# Interest Rate Futures

Lecture 6

https://fatih.ai/sjtu/derivatives/

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# Outline

1. Bond prices and delivery conventions
2. Conversion factors and CTD pricing
3. Short-rate futures and the basis trade

# Bond prices and delivery conventions

# Clean and dirty prices

Bonds earn interest continuously but pay coupons in lumps. A buyer between coupon dates owes the seller the interest accrued since the last coupon. Take a 3% semiannual bond (¥1.50 per half-year per ¥100), quoted at ¥100.40, 61 of the period's 184 days elapsed:

$$\text{accrued} = 1.50 \times \tfrac{61}{184} = ¥0.497$$


Cash price $= 100.40 + 0.497 =$ **¥100.897**.

KEY EQUATION · THE PRICE YOU PAY IS NOT THE PRICE YOU READ

$$\text{cash (dirty) price} = \text{quoted (clean) price} + \text{accrued interest}$$

Per ¥100 of face; on a ¥1 billion position the accrued alone is ¥4.97 million, paid, not quoted.

# Day-count conventions and accrued interest

A rate quote depends on its compounding convention (Lecture 4); the calendar that counts elapsed time is also a convention.

| convention | counts | typical use |
|---|---|---|
| ACT/ACT | actual days over actual days | government bonds |
| 30/360 | every month 30 days, year 360 | many corporate bonds |
| ACT/360 | actual days over a 360-day year | money markets |

Same bond, same dates. Under ACT/ACT the accrued is $1.50 \times \frac{61}{184} = ¥0.4973$; under 30/360 it is $1.50 \times \frac{60}{180} = ¥0.5000$. The difference is ¥0.0027 per ¥100, and **about ¥27,000 per ¥1 billion** of face.

— Settlement uses the day-count convention named in the contract; changing the convention changes the cash paid.

# The contract specification

## CFFEX 10-year Treasury futures (T)

CONTRACT SIZE
**face value**
¥1,000,000 per contract
quoted per ¥100 of face

PRICING GRID
**notional bond**
3% coupon
tick 0.005 = ¥50

TRADING MECHANICS
physical delivery
quarterly months
minimum margin: 2%

DELIVERABLE MENU
any CGB with original term $\leq$ 10 years and residual maturity $\geq$ 6.5 years
on the first day of the expiry month.
Family: TS, TF, T, TL; scaled windows; all use a 3% notional coupon.

Two features need explaining: a *notional* 3% bond that does not exist, and *any* qualifying CGB as deliverable. Both lead to the delivery pricing problem.

# One futures price for many bonds

The futures market quotes one number, $F$. At expiry the short can deliver bond $j$ from the eligible CGB list.

## Problem

- One futures quote.
- Many possible deliverable bonds.
- Different coupons and maturities.

— Use the conversion factor to turn one futures quote into the clean invoice for each deliverable bond.

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$\mathrm{CF}_j$ is the exchange's multiplier for bond $j$.

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The full invoice later adds accrued interest. The conversion factor handles the fact that bond $j$ is not exactly the notional 3% bond.

— Use the conversion factor to turn one futures quote into the clean invoice for each deliverable bond.

# Why the contract has a deliverable menu

Why not specify one exact CGB for delivery?

**Single-bond design**

- Every short needs the same bond.
- Holders of that bond gain market power near expiry.
- Settlement can become a squeeze.

**Deliverable menu**

- Several similar CGBs are acceptable.
- Shorts can switch if one bond becomes scarce.
- Delivery supply is deeper and safer.

— The deliverable menu reduces squeeze risk, but it gives the short a delivery choice.

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The trade-off: the short will choose the cheapest acceptable bond. That is the cheapest-to-deliver problem.

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# Conversion factors and CTD pricing

# Conversion factors

DEFINITION — The **conversion factor** of a deliverable bond is its price per ¥1 of face if the whole yield curve were flat at the notional coupon, 3%.

- A bond with coupons above 3% is worth more than par in that world: CF $> 1$. Coupons below 3%: CF $< 1$. A 3% bond: CF $= 1$.
- A conversion factor is a fixed exchange rate between each bond and the notional 3% bond.

