Risk Management and Derivatives 2026
Shanghai Jiao Tong University
Summer School 2026
Quiz 3: Topics 7–9
With Solutions

Reading time: 10 minutes. **Writing time:** 60 minutes.
Total marks: 100. **Calculators are permitted.**
Instructions: Answer **all** questions. In Section A, circle the letter of the single best answer. In Section B show your working; a correct number with no method earns little credit. Section C is a short essay. Interest rates are **continuously compounded** unless stated otherwise.

Structure:

- **Section A** — Multiple choice: 20 questions, 2.5 marks each (**50 marks**).
- **Section B** — One analytical problem in five parts (**30 marks**).
- **Section C** — One short essay (**20 marks**).

Useful formulas. Discounting $PV = CF_T\, e^{-z(T)\,T}$; discount factor $d_t = e^{-z(t)\,t}$; par swap rate $s = \dfrac{100\,(1-d_n)}{\sum_{i=1}^{n} d_i}$ per 100 of notional; swap value to the fixed payer $V = B_{\text{float}} - B_{\text{fixed}}$, and a floating-rate bond is worth par at a reset date; loss fraction of a tranche spanning (a,d) when the pool loses L: $\min\big(\max\big(\frac{L-a}{d-a},0\big),1\big)$; call intrinsic value $= \max(S-K,0)$, put intrinsic value $= \max(K-S,0)$, time value $=$ premium $-$ intrinsic value; naked-call writer margin per share $=$ the greater of $\big(0.20\,S + \text{premium} - \text{out-of-the-money amount}\big)$ and $\big(0.10\,S + \text{premium}\big)$.

Section A: Multiple Choice (20 questions × 2.5 marks = 50 marks)

1. A two-year interest-rate swap is struck today at the current market swap rate. Setting aside dealer spreads, its value at signing is:

(A) positive to the fixed payer.

(B) zero to both parties.

(C) positive to the fixed receiver.

(D) equal to the first year's net payment.

Solution. (B). The market swap rate is the par yield: it is the fixed coupon that prices the fixed bond at exactly 100, which is the value of the floating bond at a reset. With $B_{\text{float}} = B_{\text{fixed}}$, the swap is born worth zero to both sides; neither party pays the other to enter. Options (A) and (C) confuse a fair price with a forecast, and (D) confuses value with cash flow.

2. A two-year annual swap curve gives discount factors $d_1 = 0.97$ and $d_2 = 0.94$. The par swap rate is closest to:

(A) 1.57%.

(B) 3.00%.

(C) 6.00%.

(D) 3.14%.

Solution. (D). $s = \dfrac{100\,(1 - d_2)}{d_1 + d_2} = \dfrac{100 \times 0.06}{1.91} = 3.14\%$. Option (B), 3.00%, divides by the number of years instead of the sum of discount factors; option (C) is the numerator alone, forgetting the annuity in the denominator; option (A) uses $1-d_1$ in the numerator.

3. In a plain-vanilla swap on a notional of \$200 million, the fixed rate is 3.5% and this year's floating benchmark was set at 2.9%. At the year-end payment date:

(A) the fixed payer pays a single net \$1.2 million.

(B) the fixed payer pays a single net \$5.8 million.

(C) the two parties exchange \$7.0 million against \$5.8 million.

(D) the two parties also exchange the \$200 million notional.

Solution. (A). Payments net: the fixed payer owes 3.5% and is owed 2.9% on the same notional, so one transfer of $(0.035 - 0.029) \times 200\text{M} = \$1.2\text{M}$ moves. Option (C) is the gross picture that netting removes; option (B) mistakes the floating leg for the net; option (D) forgets that the notional in an interest-rate swap is recorded, never exchanged.

4. At a reset date, a floating-rate bond is worth:

(A) its face value minus the coupon just set.

(B) its face value only if the yield curve is flat.

(C) exactly its face value (par).

(D) the average of its remaining expected coupons.

Solution. (C). A floating-rate bond is a chain of one-period deposits, each refreshed at the market rate. In one period it pays $100(1+R)$ where R is the rate just set; discounting at that same rate gives $100(1 + R)/(1 + R) = 100$, and induction carries the argument back to today. The result holds for any curve shape, which is what makes option (B) wrong and the swap's floating leg priceable in one line.

5. Firm X can borrow at 3.4% fixed or benchmark +0.4% floating; firm Y at 5.2% fixed or benchmark +1.1% floating. The total comparative-advantage saving a swap between them can share out is:

(A) 1.8% per year.

