Risk Management and Derivatives 2026
Shanghai Jiao Tong University
Summer School 2026
Midterm Exam: Topics 1–6
With Solutions

Reading time: 10 minutes. **Writing time:** 120 minutes.
Total marks: 100. **Calculators are permitted.**
Instructions: Answer **all** questions. In Section A, circle the letter of the single best answer. In Section B show your working; a correct number with no method earns little credit. Section C is a short essay. Interest rates are **continuously compounded** unless stated otherwise.

Structure:

- **Section A** – Multiple choice: 20 questions, 2.5 marks each (**50 marks**).
- **Section B** – One analytical problem in five parts (**30 marks**).
- **Section C** – One short essay (**20 marks**).

Useful formulas. Hedge ratio $h^* = \rho\,\sigma_S/\sigma_F$; contracts $N^* = h^* Q_A/Q_F$; hedge effectiveness $= \rho^2$; basis $b = S - F$; conversions $R_c = m\ln(1+R_m/m)$, $R_m = m(e^{R_c/m} - 1)$; $PV = CF_T\, e^{-z(T)T}$; $f(T_1,T_2) = \dfrac{z(T_2)T_2 - z(T_1)T_1}{T_2 - T_1}$; $\Delta B \approx -B\,D\,\Delta y$; duration hedge $N = \dfrac{P\,D_P}{V_F\,D_F}$; $F_0 = S_0 e^{rT}$, $F_0 = (S_0 - I)e^{rT}$, $F_0 = S_0 e^{(r-q)T}$, $F_0 = S_0 e^{(r+u-y)T}$; delivery cost of bond i = quote$_i - F \times CF_i$; SOFR futures price $= 100-$ rate.

Section A: Multiple Choice (20 questions × 2.5 marks = 50 marks)

1. A trader is *short* a forward contract with delivery price $K = \$90$. At maturity the spot price is $S_T = \$75$. The payoff to the short position is:

(A) +\$15.

(B) −\$15.

(C) \$0.

(D) +\$75.

Solution. (A). The short's payoff is $K - S_T = 90 - 75 = +\$15$ per unit: it sells at the agreed \$90 something now worth only \$75. The long holds the mirror image, $S_T - K = -\$15$. A forward is a zero-sum transfer of price risk between the two sides.

2. An *arbitrageur* is a market participant who:

(A) reduces an existing business exposure.

(B) takes on risk to profit from a market view.

(C) locks in a riskless profit from a price discrepancy.

(D) guarantees both sides of every cleared trade.

Solution. (C). The arbitrageur trades two routes to the same asset when their prices disagree, locking a profit with no exposure to the market's direction. The hedger is (A), the speculator (B), and the clearing house (D). Arbitrage is the enforcement mechanism behind every pricing formula in this course.

3. The main purpose of daily settlement (marking to market) in futures markets is that it:

(A) stops losses accumulating unpaid over the life of the contract.

(B) guarantees each trader a small profit on every trading day.

(C) removes the need for traders to post any initial margin.

(D) fixes the futures price at its level until the delivery day.

Solution. (A). Settling gains and losses in cash every day keeps counterparty exposure to at most one day's move; nothing large can build up unpaid the way it can in a private forward. This is the key mechanical difference between futures and forwards, and its price is that a position needs cash on hand for margin calls.

4. Two traders, neither of whom has any existing position, trade one futures contract with each other. As a result:

(A) volume rises by one; open interest is unchanged.

(B) volume and open interest each rise by one.

(C) open interest rises by one; volume is unchanged.

(D) neither volume nor open interest changes.

Solution. (B). Volume counts contracts traded, so it rises by one. Open interest counts contracts outstanding: a new long and a new short have been created, one new open contract, so open interest also rises by one. Had the trade closed two existing positions, volume would still rise but open interest would fall.

5. A copper producer will sell its output in three months and fears a price fall. Its hedge is:

(A) a long futures position.

(B) a larger inventory of copper.

(C) a short futures position.

(D) no position, since producers gain from high prices.

Solution. (C). The producer is hurt when the price falls, so it needs a position that gains when the price falls: short futures. A future *buyer* of a commodity fears a rise and hedges long. Direction always comes from the underlying exposure.

6. A hedger is *short* futures against a future sale of the asset. Between opening and closing the hedge, the basis $b = S - F$ strengthens unexpectedly. The hedger's effective sale price is:

(A) lower than expected.

(B) exactly the initial futures price.

(C) unchanged from the initial spot price.

(D) higher than expected.

Solution. (D). The short hedger's effective price is $F_1 + b_2$: the futures price locked at the start plus the closing basis. A stronger-than-expected closing basis therefore raises the effective sale price, benefiting the seller-hedger (and hurting a buyer-hedger symmetrically). The basis is exactly the part the hedge cannot fix.

