# Risk Management and Derivatives 2026
# Shanghai Jiao Tong University
# Summer School 2026
# Quiz 2: Topics 4–6
# With Solutions

**Reading time:** 10 minutes. **Writing time:** 60 minutes.

**Total marks:** 100. **Calculators are permitted.**

**Instructions:** Answer **all** questions. In Section A, circle the letter of the single best answer. In Section B show your working; a correct number with no method earns little credit. Section C is a short essay. Interest rates are **continuously compounded** unless stated otherwise.

**Structure:**

- **Section A** — Multiple choice: 20 questions, 2.5 marks each (**50 marks**).
- **Section B** — One analytical problem in five parts (**30 marks**).
- **Section C** — One short essay (**20 marks**).

**Useful formulas.** Conversions $R_c = m\ln(1+R_m/m)$, $R_m = m\,(e^{R_c/m}-1)$; discounting $PV = CF_T\,e^{-z(T)\,T}$; forward rate $f(T_1,T_2) = \dfrac{z(T_2)T_2 - z(T_1)T_1}{T_2 - T_1}$; forward prices $F_0 = S_0e^{rT}$, $F_0 = (S_0 - I)e^{rT}$, $F_0 = S_0e^{(r-q)T}$, $F_0 = S_0e^{(r+u-y)T}$; currency $F_0 = S_0e^{(r_{\text{dom}}-r_{\text{for}})T}$; delivery cost of bond $i = \text{quote}_i - F\times CF_i$; invoice price $= F\times CF + \text{accrued}$; accrued interest $= \text{coupon}\times\dfrac{\text{days since last coupon}}{\text{days in period}}$.

## Section A: Multiple Choice (20 questions × 2.5 marks = 50 marks)

**1.** A bank quotes the same stated rate of 4% under four compounding conventions. Which convention gives the largest balance after one year?

(A) Annual compounding.

(B) Semiannual compounding.

(C) Quarterly compounding.

(D) Continuous compounding.

**Solution. (D).** The more often interest is credited, the sooner it starts earning interest on itself, so for the same stated rate the balance rises with the compounding frequency: annual < semiannual < quarterly < continuous. Continuous compounding, $e^{0.04}$, is the limit of compounding ever more often. This is why a rate means nothing until its convention is stated.

**2.** A rate of 6% with semiannual compounding is equivalent to a continuously compounded rate of:

(A) 6.09%.

(B) 6.00%.

(C) 5.91%.

(D) 5.83%.

**Solution. (C).** $R_c = m \ln(1 + R_m/m) = 2\ln(1.03) = 5.91\%$. The continuous equivalent of a discretely compounded rate is always a little smaller, because continuous crediting needs a lower stated rate to reach the same year-end amount. Option (A), 6.09%, is the conversion run the wrong way (the annually compounded equivalent of 6% semiannual).

**3.** The two-year *zero rate* $z(2)$ is the rate on:

(A) a new two-year bond issued at its face value.

(B) a loan repaid in equal monthly instalments over two years.

(C) money lent now and repaid in a single amount in two years.

(D) an average of this year's and next year's short-term rates.

**Solution. (C).** A zero rate belongs to a single cash flow: money out today, one repayment at the maturity date, nothing in between. That purity is what makes zeros the building blocks of all pricing: a coupon bond is valued as a bundle of single cash flows, each discounted at its own zero rate. A par bond's rate (A) mixes several payment dates into one number.

**4.** The two-year zero rate is 3%. The present value of \$100 received in two years is:

(A) \$94.18.

(B) \$94.26.

(C) \$97.04.

(D) \$106.18.

**Solution. (A).** $PV = 100\,e^{-0.03\times 2} = 100\,e^{-0.06} = \$94.18$. Option (B), \$94.26, uses discrete discounting $100/1.03^2$; option (C) discounts for only one year; option (D) compounds up instead of discounting.

**5.** The one-year zero rate is 2% and the two-year zero rate is 3%. The forward rate for the second year $\big(f(1,2)\big)$ is:

(A) 4.0%.

(B) 2.5%.