(B) 1.1% per year.

(C) 0.7% per year.

(D) 2.5% per year.

Solution. (B). The saving is the difference of the differences: the fixed gap is $5.2-3.4 = 1.8\%$, the floating gap is $1.1 - 0.4 = 0.7\%$, so the total to share is $1.8 - 0.7 = 1.1\%$ a year. Option (A) is the fixed gap alone, option (C) the floating gap alone, and option (D) adds the gaps instead of subtracting them.

6. The lecture's critique of the comparative-advantage argument is that:

(A) the weaker firm's own credit spread is not fixed by the swap and resets at every loan rollover.

(B) the two firms can never both want the side of the swap the argument assigns to them.

(C) the dealer's fee in practice absorbs more than the whole of the apparent saving.

(D) the argument counts the swap's notional twice, once on each of the two legs.

Solution. (A). The swap fixes the benchmark, not the firm's own spread: the weaker borrower's floating loan rolls, and if its credit worsens the spread widens. Much of the apparent 1.1%-type saving is therefore payment for a rollover risk that stays with the weaker firm, not a free gain. Options (B), (C), and (D) are not the textbook critique.

7. A currency swap differs from a plain-vanilla interest-rate swap in that:

(A) its floating leg must reset more frequently than once a year.

(B) it cannot be valued as the difference of two bonds.

(C) its fixed rate is set by the foreign curve alone.

(D) principals are exchanged at the start and re-exchanged at maturity.

Solution. (D). In a currency swap the two principals, set at the inception exchange rate, actually move: out at the start, back at maturity at that same original rate. Credit exposure therefore includes the principal at the final exchange, unlike an interest-rate swap where only net interest moves. Option (B) is false: a currency swap is valued as one bond per currency, with a single spot conversion.

8. A pool backs three tranches: equity absorbs losses from 0 to 5%, mezzanine from 5 to 20%, senior above 20%. The pool loses 8%. The mezzanine tranche loses:

(A) 8% of its principal.

(B) 100% of its principal.

(C) 20% of its principal.

(D) 3% of its principal.

Solution. (C). Losses fill from the bottom. The first 5 points wipe the equity tranche; the remaining 3 points fall on a mezzanine layer that is 15 points thick: $(8-5)/15 = 20\%$. Option (A) spreads the loss pro rata, which is exactly what the waterfall prevents; option (D) reports the loss in pool points rather than as a fraction of the tranche.

9. In a securitisation, the equity tranche is promised the highest yield because:

(A) it absorbs the pool's first losses, standing in front of every other tranche.

(B) it has the longest legal maturity of the three tranches.

(C) it is paid first out of the pool's cash collections each period.

(D) it is the largest tranche and so carries the most principal.

Solution. (A). Subordination is the whole technology: cash fills the senior tranche first, and losses consume the equity tranche first. The lower layers are paid more precisely because they stand in front of the losses. Option (C) inverts the waterfall, the most common error with this material; option (D) inverts the sizes, since the senior tranche is typically much the largest.

10. Two loans each default with probability 30%. A "senior" note loses only if *both* default. If the two defaults are independent, the probability the note is hit is:

(A) 15%.

(B) 9%.

(C) 30%.

(D) 60%.

Solution. (B). Independence multiplies: $0.30 \times 0.30 = 0.09 = 9\%$. Option (C), 30%, is what the note's risk becomes if the two loans default *together* (perfect correlation): more than three times larger with nothing changed but the correlation, which is the lecture's central experiment. Option (D) adds the probabilities; option (A) averages them and halves.

11. Default correlation in a loan pool rises while each loan's individual default probability is unchanged. The pool's *expected* loss:

(A) rises, and every tranche becomes riskier.

(B) falls, because diversification improves.

(C) is unchanged, and so is every tranche's risk.

(D) is unchanged, but the senior tranche becomes riskier.

Solution. (D). Expected loss is the average of the individual default probabilities, which have not moved. What correlation changes is the *shape* of the loss distribution: the tail fattens, and the tail is where the senior tranche lives. So the pool is worth the same while the split of its risk across tranches shifts toward the top, which is why (C), the subtle distractor, is wrong.

12. An ABS is tranched 5/15/80 (equity/mezzanine/senior), and an ABS CDO with the same 5/15/80 structure is built from the mezzanine tranches. The mortgage pool loses 17%. The CDO's senior tranche loses:

(A) 0%.