7. The maximum possible loss for the *buyer* of an option is:

(A) the premium paid.

(B) the strike price.

(C) unlimited.

(D) the spot price at expiry.

Solution. (A). The holder of an option has a right, not an obligation: in the worst case the option is left to expire and the loss is the premium already paid, nothing more. Unlimited losses belong to option *writers* and to naked futures positions, which is exactly why the two must never be confused.

8. A continuously compounded rate of 5% is equivalent to an annually compounded rate of:

(A) 5.13%.

(B) 5.00%.

(C) 4.88%.

(D) 5.25%.

Solution. (A). $R_m = m(e^{R_c/m} - 1)$ with $m = 1$: $e^{0.05} - 1 = 5.13\%$. Continuous compounding credits interest constantly, so the equivalent once-a-year rate must be higher to produce the same year-end amount. Option (C) runs the conversion in the wrong direction.

9. *Bootstrapping* a zero curve means:

(A) averaging the quoted yields across all the traded bonds.

(B) forecasting the path of future central bank rate decisions.

(C) solving for zero rates one maturity at a time from bond prices.

(D) fitting a straight line through two observed bond yields.

Solution. (C). Bootstrapping works from near to far: use short instruments to fix the short zeros, then strip the known coupons out of each longer bond's price and solve for the one unknown zero rate its final payment implies. Each step uses only rates already found, which is what makes the recursion work.

10. A forward rate agreement (FRA) struck at today's forward rate:

(A) costs the present value of the notional to enter.

(B) costs nothing to enter.

(C) requires physical delivery of a bond.

(D) locks the borrower's credit spread as well.

Solution. (B). At the current forward rate the two legs of the FRA have equal present value, so the contract has zero value at the start and costs nothing, like any forward struck at the forward price. It locks only the benchmark component of a future borrowing rate; the borrower's own credit spread (D) remains open.

11. A bond portfolio is worth \$200 million with duration 4. If all yields *fall* by 50 basis points, the portfolio approximately:

(A) loses \$4 million.

(B) gains \$2 million.

(C) gains \$4 million.

(D) loses \$2 million.

Solution. (C). $\Delta B \approx -B\,D\,\Delta y = -200\text{M} \times 4 \times (-0.005) = +\$4$ million. Falling yields raise bond prices; duration scales the half-point move into a 2% gain on \$200 million. Option (B) uses half the duration; option (A) has the sign backwards.

12. A share pays no dividends and trades at $S_0 = \$50$; the one-year rate is 2%. The one-year forward price is:

(A) \$51.01.

(B) \$50.00.

(C) \$49.01.

(D) \$52.04.

Solution. (A). $F_0 = S_0 e^{rT} = 50\,e^{0.02} = \$51.01$: spot plus one year of financing. Option (C) discounts instead of compounding; option (D) uses two years of carry.

13. The same share as in Question 12 will instead pay a \$2 dividend in six months (present value \$1.98). The one-year forward price becomes:

(A) \$51.01.

(B) \$53.05.

(C) \$50.02.

(D) \$48.99.

Solution. (D). With known income, $F_0 = (S_0 - I)e^{rT} = (50 - 1.98)\,e^{0.02} = 48.02 \times 1.0202 = \$48.99$. The holder of the share collects the dividend; the forward buyer does not, so the forward price hands that income back. Whatever the asset pays its holder, the forward price subtracts.

14. A long forward position was entered some time ago at delivery price K. Today the forward price for the same maturity is F. The current value of the position is:

(A) always zero.

(B) $(K - F)$ discounted to today.

(C) $S_0 - K$.

(D) $(F - K)\,e^{-rT}$.

Solution. (D). The old contract locks a purchase at K; a new one would lock F. The old long is better off by $(F - K)$ at maturity, which is worth $(F - K)e^{-rT}$ in today's money. A forward is born worth zero (when $K = F$) but drifts into positive or negative value as the forward price moves away from the old delivery price.

15. The dollar interest rate is 4.5% and the yuan interest rate is 2%. In yuan terms, the one-year forward dollar trades:

(A) at a premium to spot.

(B) at a discount to spot.

(C) exactly at spot.

(D) wherever the market forecasts.

Solution. (B). Covered interest parity: $F_0 = S_0 e^{(r_{¥} - r_{\$})T}$ with $r_{¥} - r_{\$} = -2.5\%$, so $F_0 < S_0$: the higher-yielding currency trades at a forward discount. The discount offsets the extra dollar interest, leaving no arbitrage between the two deposits. It is parity, not a prediction (D).

16. In April 2020 a crude oil futures contract settled at a *negative* price while the next month's contract stayed positive. The episode showed that:

(A) the cost-of-carry pricing relation had simply failed.

(B) the exchange had published an erroneous settlement price.

(C) with storage full, the carry arbitrage could not run.

(D) crude oil had permanently lost its economic value.