(C) 3.0%.

(D) 5.0%.

**Solution. (A).** $f(1,2) = \dfrac{z(2)\times 2 - z(1)\times 1}{2-1} = \dfrac{0.06-0.02}{1} = 4\%$. The two-year zero is an *average* over both years; if the first year costs only 2%, the second must cost 4% for the average to come out at 3%. The forward is the marginal cost of the next period, and on a rising curve it sits above the zeros.

**6.** Yield curves usually slope upward. The standard explanation from the lecture is that:

(A) investors always expect inflation to rise steadily over time.

(B) central banks set long rates above the short rates they control.

(C) long bonds trade more actively and are easier to sell quickly.

(D) lenders prefer to lend short, so long rates carry a premium.

**Solution. (D).** This is liquidity preference: depositors like to keep their money accessible (lend short), while borrowers like certainty (borrow long). Reconciling the two requires long rates to pay a premium, which pushes forward rates above expected future short rates and tilts the curve upward even when no rate rise is expected. Central banks (B) control only the short end.

**7.** A stock trades at \$50 and will pay a dividend whose present value is \$2 before a one-year forward contract matures. The one-year rate is 4%. The forward price is closest to:

(A) \$52.04.

(B) \$49.96.

(C) \$50.00.

(D) \$48.00.

**Solution. (B).** With known income, $F_0 = (S_0 - I)e^{rT} = (50 - 2)e^{0.04} = 48 \times 1.0408 =$ \$49.96. The dividend is cash the forward holder forgoes, so its present value is stripped from the spot price before the rest is carried forward. Option (A), \$52.04, forgets the income and carries the full spot; option (D) strips the income but forgets to carry it forward.

**8.** A futures and a forward on the same asset share the same maturity. The futures price tends to *exceed* the forward price when:

(A) the asset pays a large known income.

(B) the asset's price is strongly *negatively* correlated with interest rates.

(C) the asset's price is strongly *positively* correlated with interest rates.

(D) interest rates are known and constant over the contract's life.

**Solution. (C).** Futures are settled daily. When the asset's price rises just as rates rise, the long's daily gains arrive exactly when they can be reinvested at the new higher rate, while losses are financed when rates are low; that timing advantage makes the futures worth a little more than the otherwise identical forward. If rates are known and constant (D), daily settlement carries no such advantage and the two prices are equal.

**9.** A government bond pays a semiannual coupon of \$4.00. Ninety days have passed since the last coupon in a 180-day coupon period. The accrued interest the buyer adds to the clean price is:

(A) \$1.00.

(B) \$2.00.

(C) \$4.00.

(D) \$8.00.

**Solution. (B).** Accrued interest = coupon × (days since last coupon)/(days in period) $= 4.00 \times 90/180 = \$2.00$: the seller is owed half of the current coupon for the half-period already held. The buyer pays the clean quote *plus* this accrued amount, which is the dirty (cash) price. Option (C) hands over a whole coupon; option (D) uses the annual coupon.

**10.** Gold trades at \$2,400 per ounce, the six-month interest rate is 3%, and gold pays no income and (here) costs nothing to store. The six-month forward price is:

(A) \$2,436.27.

(B) \$2,400.00.

(C) \$2,473.10.

(D) \$2,364.27.

**Solution. (A).** $F_0 = S_0 e^{rT} = 2{,}400\, e^{0.03\times 0.5} = 2{,}400\, e^{0.015} = \$2{,}436.27$. The forward price is the spot price plus the cost of carrying the ounce for six months, which here is only the financing. Option (C) carries it for a full year; option (D) discounts instead of compounding.

**11.** A stock-index futures price can sit *below* the index itself, with no arbitrage available, when:

(A) the dividend yield exceeds the interest rate.

(B) the market expects the index to fall.

(C) the futures contract is mispriced.

(D) index funds are selling futures heavily.

**Solution. (A).** With $F_0 = S_0 e^{(r-q)T}$, a dividend yield $q$ above the interest rate $r$ makes the exponent negative, so $F_0 < S_0$. The holder of the shares collects dividends the futures holder forgoes, and the futures price hands that carry back. It is lawful carry, not a forecast (B) and not a mispricing (C).

