent fixed, confirming the swap is in the money for the fixed payer.)

(e) Comparative advantage with an intermediary bank

Intuition. The shareable gain is the difference between how much cheaper the strong firm is in fixed and how much cheaper it is in floating. Each firm borrows where it is *relatively* cheaper, then the swap moves them to the funding they actually want. The bank takes a slice for standing in the middle.

Calculation.

- Total gain = (fixed gap) minus (floating gap) $= (5.8 - 4.0) - (1.20 - 0.30) = 1.80 - 0.90 = \mathbf{0.90}\%$.
- The bank keeps 0.10 per cent, so 0.80 per cent is left to split: **0.40 per cent to each firm**.
- AlphaCorp is 1.80 per cent cheaper in fixed but only 0.90 per cent cheaper in floating, so it has the advantage in **fixed**. AlphaCorp borrows **fixed at 4.0%**; BetaCo borrows **floating at SOFR** $+1.20\%$. Then they swap.

One clean set of cash flows through the bank:

- **AlphaCorp**: borrows fixed 4.0% from the market; pays the bank SOFR; receives 4.10% fixed from the bank. All-in $= 4.0 + \text{SOFR} - 4.10 = \text{SOFR} - 0.10\%$.
- **BetaCo**: borrows floating SOFR + 1.20% from the market; pays the bank 4.20% fixed; receives SOFR from the bank. All-in $= (\text{SOFR} + 1.20) + 4.20 - \text{SOFR} = 5.40\%$.

- **Bank**: receives SOFR from Alpha, pays 4.10 to Alpha, receives 4.20 from Beta, pays SOFR to Beta. Net = +0.10%.

Check the savings. AlphaCorp wanted floating, would have paid SOFR + 0.30%, now pays SOFR − 0.10%, saving 0.40 per cent. BetaCo wanted fixed, would have paid 5.8 per cent, now pays 5.40 per cent, saving 0.40 per cent. The bank earns 0.10 per cent. Total $0.40 + 0.40 + 0.10 = 0.90$ per cent. **Consistent.**

The critique (this is the answer to Concept check 2). The 0.90 per cent is not a pure free lunch. The fixed-rate gap is wide partly because lenders charge BetaCo a large premium to lock in fixed for its weaker credit. In the swap, BetaCo still rolls short-term floating debt and keeps the rollover and default risk that the wide fixed spread was pricing. If BetaCo's credit worsens and it cannot refinance, the saving can evaporate. The apparent advantage partly compensates for risk that does not disappear.

Extension: fixed GBP for fixed USD currency swap

Intuition. A currency swap looks like the single-currency swap, but with two new features: principal *is* exchanged, and the two legs are in different currencies. So value now comes from interest-rate moves *and* the exchange-rate move.

Calculation. Spot 1.25 USD per GBP; principals £80 million against $100 million.

(1) Annual USD interest = 5% × 100 = $**5.00** million.

(2) Annual GBP interest = 3% × 80 = £**2.40** million.

(3) Value the swap as a USD bond minus a GBP bond converted at spot. The two drivers are the two interest-rate curves and the GBP/USD exchange rate. The "no principal so no value" intuition of Concept check 3 is specific to single-currency swaps, where the principals match and cancel; here they are in different currencies and stay on the books.

The one idea behind every part: the fixed rate is set so the two legs are equal at the start, so the swap is worth about zero then. Once the curve moves, the swap becomes an asset to one side and a liability to the other, and it only protects you if it offsets the exact exposure you actually hold.

Phase 3: Case, what happened

You were stopped in the mid-2000s, with Jefferson County's sewer debt structured as auction-rate bonds plus large swaps, looking hedged on paper. The county is **Jefferson County, Alabama**. Here is what happened.

In 2008 the credit crisis hit exactly the two weak points in this structure at once. First, the bond insurers that guaranteed Jefferson County's debt were downgraded, and the auction-rate market froze: auctions failed, and under the contract rules the county's borrowing rate jumped to penalty levels. So the floating cost the county *paid* shot up. Second, the broad market rates that the county *received* in its swaps fell as central banks cut rates. The two floating legs moved in *opposite* directions. This is **basis risk**, and it turned the "synthetic fix" into a trap: the swaps were a hedge against the wrong exposure.

To make it worse, the swaps had moved against the county, so they were now a liability the county would have to pay to terminate, on top of soaring bond costs. The deals themselves were corrupt. In November 2009 the US Securities and Exchange Commission announced a settlement in which JPMorgan agreed to pay a $25 million civil penalty, pay $50 million to Jefferson County, and forgo more than $647 million of swap termination fees it claimed the county owed, a package the press valued at about $722 million. The SEC charged that JPMorgan had made undisclosed payments to friends and associates of county commissioners to win the bond and swap business. Two former JPMorgan managing directors were charged. More than 20 people connected to the county's finances, including a former county commission president, were later convicted on bribery and fraud charges.

The county could not climb out. On **9 November 2011**, Jefferson County filed for Chapter 9 bankruptcy with about \$4.2 billion of debt, of which roughly \$3.14 billion was the sewer debt. At the time this was the **largest municipal bankruptcy in US history**; it was surpassed only when Detroit filed in 2013. The county exited bankruptcy in late 2013.

The lesson, stated plainly. This is Concept check 3 and desk-problem part (d) in the real world. A swap took on value after rates moved, it became a liability the county had to pay to escape (termination risk), the in-the-money banks were owed money (counterparty risk), and above all the swap hedged a *different* floating index than the county actually owed (basis risk). The structure was called a hedge, but it offset the wrong exposure.

A swap protects you only if it offsets the exact exposure you actually hold. Called a hedge, the Jefferson County swaps offset the wrong floating index, so basis, counterparty and termination risk all remained, and a "synthetic fix" ended in the largest US municipal bankruptcy of its time.

End of solutions.