Hedging Strategies Using Futures

Lecture 3

https://fatih.ai/sjtu/derivatives/

whether → with what → how much → for how long

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Outline

1. Whether to hedge, and with what
2. How much to hedge: the minimum-variance hedge ratio
3. Hedging over time
4. Equity hedging with index futures

Whether to hedge, and with what

The hedger's problem

Lecture 2 made the futures position enforceable. It did not make the hedge automatic: the airline buys **jet fuel**, while the liquid contract is **crude oil**.

whether hedge or stay exposed? → **with what** which tradable risk? → **how much** hedge ratio h^* → **for how long** roll and fund

Block A answers the first two questions; Block B derives the hedge ratio; Block C studies the cash and rolling problem over time; the final section applies the same machinery to equity portfolios.

Whether to hedge: the case for

- **Planning.** A known fuel cost lets the firm price tickets, plan routes, and budget a year ahead, instead of leaving the result to the oil market.

- **Financial-distress costs.** The danger in the bad tail is not a low profit but distress: cancelled orders, forced asset sales, nervous lenders. Reducing that tail has value even to diversified shareholders.

- **Comparative advantage.** The airline's expertise is in operations, not in forecasting oil. Carrying an oil view is a position in a market where it has no edge.

The competitive-position argument

Suppose all competitors fly unhedged, and ticket prices move with industry-wide fuel costs.

fuel at T	rivals' fuel cost	our (hedged) cost	relative position
low	low — fares fall	fixed	worse
high	high — fares rise	fixed	better

The hedge fixes the **absolute** cost but unfixes the **relative** one. When industry prices track costs, an unhedged firm is partly hedged through its revenue, and the hedger has taken a position.

Hedging as a policy

The arguments do not settle "always hedge" or "never hedge." They define *when*:

- hedge exposures that can reach financial distress, where the firm has no comparative advantage, and where competitors' exposures do not already offset the risk;
- decide it once, in writing, at board level — as a policy, not trade by trade.

The decision to hedge is a stated risk-management policy, not a reaction to the latest price.

With what: cross-hedging and four mismatches

There is no liquid renminbi jet-fuel futures contract. The airline therefore **cross-hedges**: it uses the nearest liquid traded risk, INE crude oil.

business exposure
jet fuel

↓

tradable hedge
crude futures

related,
not identical

mismatch	residual risk
asset	jet fuel versus crude oil
quantity	800,000 barrels versus 1,000-barrel lots
timing	June purchase versus listed contract months
location	airport delivery versus bonded tanks

These residual risks are not mistakes in the hedge. They are the basis risk created by using a liquid proxy instead of the exact exposure.

Basis and basis risk

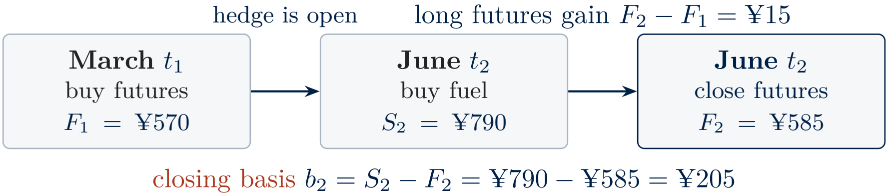
$$b \;=\; S_{\text{(asset hedged)}} \;-\; F_{\text{(futures used)}} \;=\; S_{\text{jet fuel}} - F_{\text{crude}}$$

- Both prices move daily; they are related (the same crude is upstream of jet fuel) but not identical (refining margins, season, location, delivery month).
- Lecture 2 showed the futures price converges to *crude* spot, not to jet fuel, so the gap survives convergence.

A futures hedge replaces price risk with basis risk; it does not remove risk, it changes its form.

The long hedge

The airline will **buy fuel in June**. A higher June spot price hurts, so it buys crude futures in March (one futures barrel per exposure barrel, for now).



The basis is always $b_t = S_t - F_t$, so $b_1 = S_1 - F_1$ and $b_2 = S_2 - F_2$. Here the cost is driven by the closing basis b_2, because the fuel is bought and the futures hedge is closed in June.

Long hedge: effective purchase cost

In June the airline pays the spot price for fuel and closes the long futures position.