**12.** A one-year forward on a non-dividend share is quoted well *above* $S_0 e^{rT}$. The arbitrage is to:

(A) buy the forward and short the share.

(B) buy both the share and the forward.

(C) short the share and lend the proceeds.

(D) borrow, buy the share, and sell the forward.

**Solution. (D).** Cash and carry: borrow $S_0$, buy the share today, sell it forward at the rich quote. At maturity, deliver the share, repay $S_0e^{rT}$, and keep the difference with no risk. Selling the expensive route to owning the share and buying the cheap one is exactly the trading pressure that pins the forward to $S_0e^{rT}$.

**13.** The dollar interest rate is above the yuan interest rate, and the one-year forward price of the dollar (in yuan) is below today's spot rate. This forward discount tells us that:

(A) the market is forecasting a weaker dollar over the year.

(B) the interest gap between the two currencies is being priced in.

(C) the central bank has fixed the forward rate by decree.

(D) dollar deposits carry more default risk than yuan deposits.

**Solution. (B).** Covered interest parity: $F_0 = S_0e^{(r_{¥}-r_{\$})T}$, and with $r_{\$} > r_{¥}$ the exponent is negative, so the dollar must trade at a forward discount. The discount exactly offsets the extra interest a dollar deposit earns over the year; locking the forward is not free money, and the forward is not a forecast. Reading forward points as an exchange-rate prediction is the most common error with this material.

**14.** For a consumption asset such as crude oil, only the upper bound $F_0 \leq S_0e^{(r+u)T}$ can be enforced by arbitrage, because:

(A) storage costs are too high and too variable to measure.

(B) firms will not sell or lend the physical asset they need.

(C) short selling commodities is prohibited everywhere.

(D) futures markets in consumption assets are too small to use.

**Solution. (B).** The ceiling is policed by anyone with storage: if the futures is too high, buy spot, store, deliver. The floor would need holders to sell their inventory and buy it back forward, but refiners and manufacturers hold the physical asset to *use* it and will not part with their feedstock. One arbitrage arm fails, so the pricing relation becomes one-sided.

**15.** The *convenience yield* on a commodity is:

(A) the interest earned on the margin account.

(B) the dividend the commodity pays its holder.

(C) the benefit of holding the physical asset itself.

(D) the fee charged by the warehouse.

**Solution. (C).** The convenience yield is what physically holding the barrel or tonne is worth to a user: keeping a refinery running, meeting a delivery when supplies are tight. It is paid in availability rather than cash, and it is *implied* from the gap between spot and futures prices rather than quoted anywhere, the same inversion we will meet again with implied volatility.

**16.** A commodity futures market is in contango ($F_0 > S_0$) when:

(A) the convenience yield exceeds interest plus storage.

(B) the spot price is expected to rise.

(C) interest plus storage exceeds the convenience yield.

(D) the commodity pays its holder a high income.

**Solution. (C).** From $F_0 = S_0 e^{(r+u-y)T}$: the futures sits above spot when the cost of carrying the asset forward $(r + u)$ outweighs the benefit of holding it now $(y)$. Backwardation is the reverse, $y > r + u$: the market pays a premium to have the barrel today. The curve shapes from Lecture 2 now have a cause.

**17.** A government bond is quoted at 100.40. The buyer actually pays:

(A) the quoted price minus accrued interest.

(B) the quoted price plus accrued interest.

(C) exactly the quoted price.

(D) the face value plus one coupon.

**Solution. (B).** Bonds are quoted *clean* but settle *dirty*: cash price = quoted price + interest accrued since the last coupon. Clean quoting keeps the published price from jumping mechanically at each coupon date, but the seller must still be paid for the coupon interest already earned while holding the bond.

**18.** In a Treasury bond futures contract with a 3% notional coupon, a deliverable bond has a conversion factor greater than 1 exactly when:

(A) its remaining maturity exceeds ten years.

(B) it currently trades above its face value.