(B) 17%.

(C) 75%.

(D) 80%.

Solution. (C). Two passes through the waterfall. The ABS mezzanine loses $(17-5)/15 = 80\%$; that 80% is the CDO's pool loss, so the CDO senior tranche loses $(80-20)/80 = 75\%$. Option (D) stops after the first pass; option (A) is the intuition the re-tranching destroyed: a AAA label two floors up from a 17% pool loss still loses three quarters of its principal.

13. A AAA senior tranche and a AAA corporate bond carry the same estimated default probability. The argument from the lecture that the tranche should nonetheless *yield more* is that:

(A) it fails only in a systematic catastrophe, while the corporate bond fails idiosyncratically.

(B) its promised cash flows arrive later on average than the corporate bond's coupons.

(C) its rating is reviewed less often than the rating of a comparable corporate bond.

(D) it is exempt from the risk-retention rules that cover ordinary corporate issuers.

Solution. (A). The mispricing was about *when* an asset fails, not how often. A corporate AAA defaults as one weak firm in an ordinary world; a senior tranche defaults only when housing, jobs, and markets fail together, precisely the states in which a dollar is most valuable. An asset that pays nothing in the worst states should be cheaper, that is, yield more; the two paid almost the same.

14. A stock trades at \$73. A call with strike \$70 is quoted at \$5.20. Its time value is:

(A) \$3.00.

(B) \$2.20.

(C) \$5.20.

(D) \$8.20.

Solution. (B). Intrinsic value is $\max(73-70, 0) = \$3.00$; time value is the rest of the premium, $5.20 - 3.00 = \$2.20$. Option (A) is the intrinsic value itself; option (C) treats the whole premium as time value, true only for an out-of-the-money option; option (D) adds where it should subtract.

15. Across the strikes of one expiry, time value is largest:

(A) deep in the money.

(B) deep out of the money.

(C) at the money.

(D) at whichever strike has the highest open interest.

Solution. (C). Time value is the price of what might still happen. Far in the money the outcome is nearly settled; far out of the money it is nearly hopeless; at the money the remaining life has the greatest power to change the contract's fate, so the hump of time value peaks there. Liquidity tends to follow time value, so (D) reverses cause and effect.

16. You paid \$4 for a call with strike \$50. At expiry the stock trades at \$52. You should:

(A) abandon the option, since the trade has lost money overall.

(B) exercise only if the stock is above \$54, the breakeven price.

(C) abandon the option and write off the premium against the loss.

(D) exercise, collect the \$2 payoff, and accept a \$2 overall loss.

Solution. (D). At expiry the premium is sunk; the only live comparison is exercise value against nothing. Exercising pays $52 - 50 = \$2$, so the trade ends at $2 - 4 = -\$2$ instead of $-\$4$. Options (A) and (C) throw away \$2 to avoid admitting a loss; option (B) confuses the breakeven of the whole trade with the exercise decision, the classic payoff-versus-profit error.

17. Which position has a *bounded* maximum gain but an *unbounded* maximum loss?

(A) A long call.

(B) A naked short call.

(C) A long put.

(D) A short put.

Solution. (B). The naked call writer's best case is keeping the premium; the worst case grows without limit as the stock rises, since delivery at the strike is owed however high the price goes. The short put (D) is the near miss: its gain is also capped at the premium, but its loss is bounded because the stock cannot fall below zero. The long positions (A), (C) risk only the premium.

18. A stock trades at $S = \$56$. A client writes one naked call, strike \$60, premium \$2.10, contract size 100 shares. Using the margin rule in the formulas box, the margin per contract is:

(A) \$930.

(B) \$770.

(C) \$210.

(D) \$1,330.

Solution. (A). The out-of-the-money amount is $60 - 56 = \$4$. Method one: $0.20 \times 56 + 2.10 - 4.00 = \$9.30$ per share; method two: $0.10 \times 56 + 2.10 = \$7.70$. The greater is 9.30, so the margin is $9.30 \times 100 = \$930$. Option (B) takes the smaller leg; option (D) forgets the out-of-the-money deduction; option (C) is merely the premium, which is the writer's income, not the writer's cover.

19. You hold one call, strike \$80, on 100 shares. The company announces a 2-for-1 stock split and the share price halves from \$88 to \$44. After the standard adjustment, your position is:

(A) worthless, since \$44 is below the \$80 strike.

(B) an \$80 strike on 200 shares.