It is not a market price. It is computed once, by formula, at an assumed flat 3% yield: a convention, like the day-count rules above.

— A conversion factor is a fixed ruler: it expresses each deliverable bond in notional 3% bond units.

# Conversion factors for three bonds

Value each bond's cash flows at a flat 3% (annual coupons, whole years for simplicity):

$$\text{value at }3\% = \sum_{t=1}^{T} \frac{\text{coupon}}{(1.03)^t} + \frac{100}{(1.03)^T}, \qquad \text{CF} = \frac{\text{value at }3\%}{100}.$$

| bond | coupon | maturity | value at 3% | CF |
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| B | 2.4% | 9 years | ¥95.33 | 0.9533 |
| C | 3.0% | 7 years | ¥100.00 | 1.0000 |

Above the notional coupon, below it, and exactly on it.

# The invoice price

When the short delivers bond $i$ against a futures settlement price $F$, the long pays

KEY EQUATION · THE INVOICE FOR DELIVERING BOND $i$

$$\text{invoice} = F \times \mathrm{CF}_i + \text{accrued}_i.$$

- A more valuable bond has a larger conversion factor and a larger invoice: the design pays the short more for delivering more.
- $F \times \mathrm{CF}_i$ is the clean invoice component; adding accrued interest gives the cash settlement amount.
- If the curve were genuinely flat at 3%, every deliverable bond would net the short the same amount, and the menu would be value-neutral.

# When market yields differ from the 3 percent assumption

The conversion factor fixes relative values at 3%. The market is nowhere near it.

- CGB 10-year yields have recently been around **1.7 to 1.8 per cent**, far below the 3% ruler.
- At those yields the bonds' true relative values differ from their conversion-factor ratios.
- Delivery is therefore not value-neutral: one bond is systematically the cheapest to deliver.

The short, holding the menu, will find that bond.

— The CTD bond appears because conversion factors use the 3% ruler while market prices use actual yields.

# The cheapest-to-deliver

With the futures at $F = 105.29$ and the quoted bond prices below, the short's delivery cost for bond $i$ is $\text{quote}_i - F \times \text{CF}_i$:

| bond | quote | CF | $F \times$CF | cost |
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— The short chooses the bond with the lowest delivery cost; that delivery cost is the basis.

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| C | 106.90 | 1.0000 | 105.29 | 1.61 |

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Bond A is cheapest to deliver, narrowly over C (1.27 versus 1.61). This net delivery cost has a market name used for the rest of the hour: the **basis**.

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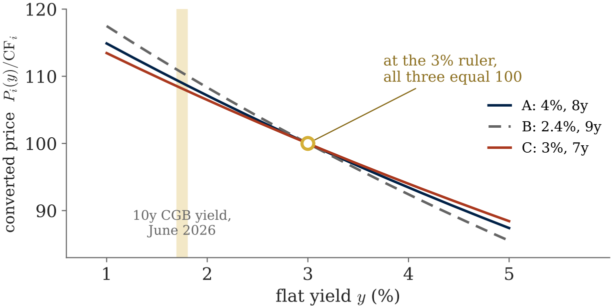
# CTD switch risk




Each bond's price divided by its CF is a converted price. All three meet at **100** at the 3% ruler: the neutral point.

- below 3%, the lower-duration bond is cheapest;
- above 3%, the higher-duration bond takes over;
- near the ruler the curves sit close, so a modest yield move can **switch** the CTD.

The short also chooses *when* in the delivery month to deliver. These embedded options have value, paid for by a slightly lower futures price (appendix A8).

# Pricing the futures contract through the CTD

A bond future is a forward on the cheapest-to-deliver bond, expressed in conversion-factor units. Price it with the cost-of-carry formula, in four steps, each tagged to the tool being used:

1. start from the CTD's dirty price today; (clean/dirty prices)
2. carry it to delivery: finance at $r$, less any coupons received on the way; (cost of carry)
3. strip the accrued interest at delivery, back to a clean forward; (accrued interest)
4. divide by the conversion factor, into contract units. (conversion factor)

Each step uses a tool already built.