(C) its yield exceeds the notional coupon.

(D) its coupon exceeds the notional coupon.

**Solution. (D).** The conversion factor is the bond's price per unit of face value if the entire curve were flat at the notional coupon (3% at CFFEX, 6% at CBOT). At a flat 3%, a bond paying more than 3% is worth more than face, so $CF > 1$; paying less, worth less than face, $CF < 1$. It is a fixed convention for making unlike bonds comparable, not a market price (B).

**19.** The futures price is $F = 100$. Two bonds are deliverable: bond X, quoted at 110.00 with conversion factor 1.05; and bond Y, quoted at 98.00 with conversion factor 0.95. The cheapest to deliver is:

(A) bond Y, at a delivery cost of 3.00.

(B) bond X, at a delivery cost of 5.00.

(C) bond X, at a delivery cost of 3.00.

(D) bond Y, at a delivery cost of 5.00.

**Solution. (A).** The short's cost of delivering bond $i$ is $\text{quote}_i - F \times CF_i$. Bond X: $110.00 - 100 \times 1.05 = 5.00$. Bond Y: $98.00 - 100 \times 0.95 = 3.00$. The short delivers the bond with the smaller gap, bond Y. A large conversion factor alone says nothing about cheapness: it must be compared with the bond's market price.

**20.** A Treasury-bond futures contract settles at a futures price of 98.00. The short delivers a bond with conversion factor 1.10 and accrued interest \$0.50 (per \$100 face). The invoice price the short receives is:

(A) \$98.50.

(B) \$98.00.

(C) \$107.80.

(D) \$108.30.

**Solution. (D).** The invoice price is $F \times CF + \text{accrued} = 98.00 \times 1.10 + 0.50 = 107.80 + 0.50 =$ \$108.30. The conversion factor scales the single futures price to the particular bond delivered, and the short is still owed the coupon interest accrued since the last payment. Option (C) forgets the accrued interest; option (A) adds accrued to the raw futures price without applying the conversion factor.

## Section B: Analytical Problem (30 marks)

A treasury desk faces the following market this morning. Zero rates, continuously compounded: $z(0.5) = 2.0\%$, $z(1) = 2.5\%$, $z(2) = 3.0\%$. Gold trades at $S_0 = \$2{,}400$ per ounce, pays no income, and (for this problem) costs nothing to store. The desk also holds a government bond portfolio.

(a) **[6 marks]** Compute the present value of \$104 received in two years. Then compute the forward rate for the second year, $f(1,2)$, and explain in one sentence why it is higher than both zero rates.

(b) **[6 marks]** The desk can buy a government bond today for \$100.00. The bond will pay a \$3 coupon in six months, and nothing else before a one-year forward on it matures. Using the zero rates above, compute the one-year forward price of the bond, and explain in one sentence why the coupon lowers the forward.

(c) **[6 marks]** Compute the six-month forward price of gold.

(d) **[6 marks]** A dealer quotes a six-month gold forward at \$2,460. Describe the arbitrage step by step, and compute the riskless profit per ounce at delivery.

(e) **[6 marks]** A colleague argues: "the six-month forward price is the market's forecast of the gold price in six months." Using your answers to parts (c) and (d), explain in two or three sentences why this reading is wrong.

**Solution. (a)** $PV = 104\,e^{-z(2)\times 2} = 104\,e^{-0.06} = \$\mathbf{97.94}$. The forward rate is

$$f(1,2) = \frac{z(2)\times 2 - z(1)\times 1}{2-1} = \frac{0.060 - 0.025}{1} = \mathbf{3.5}\%.$$

The two-year zero is the *average* cost of money over two years; with the first year costing only 2.5%, the second year alone must cost 3.5% to pull the average up to 3.0%. On a rising curve the marginal rate always sits above the average it is lifting.