(C) a \$40 strike on 100 shares.

(D) a \$40 strike on 200 shares.

Solution. (D). Adjustments neutralise corporate arithmetic: on a 2-for-1 split the strike halves and the contract size doubles, so the claim's economics are identical before and after. At \$44 against a \$40 strike on 200 shares, the position is still in the money by the same \$800 of intrinsic value as before. Options (B) and (C) each apply half the adjustment; option (A) applies none.

20. Option buyers post no margin while option writers post margin daily. The reason is that:

(A) buyers are screened for creditworthiness before they are allowed to trade.

(B) buyers' positions are individually too small for the clearing house to track.

(C) the buyer's worst case is pre-paid in full; the writer's has no ceiling.

(D) exchanges keep a convention inherited from the era of paper certificates.

Solution. (C). A bought option can hurt its holder only once: the premium leaves the account on day one and nothing more is ever demanded, so the clearing house has no further claim. The writer has pre-sold risk instead: the obligation has no fixed ceiling, so it is collateralised nightly, and the margin grows as the market moves toward the strike.

Section B: Analytical Problem (30 marks)

A markets desk at a mid-sized bank faces three books this morning. Zero rates, continuously compounded: $z(1) = 2.0\%$, $z(2) = 3.0\%$. Swap payments are annual. The desk must price a client swap, assess a securitisation offer, and clear a client's option orders.

(a) **[6 marks]** Compute the discount factors d_1 and d_2 and the two-year par swap rate. Explain in one sentence why a swap struck at that rate is worth zero at signing.

(b) **[6 marks]** A client swap struck some time ago pays 4.0% fixed annually against the floating benchmark on a notional of \$100 million. Exactly one exchange remains, in one year, and the floating rate has just been reset. Value the swap to the fixed *payer* as the difference of two bonds, state the dollar value on the full notional, and interpret the sign in one sentence.

(c) **[6 marks]** The desk is offered the mezzanine tranche of a mortgage securitisation tranched equity 0–5%, mezzanine 5–20%, senior 20–100% of pool losses. In the stress scenario the pool loses 11%. Compute the percentage loss of each tranche in that scenario, and state the pool loss at which the mezzanine tranche is wiped out.

(d) **[6 marks]** On the options book, the underlying stock trades at \$62. Two calls are quoted: strike \$60 at \$4.10, and strike \$65 at \$1.60 (contract size 100 shares). Decompose each premium into intrinsic value and time value, and state each option's moneyness.

(e) **[6 marks]** The client now writes one naked call, strike \$65, at the quoted \$1.60. Using the margin rule in the formulas box, compute the margin per contract and compare it with the premium received. Then explain in one or two sentences why the *buyer* of the same call posts no margin at all.

Solution. (a) $d_1 = e^{-0.02} = 0.98020$ and $d_2 = e^{-0.03\times 2} = e^{-0.06} = 0.94176$. The par swap rate is

$$s = \frac{100\,(1-d_2)}{d_1+d_2} = \frac{100\times 0.05824}{1.92196} = \frac{5.8235}{1.92196} = \mathbf{3.03}\%.$$

This is the par yield: the fixed coupon that prices the fixed bond at exactly 100, which is what the floating bond is worth at a reset, so $B_{\text{float}} = B_{\text{fixed}}$ and the swap is born worth zero.

(b) At a reset the floating bond is worth par: $B_{\text{float}} = 100$. The fixed leg is one remaining payment of coupon plus principal:

$$B_{\text{fixed}} = 104\,e^{-0.02} = 104\times 0.98020 = 101.941.$$

So, per 100 of notional,

$$V_{\text{pay fixed}} = B_{\text{float}} - B_{\text{fixed}} = 100 - 101.941 = \mathbf{-1.94},$$

which is $-\$$**1.94 million** on the \$100 million notional. The client locked 4.0% when it entered, but one-year money now costs about 2%, so paying fixed above the market makes the swap a liability to the fixed payer. (Cross-check by the FRA view: the just-set benchmark is $e^{0.02} - 1 = 2.0201\%$, and $(2.0201 - 4.0) \times 0.98020 = -1.94$ per 100. The two decompositions agree, as the law of one price requires.)

(c) Losses fill from the bottom. At a pool loss of 11%: the **equity** tranche (0–5) is wiped out, a **100%** loss; the **mezzanine** (5–20) takes the remaining 6 points on a 15-point layer, $(11 - 5)/15 = \mathbf{40}\%$; the **senior** (20–100) is untouched, **0**%, since $11 < 20$. The mezzanine is wiped out when the pool loss reaches its detachment point, $L = \mathbf{20}\%$.