Cash-flow view

June item	effect
buy fuel at S_2	$+¥790$
long futures gain $F_2 - F_1$	$-¥15$
effective purchase cost	¥775

Basis view

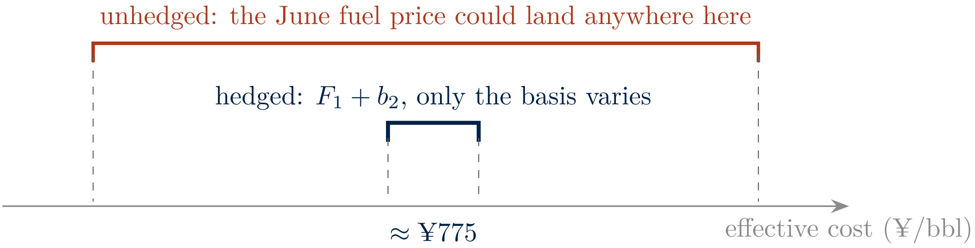
$$\begin{aligned} \text{effective cost} &= S_2 - (F_2 - F_1) \\ &= S_2 + F_1 - F_2 \\ &= F_1 + (S_2 - F_2) \\ &= F_1 + b_2. \end{aligned}$$

With the numbers:

$$570 + (790 - 585) = 570 + 205 = ¥775.$$

TAKEAWAY — The hedge fixes F_1, not the future spot price. The remaining uncertainty is the closing basis b_2.

What the hedge achieves



A wide uncertainty is exchanged for a narrow one. The hedge does not deliver certainty; it delivers a smaller, residual uncertainty in the basis.

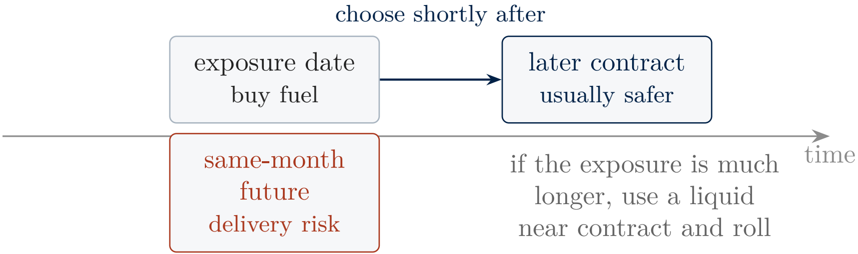
Which way does the basis hurt?

Effective cost $= F_1 + b_2$, so for a **buyer** hedging with a long futures position:

June basis b_2 turns out	buyer-hedger	seller-hedger
higher (strengthens)	worse	better
lower (weakens)	better	worse

The effect is symmetric, usually small, and cannot be removed with this contract: the basis is exactly the part the futures does not track.

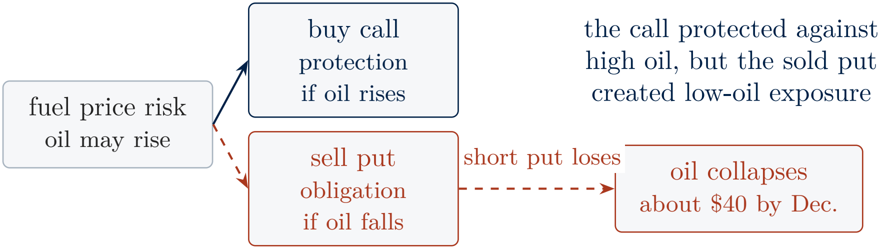
Choosing the contract month



Avoid the delivery month unless the firm is operationally ready for delivery. Liquidity and delivery risk are part of contract choice, not afterthoughts.

A 2008 case: hedges that re-created the exposure

Oil rose above **$147** per barrel in July 2008. Buying ordinary call options for protection was expensive at that level, so several Chinese airlines used a **collar**: buy a call, and finance it by **writing (selling) a put** below the market — “zero-cost” protection.



The 2008 case: the loss, and the lesson

When oil collapsed, the written puts moved deep into the money:

- China Eastern reported a fuel-hedging **fair-value** loss of about **¥6.3 billion** for 2008; Air China about **¥7.5 billion**.
- These were **mark-to-market** (fair-value) losses on open contracts, not realised cash losses — the Lecture 1-2 distinction between value and notional, in a balance sheet.
- Both are state-owned enterprises; regulators tightened state-owned firms' use of derivatives afterwards.

A hedging instrument with an embedded short option can re-create, and amplify, the exposure it was meant to reduce.