# Four-step futures price calculation

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$$\text{carry to delivery} = 114.45\, e^{0.02\times 0.75} = 116.18 \qquad \text{finance nine months, no coupon}$$

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KEY EQUATION · THE FUTURES PRICE FROM THE CTD

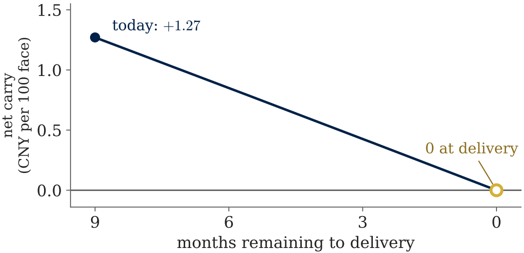
$$F = \frac{S_{\text{dirty}}\, e^{rT} - (\text{accrued at delivery})}{\text{CF}}$$

This matches the $F = 105.29$ used on frame 13: price and carry are mutually consistent.

# The basis is the net carry

Bond A's delivery cost, $113.95 - 112.68 = ¥1.27$, is the basis. Audit the holder's nine months:

| | |
|---|---|
| coupon accrued $(3.50 - 0.50)$ | $+¥3.00$ |
| financing $(114.45(e^{0.015} - 1))$ | $-¥1.73$ |
| **net carry** | **+¥1.27** |




— For the CTD, the basis records net carry and should shrink as delivery approaches.

# Contract sensitivity and hedge ratio

One T contract inherits the CTD's sensitivity:

| | |
|---|---|
| value at $F = 105.29$ | ¥1,052,900 |
| CTD duration | $\approx 7.1$ |
| per 50bp move | $\approx$ ¥37,400 |

Lecture 4 used $N = \dfrac{P\,D_P}{V_F\,D_F}$ with $V_F \approx$ ¥1M and $D_F \approx 8$, giving 60. With the true numbers:

$$N = \frac{80\text{M} \times 6}{1.0529\text{M} \times 7.1} = 64.2 \;\Rightarrow\; \textbf{64 short.}$$

The two approximations did not cancel: $D_F$ was too high (8 versus 7.1) and $V_F$ too low (1.00 versus 1.05). Stress a $+50$bp move:

| | |
|---|---|
| bond portfolio $-80\text{M} \times 6 \times 0.005$ | $-$¥2.40M |
| futures $64 \times 1.0529\text{M} \times 7.1 \times 0.005$ | $+$¥2.39M |

— Use the CTD's value and duration in the hedge ratio; redo the hedge if the CTD can switch.

# Practice problems

PROBLEM A — A 4% semiannual bond (¥2.00 per period) is quoted at 102.30, with 77 days elapsed of a 182-day period. Cash price per ¥100? Per ¥500M face? Answer: accrued $2 \times \frac{77}{182} = 0.846$; cash **103.146**; ¥515.73M.

PROBLEM B — Futures at 104.20. Bond X quote 109.85, CF 1.0540; bond Y quote 99.40, CF 0.9520. Which is CTD, and its basis? Answer: X cost 0.024, Y cost 0.20; CTD **X**, basis **0.02**.

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Both use the mechanics of frames 4 and 13. Full step-by-step solutions are in appendix A12–A13.

# Short-rate futures and the basis trade

# SOFR futures and interest-rate locks

The T contract trades long-term rates. Short-term rates use the 3-month SOFR future: notional \$1M, quoted as $\text{price} = 100 - \text{rate}$, so \$25 per basis point ($1\text{M} \times 0.0001 \times 0.25$). A borrower fearing next quarter's financing **sells** one at 96.80, locking 3.20%:

| rate sets at | future settles | futures P&L | all-in rate |
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— Selling a SOFR future locks the borrowing rate: futures gains offset higher interest costs.

# The Eurodollar conversion: 26.161 basis points

For four decades the dominant short-rate contract was the **Eurodollar** future: three-month *LIBOR* futures. When LIBOR was discontinued (Lecture 4), its large open interest referenced a rate that would cease to exist.

- On **14 April 2023**, CME converted every open Eurodollar position into the equivalent SOFR contract, about **7.5 million contracts**, at a fixed spread of **26.161 basis points**.
- That spread is the industry fallback: the priced credit gap between unsecured LIBOR and secured SOFR, fixed once.