**(b)** The coupon is known income, so strip its present value from the spot price before carrying the rest forward. The \$3 coupon paid at six months is worth $I = 3\,e^{-z(0.5)\times 0.5} = 3\,e^{-0.010} = \$2.97$ today. Then

$$F_0 = (S_0 - I)\,e^{z(1)\times 1} = (100.00 - 2.97)\,e^{0.025} = 97.03\,e^{0.025} = \$\mathbf{99.49}.$$

The coupon is cash the forward buyer will not receive, so it is removed from the amount that has to be financed to maturity; known income always lowers the forward price.

**(c)** Six-month carry at the six-month zero:

$$F_0 = S_0\,e^{z(0.5)\times 0.5} = 2{,}400\,e^{0.020\times 0.5} = 2{,}400\,e^{0.010} = \$\mathbf{2{,}424.12}.$$

**(d)** The quote \$2,460 is above the fair value \$2,424.12, so run cash and carry: **(1)** borrow \$2,400 for six months at 2.0%; **(2)** buy one ounce today; **(3)** sell it six months forward at \$2,460. At delivery: hand over the ounce, collect \$2,460, repay the loan $2{,}400\,e^{0.010} = \$2{,}424.12$. Riskless profit $= 2{,}460.00 - 2{,}424.12 = \$\mathbf{35.88}$ per ounce, locked in today whatever the gold price does.

**(e)** The forward price is pinned by **arbitrage** to spot plus the cost of carry, $S_0 e^{rT}$; no forecast enters that calculation anywhere. Part (d) shows the enforcement: when the quote drifted above spot-plus-carry, a riskless trade appeared, and such trading pushes the price back regardless of anyone's view on gold. Expectations about gold live in today's *spot* price; the forward simply adds the financing cost of waiting.

## Section C: Essay (20 marks)

Read the scenario and write a short essay of **about half a page** (roughly 5 to 8 sentences). Marks are for clear economic reasoning, not length.

*A trader is **short** one Treasury-bond futures contract. As delivery nears, she realises the contract lets her choose which of several eligible bonds to hand over, and that the choice is worth real money. A colleague objects: "the conversion-factor system exists precisely to make every deliverable bond equally good to deliver, so your choice cannot matter."*

**Question [20 marks].** Using the tools of this course, explain (i) what the conversion factor is trying to do and why it does *not* make all bonds equally cheap to deliver; (ii) how the short decides which bond is cheapest to deliver; and (iii) why this delivery choice is valuable to the short and how it feeds into the futures price.

**Solution. Model answer (the cheapest-to-deliver option).** The conversion factor prices each eligible bond as if the whole yield curve were flat at the contract's notional coupon, turning one futures price into a fair invoice price for very different bonds; it is a fixed convention, not a market price. Because real yields are not flat at the notional coupon, those factors are only approximate, so one bond always ends up cheaper to deliver than the rest. The short compares the net cost of delivering each candidate, $\text{quote}_i - F \times CF_i$, and delivers the bond with the smallest gap, the cheapest-to-deliver. That freedom is a genuine **option**: whatever the curve does, the short hands over whichever bond is cheapest at the time, and the option is worth more when yields move enough to switch which bond wins. Because the short holds this valuable option, the market prices the futures off the cheapest-to-deliver bond and a little *lower* than it would be without the choice; the long accepts the lower price as payment for granting the delivery flexibility. So the colleague is wrong: the conversion factor narrows the differences between bonds but cannot erase them, and the gap that remains is exactly what makes the short's choice worth money.

**Marking guide (20 marks).**

- **8 marks** — explains the **conversion factor**: it prices each bond as if the curve were flat at the notional coupon, a convention to compare unlike bonds; being only approximate, it does not make them equally cheap.
- **6 marks** — states the **cheapest-to-deliver rule**: the short minimises $\text{quote}_i - F \times CF_i$ and delivers that bond.
- **4 marks** — identifies the **delivery option's value**: the short always delivers the cheapest bond, and the option is worth more when yield moves switch which bond is cheapest.
- **2 marks** — links to the **futures price**: it is set off the cheapest-to-deliver and sits a little lower to compensate the long for the short's option; clear writing within the

length limit.

Answers that stop at "the conversion factor makes them equal" earn little: that is exactly the misconception the question is built to test.