The instrument test, and where we are

Before putting on a hedge, test the *instrument*:

- in the scenario the business fears, does it pay?
- in the scenario the business benefits from, what does it owe?
- can it be valued and its margin funded in a stressed week (Lecture 2)?

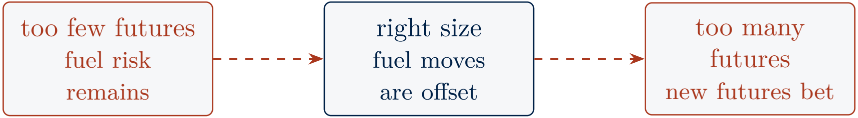
The airline's choice here is plain crude futures: symmetric and fundable.

whether ✓ → with what ✓ → **how much** → for how long

How much to hedge: the minimum-variance hedge ratio

The sizing problem

We have decided to hedge a future jet-fuel purchase with crude futures. The next decision is not the direction of the hedge; it is the **size**.



The tempting answer is $h = 1$: one futures barrel for one exposure barrel. That is only right if crude futures and jet fuel move roughly one-for-one.

The problem is not "how many barrels?" It is "how much price risk does one futures barrel remove?"

Turn the sizing problem into data

Let h be the number of futures barrels used for each barrel of jet-fuel exposure.

One month of price movement

$$\Delta S = \text{jet-fuel spot change}$$

$$\Delta F = \text{crude futures change}$$

After hedging

$$\text{remaining change} = \Delta S - h\,\Delta F.$$

Choose h to make this remaining change as stable as possible:

$$\min_h \operatorname{Var}(\Delta S - h\,\Delta F).$$

Data answer the two sizing questions:

- **Scale:** does crude futures move more or less than jet fuel?
- **Fit:** how tightly do the two price changes move together?

The next slides estimate scale and fit from price changes.

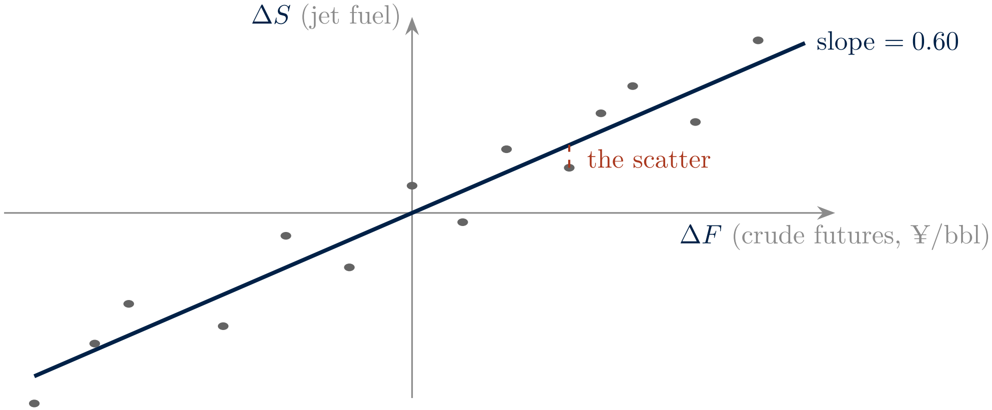
The data

Twenty-four months of *monthly changes*, in renminbi per barrel: ΔS (jet fuel spot) and ΔF (crude futures). Sample statistics:

std. deviation of jet-fuel changes, σ_S	¥30
std. deviation of futures changes, σ_F	¥40
correlation, ρ	0.80

Everything in this block is built from these three numbers — and we look at the data before using a formula.

Look at the data first



The monthly changes line up roughly along a line, tilted well below $45°$, with scatter around it. Both features matter: the slope sets the hedge ratio, and the scatter is the risk the hedge cannot remove.

What are we minimising?

Work per exposure barrel. A long futures hedge gains when the futures price rises.

hedge size	remaining change	interpretation
no hedge: $h = 0$	ΔS	all jet-fuel price risk remains
one-for-one: $h = 1$	$\Delta S - \Delta F$	assumes futures and fuel move one-for-one
general hedge: h	$\Delta S - h\,\Delta F$	choose the size from data

Why $h\,\Delta F$? If one futures barrel gains ΔF, then h futures barrels gain $h\,\Delta F$ per exposure barrel.

$$\boxed{h^* = \arg\min_h \operatorname{Var}(\Delta S - h\,\Delta F)}$$

Choose h, the hedge size, to make the remaining change as stable as possible.