Solution. (C). The cash-and-carry ceiling requires somewhere to put the barrels. With storage at the delivery point full, that precondition failed, so the relation could not be enforced and the expiring contract collapsed below zero while later months held. The huge spread between the two months was, in effect, the market price of one month of storage capacity.

17. In a Treasury bond futures delivery, the invoice amount the long pays the short is:

(A) $F \times CF$ minus accrued interest.

(B) F plus accrued interest.

(C) the quoted bond price times CF.

(D) $F \times CF$ plus accrued interest.

Solution. (D). The invoice is the futures price scaled by the delivered bond's conversion factor, plus the accrued interest on that bond: settlement is at a dirty price, like every bond transaction. The conversion factor adjusts the single futures price to the particular bond the short chose to deliver.

18. The futures price is $F = 95$. Bond P is quoted at 101.00 with conversion factor 1.06; bond Q at 93.00 with conversion factor 0.97. The cheapest to deliver is:

(A) bond Q, at a cost of 0.85.

(B) bond P, at a cost of 0.85.

(C) bond Q, at a cost of 0.30.

(D) bond P, at a cost of 0.30.

Solution. (D). Delivery cost = quote $- F \times CF$. Bond P: $101.00 - 95 \times 1.06 = 101.00 - 100.70 = 0.30$. Bond Q: $93.00 - 95 \times 0.97 = 93.00 - 92.15 = 0.85$. The short delivers bond P. Note that the bond with the *higher* price is the cheaper delivery once the conversion factor is applied: cheapness is about the gap, not the level.

19. A fund manager wants to hedge a long government bond portfolio (value P, duration D_P) using bond futures (contract value V_F, duration D_F). The hedge is:

(A) long futures, $N = P\,D_P/(V_F\,D_F)$ contracts.

(B) short futures, $N = P\,D_P/(V_F\,D_F)$ contracts.

(C) short futures, $N = V_F\,D_F/(P\,D_P)$ contracts.

(D) long futures, $N = P/V_F$ contracts.

Solution. (B). The portfolio loses when yields rise, so the hedge must gain then: short futures. Matching money durations gives $N = P\,D_P/(V_F\,D_F)$: the ratio of the portfolio's money duration to one contract's. Option (D) matches values but ignores that the two sides respond differently to the same yield move.

20. In March 2020, funds running the Treasury cash-futures basis trade suffered forced unwinds even though the trade holds government bonds and is close to duration-neutral. The episode showed that:

(A) government bonds carry more default risk than markets assume.

(B) leveraged positions can be forced to unwind when margins tighten.

(C) the basis trade is riskless once its duration is hedged.

(D) futures prices always track the cash bonds exactly, at all times.

Solution. (B). The trade's exposure is not direction or default; it is *leverage meeting liquidity*. When volatility spiked, margins and haircuts rose together, and funds financed at fifty-to-one had to unwind into a falling market until the central bank stepped in. Daily settlement converts credit risk into liquidity risk, and that applies even in the world's safest market.

Section B: Analytical Problem (30 marks)

An airline will buy **500,000 barrels** of jet fuel in three months and hedges with crude-oil futures; each contract covers **1,000 barrels**. Current crude futures price: $F_1 = \$85$ per barrel. From monthly price-change data: $\sigma_S = 2$ (jet-fuel spot changes), $\sigma_F = 3$ (crude-futures changes), $\rho = 0.9$. Margin per contract: initial \$6,000, maintenance \$4,500.

(a) **[6 marks]** Compute the minimum-variance hedge ratio h^* and the number of contracts N^*. Should the airline be long or short, and why?

(b) **[6 marks]** What fraction of the variance of the fuel cost does this hedge remove, and what is the remaining risk called?

(c) **[6 marks]** On the first day of the hedge the crude futures price *falls* by \$2, to \$83. For one contract, compute the variation-margin flow and the margin balance at the end of the day. Does a margin call occur, and if so, how much cash must the airline post in total across all contracts?

(d) **[6 marks]** Three months later the airline buys its fuel and closes the hedge. Crude futures have risen from \$85 to \$95, and jet fuel spot is \$97 per barrel. Compute the total futures profit, the airline's net fuel bill, and the effective price paid per barrel.

(e) **[6 marks]** To finance the fuel purchase, the airline plans to borrow for six months starting in three months. The three-month zero rate is 4.2% and the nine-month zero rate is 4.8%, both continuously compounded. Compute the forward rate for the period between month three and month nine, and name the contract the airline could enter today to lock in this borrowing rate.

Solution. (a) $h^* = \rho \dfrac{\sigma_S}{\sigma_F} = 0.9 \times \dfrac{2}{3} = \mathbf{0.6}$, and $N^* = h^* \dfrac{Q_A}{Q_F} = 0.6 \times \dfrac{500{,}000}{1{,}000} =$ **300** contracts. The airline will *buy* fuel and is hurt by a price rise, so it goes **long**: the futures must gain when prices rise.