(d) With $S = 62$: the **60 call** has intrinsic value $\max(62 - 60, 0) = \$2.00$ and time value $4.10 - 2.00 = \$2.10$; it is **in the money**. The **65 call** has intrinsic value $\max(62 - 65, 0) = \$0$ and time value equal to its whole premium, \$1.60; it is **out of the money**. Both time values are positive, as they must be for a live option, and the larger time value sits at the strike nearer the money.

(e) The out-of-the-money amount of the 65 call is $65 - 62 = \$3.00$. Per share: method one gives $0.20 \times 62 + 1.60 - 3.00 = 12.40 + 1.60 - 3.00 = \$11.00$; method two gives $0.10 \times 62 + 1.60 = \$7.80$. The greater is 11.00, so the margin is $11.00 \times 100 = \$\mathbf{1{,}100}$ per contract, against a premium received of only \$160: roughly seven times the income, posted as cover. The buyer posts nothing because the buyer's worst case, the premium, was paid in full on day one, so the clearing system has no further claim; the writer's loss has no ceiling, so the obligation is collateralised daily.

Section C: Essay (20 marks)

Read the scenario and write a short essay of **about half a page** (roughly 5 to 8 sentences). Marks are for clear economic reasoning, not length.

An investment committee is offered the senior tranche of a residential-mortgage securitisation, rated AAA and attaching at 20% of pool losses. One member argues: "this is as safe as a AAA corporate bond, and it pays the same. The pool holds ten thousand loans; they cannot all fail at once, and losses must burn through the equity and mezzanine tranches before they ever reach us."

Question [20 marks]. Using the tools of this course, explain (i) how pooling and tranching manufacture a AAA security out of risky loans, and what the 20% attachment point does; (ii) why the senior tranche's safety is governed by *default correlation* rather than by the average quality of the loans; and (iii) what the 2007–08 episode revealed about the committee member's argument.

Solution. Model answer (the senior tranche and the correlation assumption). Pooling gathers the loans into a bankruptcy-remote vehicle, and tranching orders the claims on that pool: cash fills the senior tranche first while losses consume the equity tranche first, so the 20% attachment point means the senior holder is untouched until more than a fifth of the pool is lost. That subordination is real protection, but it is a conditional promise: it holds only if pool losses rarely exceed 20%, and how often they do is set by default correlation, not by average loan quality. If defaults are independent, ten thousand loans almost never fail together and the tail beyond 20% is vanishingly thin; if the loans share one common factor, the pool behaves like one large bet, the loss distribution becomes fat-tailed, and the tail is exactly where the senior tranche lives, even though the pool's *expected* loss is identical in both worlds. In 2007–08 the common factor was the national house price: teaser borrowers all depended on refinancing against rising prices, so when prices fell every exit closed in the same season, and the correlation parameter, estimated from decades in which national prices had never fallen, had priced the senior tranches as if defaults were independent. The committee member's analogy also fails on timing: a AAA corporate bond defaults idiosyncratically, while a senior tranche defaults only in a systematic catastrophe, when cash is dearest, so equal ratings do not justify equal yields. The tranche is, in effect, a short position in the assumption that defaults are independent, and "they cannot all fail at once" is precisely the sentence that assumption writes.

Marking guide (20 marks).

- **8 marks**: explains **pooling and the waterfall**: cash fills from the top, losses rise from the bottom; subordination and the 20% attachment point shield the senior tranche; and identifies the safety claim as *conditional* on pool losses staying below the attachment point.

- **6 marks**: makes the **correlation argument**: rising default correlation leaves the pool's expected loss unchanged but fattens the loss tail, which is where the senior tranche sits, so senior risk is set by correlation, not average loan quality (credit for a small illustration, such as two loans at 10% each: 1% hit probability if independent versus 10% if perfectly correlated).
- **4 marks**: applies the **2007–08 lesson**: the common factor was the national house price via the refinancing exit; models were calibrated to data in which national prices had never fallen; and a tranche that fails only in the worst states should yield more than a bond that fails at random.
- **2 marks**: clear writing within the length limit, ending with a stated verdict on the committee member's claim.

Answers that stop at "ten thousand loans are diversified, so the tranche is safe" earn little: that is exactly the misconception the question is built to test.