Deriving the minimum-variance hedge ratio

We minimise the variance of the remaining price change:

$$v(h) = \mathrm{Var}(\Delta S - h\,\Delta F).$$

Use the variance rule:

$$\mathrm{Var}(X - Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) - 2\,\mathrm{Cov}(X, Y).$$

$$\begin{aligned} v(h) &= \mathrm{Var}(\Delta S) + \mathrm{Var}(h\Delta F) - 2\,\mathrm{Cov}(\Delta S, h\Delta F) \\ &= \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F. \end{aligned}$$

- σ_S^2: original fuel-price risk.
- $h^2\sigma_F^2$: risk introduced by the futures leg.
- $-2h\rho\sigma_S\sigma_F$: the offset when futures and spot move together.

$$\begin{aligned} v'(h) = 0 &\Longrightarrow 2h\sigma_F^2 = 2\rho\sigma_S\sigma_F \\ &\Longrightarrow h^* = \frac{\rho\sigma_S\sigma_F}{\sigma_F^2} = \boxed{\rho\frac{\sigma_S}{\sigma_F}}. \end{aligned}$$

Reading and computing the hedge ratio

$$h^* = \underbrace{\frac{\sigma_S}{\sigma_F}}_{\text{relative volatility}} \times \underbrace{\rho}_{\text{co-movement}} = \frac{30}{40} \times 0.80 = \mathbf{0.60}$$

- The volatility ratio scales the position: the futures moves ¥40 when the exposure moves ¥30, so each futures barrel does more than one barrel of work.
- The correlation discounts it: only the part of the futures move that tracks the exposure is useful; the rest would add risk.

The optimal hedge here is 0.60 *— less than one-for-one; matching barrels would over-hedge.*

Regression view: the line through the cloud

Each month gives one point:

$$(\Delta F_t, \Delta S_t) = (\text{crude futures move},\ \text{jet-fuel spot move}).$$

A regression line summarises how the exposure moves with the hedge instrument:

$$\Delta S_t = a + \beta\,\Delta F_t + e_t.$$

symbol	meaning
β	slope: expected jet-fuel move for a one-unit futures move
e_t	residual: the part crude futures do not explain, i.e. basis noise

The slope tells us how many futures barrels are useful per exposure barrel; the residual is the risk the hedge cannot remove.

The hedge ratio is the regression slope

For the regression of ΔS on ΔF, the slope is

$$\beta = \frac{\mathrm{Cov}(\Delta S, \Delta F)}{\mathrm{Var}(\Delta F)}.$$

Now use

$$\mathrm{Cov}(\Delta S, \Delta F) = \rho \sigma_S \sigma_F, \qquad \mathrm{Var}(\Delta F) = \sigma_F^2.$$

Therefore

$$\beta = \frac{\rho \sigma_S \sigma_F}{\sigma_F^2} = \rho \frac{\sigma_S}{\sigma_F} = h^*.$$

In practice, estimate the hedge ratio by regressing the exposure change on the futures change.

How much risk remains?

Compare the variance before and after the optimally sized hedge.

$$\text{before hedge:} \quad \mathrm{Var}(\Delta S) = \sigma_S^2.$$

$$\text{after hedge:} \quad \mathrm{Var}(\Delta S - h^* \Delta F) = \sigma_S^2(1-\rho^2).$$

The fraction of variance removed is

$$1 - \frac{\sigma_S^2(1-\rho^2)}{\sigma_S^2} = \rho^2.$$

removed by futures: $\rho^2 = 64\%$	basis risk: 36%

$$\rho = 0.80 \Rightarrow \rho^2 = 0.64$$

The remaining 36% is the scatter around the regression line: basis risk, now measured.

From hedge ratio to number of contracts

The hedge ratio is per barrel. Exchanges trade contracts, so convert barrels into lots.

$$\begin{aligned} N^* &= \text{number of futures lots}, \\ Q_A &= \text{exposure size} = 800{,}000 \text{ bbl}, \\ Q_F &= \text{one futures lot} = 1{,}000 \text{ bbl/lot}. \end{aligned}$$

$$N^* = h^* \frac{Q_A}{Q_F} = 0.60 \times \frac{800{,}000}{1{,}000} = \mathbf{480} \text{ lots}.$$

Not 800 lots but 480: matching barrels would over-hedge and add margin, funding pressure, and noise.