(b) Effectiveness $= \rho^2 = 0.9^2 = \mathbf{0.81}$: the hedge removes about 81% of the variance, leaving 19%. The remainder is **basis risk**: jet fuel and crude are related but not identical, so no crude position of any size can track the fuel bill exactly.

(c) The airline is long, so a \$2 fall loses $2 \times 1{,}000 = \$2{,}000$ per contract in variation margin. The balance falls from 6,000 to \$**4,000**, which is below the maintenance level of 4,500, so a **margin call** occurs and the account must be restored to the *initial* level: a top-up of \$2,000 per contract, or $300 \times 2{,}000 = \$\mathbf{600{,}000}$ in cash that day.

(d) Futures profit $= (95 - 85) \times 1{,}000 \times 300 = +\$\mathbf{3.0}$ million. The fuel bill is $500{,}000 \times \$97 = \$48.5$ million, so the net bill is $48.5 - 3.0 = \$\mathbf{45.5}$ million: an effective price of $45.5\text{M}/500{,}000 = \$\mathbf{91}$ per barrel. The futures gain of $h^* \times \Delta F = 0.6 \times \$10 = \$6$

per barrel reduced the effective price from \$97 to \$91; the gap that remains reflects the partial hedge ratio and the basis.

(e) Using the forward-rate formula,

$$f(0.25, 0.75) = \frac{z(T_2)T_2 - z(T_1)T_1}{T_2 - T_1} = \frac{0.048 \times 0.75 - 0.042 \times 0.25}{0.75 - 0.25} = \frac{0.0255}{0.50} = \mathbf{5.1\%}$$

(continuously compounded). The curve slopes upward, so the forward rate sits above both zero rates: the nine-month rate can only average 4.8% if the later stretch earns more than 4.8%. The airline locks this rate today with a **forward rate agreement (FRA)** on the three-to-nine-month period, taking the borrower's side; the FRA fixes the benchmark rate only, so the airline's own credit spread remains open.

Section C: Essay (20 marks)

Read the scenario and write a short essay of **about half a page** (roughly 5 to 8 sentences). Marks are for clear economic reasoning, not length.

A fuel-supply company signs contracts to deliver fuel to customers at fixed prices for the next ten years. To hedge this long-term commitment it buys short-dated fuel futures, rolling them into the next month as each contract expires. The direction is right: if fuel prices rise, the futures gain what the customer contracts lose. Two years in, fuel prices fall steadily instead. The futures positions bleed cash through daily settlement month after month, while the offsetting gains on the customer contracts will arrive only over the coming decade. Facing mounting margin payments, the parent company loses patience and closes the entire position near the bottom of the market, at a loss of more than a billion dollars.

Question [20 marks]. The hedge had the right direction, yet the outcome was a disaster. Using the tools of this course, explain what was wrong with the hedge's design, why falling prices caused a cash crisis even though the firm's overall economic position was sound, and what lesson this holds for how hedging programmes should be built.

Solution. Model answer (maturity mismatch plus margin: solvency is not liquidity). The direction was right but the **maturities were not**: a ten-year exposure was hedged with one-month futures rolled again and again, which replaces one long-dated risk with a chain of roll costs, repeated basis bets, and a permanent funding requirement. The two legs also pay out on different clocks: futures losses are settled in **cash every day** through marking to market, while the offsetting gains on the fixed-price customer contracts arrive only as deliveries happen over a decade. Falling prices therefore drained real cash immediately against paper gains far in the future; the firm's overall economic position was sound (**solvent**), but it could not fund the margin calls (**illiquid**). Closing the position near the bottom crystallised exactly the loss the hedge existed to offset, converting a liquidity squeeze into a permanent one. The lessons: match the hedge's horizon to the exposure where possible; before trading, stress the *cash* path of the hedge, not just its final payoff; and secure funding lines and management commitment in advance, because a hedge abandoned in mid-storm is worse than no hedge at all.

Marking guide (20 marks).

- **8 marks** – identifies the **maturity mismatch**: long-dated exposure hedged by rolling short-dated futures (stack-and-roll), leaving roll costs and calendar-basis risk.
- **6 marks** – explains the **cash-timing asymmetry**: daily variation margin drains cash now, offsetting customer-contract gains arrive over years; sound position, unfundable path (**solvency $\neq$ liquidity**).
- **4 marks** – draws the design lessons: stress the funding path, arrange cash lines and

board commitment in advance, avoid abandoning the hedge at the worst moment.

- **2 marks** – clear, well-organised writing within the length limit.

Answers blaming the direction of the hedge, or "futures are too risky," earn little: the position lost cash precisely because it was hedging, and the failure was in design and funding, not in intent.