What the hedge costs to fund

A futures hedge has low upfront price, but it still needs cash capacity.

funding object	amount
notional: $480 \times 1{,}000 \times ¥560$	$\approx ¥269\text{M}$
initial margin: cash posted to open the hedge, about 10%	$\approx ¥27\text{M}$
variation margin: cash needed after a 5% adverse week	$\approx ¥13\text{M}$

The statistical answer, 480 lots, becomes a treasury question: can the firm fund margin calls while the hedge is open?

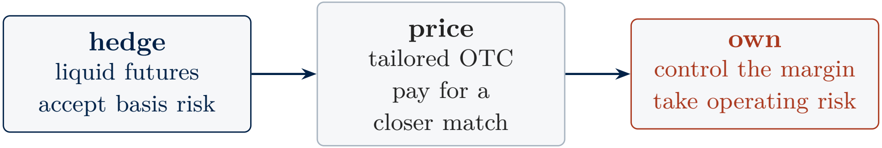
What can make h^* unreliable?

- **Estimated inputs:** ρ, σ_S, and σ_F come from past data, not from a law of nature.
- **Changing relationships:** crude and jet fuel can decouple in stressed markets.
- **Tail behaviour:** the average slope may not describe extreme months, exactly when the hedge matters most.
- **Estimation error:** $h^* = 0.60$ means "about 0.6," not a fixed constant.

In practice, re-estimate on a schedule, stress the inputs, and treat h^* as an adjustable risk-management parameter.

Basis risk: three responses

After the best futures hedge, some basis risk remains. The firm has three broad choices:



Delta Air Lines bought the Trainer refinery in 2012 to address the jet-fuel-over-crude margin directly. The result was mixed, which is the point: owning the basis creates operating risk.

Where we are

whether ✓ → with what ✓ → how much ✓ → **for how long**

We now know whether to hedge, what to use, and how many lots to trade. The remaining question is time: can the hedge be maintained for as long as the business exposure lasts?

Hedging over time

When the exposure outlives the contract

Fuel is purchased every season, but liquid futures reach only a few months ahead. Two ways to bridge the gap:

Far months are illiquid; rolling near months keeps liquidity but turns the hedge into a position that must be maintained, trade after trade.

The cost of rolling

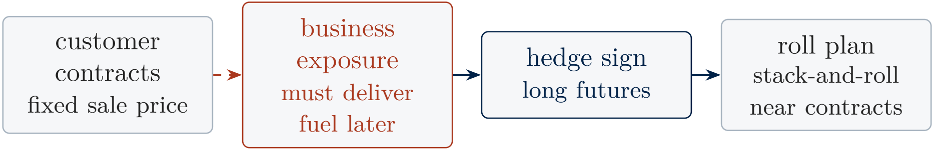
Each roll closes the expiring month and reopens the next, crossing the price gap between them.

contango	**backwardation**
sell near low	sell near high
buy next high	buy next low
pay roll toll	earn roll benefit

Rolling converts one long-dated basis risk into repeated roll costs, repeated basis bets, and a continuous funding requirement.

Metallgesellschaft, 1993: the strategy

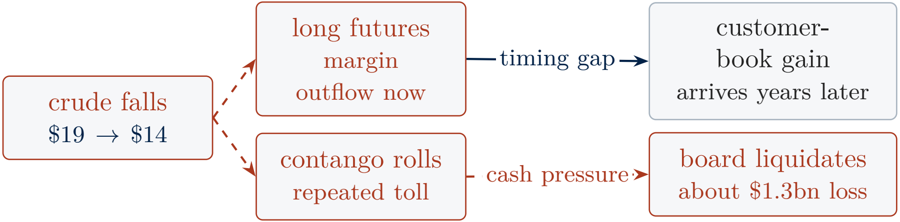
MG Refining and Marketing (MGRM), the US subsidiary of the German group Metallgesellschaft, sold customers **fixed-price supply contracts of up to ten years** — on the order of **150 million barrels** of long-dated delivery commitments.



The hedge direction was economically sensible. The risk was the mismatch between a long-dated exposure and a short-dated rolling hedge.

Metallgesellschaft, 1993: the cash crisis

Through 1993, crude fell from about \$19 toward \$14, which was good for the customer book years out. The cash problem arrived first.



The reported cash drain on the hedge was on the order of **\$1 billion**; liquidation near the price low crystallised the loss.

Metallgesellschaft: the debate, and the lesson

Economists still disagree, and both readings are instructive:

SOUND BUT ILLIQUID — At maturity the contracts could have offset. The failure was funding and the decision to unwind early.

Culp & Miller reading: liquidity failure.

FLAWED BY MISMATCH — Ten-year exposure hedged with one-month contracts is a calendar-basis position plus recurring roll cost.

Mello & Parsons reading: hedge-design failure.

Both readings lead to the same governance rule: a board that must fund a hedge has to understand its cash calendar before the market moves, not after.

For how long: the answer

- Match the hedge's life to the exposure where liquidity allows; where it does not, roll, and budget the roll cost.
- Keep two calendars: a roll calendar and a cash calendar (worst-week variation margin, pre-funded).
- Size the hedge so that the stressed-week cash need cannot exceed the funding line. If it can, the hedge is too large, whatever h^* says.

The four decisions now all have answers.

Equity hedging with index futures

The same machinery: hedging equity risk

An investment portfolio holds **¥60 million** of A-shares. The manager expects a possible market fall this quarter but does not want to sell.

This is the same problem as the fuel hedge, with the market index replacing crude futures as the tradable proxy.

Beta is a hedge ratio

For a portfolio against the CSI 300 index,

$$\Delta(\text{portfolio}) \;\approx\; \beta \times \Delta(\text{index}) \;+\; \text{residual},$$

and β is estimated the same way as the hedge ratio: regress portfolio changes on index changes; the slope is β.

- $\beta = 1.2$: the portfolio tends to move 20% more than the market.
- The residual is the stock selection — the part to keep.

The equity hedge, worked

CSI 300 futures (IF): index 4,000, multiplier ¥300, so one contract is worth ¥1.2M (Lecture 2). Portfolio ¥60M, $\beta = 1.2$:

$$N^* = \beta \frac{V_P}{V_F} = 1.2 \times \frac{60\text{M}}{1.2\text{M}} = \mathbf{60} \text{ contracts, short.}$$

Stress-test a 10% market fall (4,000 $\to$ 3,600):

portfolio: $-1.2 \times 10\% \times$ ¥60M	$-$¥7.2M
futures: $60 \times 400 \times$ ¥300	$+$¥7.2M
net	$\approx$ ¥0

Adjusting beta to a target

The target need not be zero. To move from β to a target β^*, trade

$$(\beta - \beta^*)\,\frac{V_P}{V_F} \quad \text{contracts (short if positive).}$$

- Halving exposure, $1.2 \to 0.6$: short $0.6 \times 50 = 30$ contracts.
- Hedging fully, $\beta^* = 0$: the portfolio's market exposure is removed and only the stock selection remains.

Index futures let a manager set the portfolio's market exposure to a chosen level, not only to zero.

The practical cost of index hedging in China

The hedge ratio assumes the futures can be traded freely. That is not always so.

- After the 2015 market fall, CFFEX sharply restricted stock-index futures: margins on speculative positions were raised to about **40%** of contract value, fees roughly tenfold, and new positions limited to about **10 lots** per day.
- Trading volume fell by more than **90%**; hedging became far costlier and harder. The restrictions were relaxed in steps from 2017.

The feasibility and cost of a hedge depend on market design and regulation, not only on the hedge ratio.

Limitations of the index hedge

- β is estimated from history, and in a crash individual stocks' betas tend to move toward one — estimates from calm periods, used in stressed ones.
- A concentrated portfolio has a large residual: the index removes the market and leaves the names, which is the intended outcome only if it was understood.
- The short futures leg posts margin, so a market *rise* — good for the portfolio — still produces a cash outflow on the hedge.

Summary

- Lecture 1: a derivative transfers risk; it does not remove it.
- Lecture 2: central clearing concentrates risk so it can be managed.
- Lecture 3: a futures hedge **exchanges** a large price risk for a smaller, residual basis risk.

A hedge is itself a position: it has an instrument, a size, and a cash calendar.

The hedging decision, on one page

whether	yes: fuel can create distress; no oil edge
with what	crude futures — liquid, symmetric, and fundable
how much	$h^* = 0.60 \Rightarrow 480$ lots; effectiveness 64%
for how long	roll near contracts; fund ≈ ¥13M stress margin

Each entry is a decision with a number behind it, not a forecast.

Fuel is a visible exposure. But a firm that borrows to buy aircraft is also exposed to the price of money itself, which moves every day.

How is that price set, and how is it quoted across maturities?

Lecture 4: interest rates — the term structure, and the replacement of the market's main reference rate.