— Benchmark reform keeps old contracts alive by converting LIBOR exposure into SOFR plus a fixed spread.

# Convexity between futures and forward rates

A SOFR future settles daily; a forward rate agreement settles once. That daily settlement is not neutral:

- when rates rise, the short is paid, and paid exactly when the cash can be reinvested at the new high rate; when rates fall, the loss is financed cheaply. The effect is systematic.
- so the futures-implied rate sits slightly *above* the true forward rate.

KEY EQUATION · THE CONVEXITY ADJUSTMENT

$$\text{forward rate} = \text{futures rate} - \tfrac{1}{2}\,\sigma^2 T_1 T_2.$$

With rate volatility near 1.2%, the adjustment is about **0.3bp** at six months and about **19bp** at five years.

# Who supplies the short futures position

Every rate hedge in this course has been short futures: the portfolio's 64 contracts, and many institutional books like it. Yet hedgers and asset managers are, on balance, net **long** duration in futures, holding them as a cash-efficient way to own bonds. So the market needs institutions willing to sell futures, while arbitrage keeps the futures price tied to the cash bond.

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That short side is supplied by leveraged funds running the **Treasury cash-futures basis trade**.

— A large short futures position can supply duration to the market, not simply bet on higher yields.

# Treasury basis trade mechanics

LEG 1

## Buy CTD bond

Own the cash bond that is cheapest to deliver.

spot asset, cost-of-carry logic (L5)

LEG 2

## Short futures

Sell futures on the same CTD bond.

forward promise (L1), CF ratio (L3)

LEG 3

## Finance in repo

Borrow against the bond and roll the funding.

funding curve (L4), margin risk (L2)

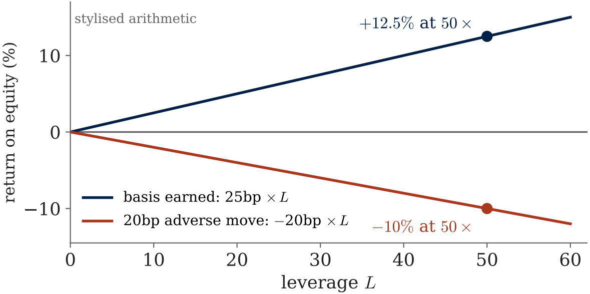
WHAT REMAINS AFTER THE HEDGE

$$\text{long CTD bond} \ - \ \text{short its futures claim} \ \approx \ \text{basis, not rate direction}$$

The basis converges toward zero at delivery, collecting net carry plus any futures richness.

This is cash-and-carry financed in repo: rate exposure is mostly hedged, but funding and margin liquidity are not.

# Leverage in the basis trade



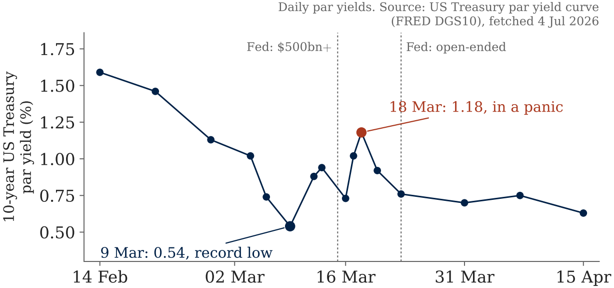

The basis is tens of basis points a year, so the trade runs at high leverage through repo. At 50 times:

- 25bp earned → +12.5% on equity;
- a 20bp adverse move → −10% of equity.

Stylised arithmetic. Repo haircuts on Treasuries run about 2% or lower, so 50 times is realistic.

At scale, leveraged funds' short Treasury-futures positions have exceeded **$1 trillion** in notional (the Chicago Fed figure): the market regarded as risk-free is held in place, in part, by highly leveraged balance sheets.

# March 2020 Treasury market stress




The safe asset **sold off** in the panic (yields rose from 0.54 to 1.18 in nine days). Then:

- margins rose, repo tightened (L2);
- basis positions unwound: sell the bonds, buy back the futures;
- the selling hit the Treasury market itself, until the Fed bought at up to **$75bn a day**.

— Daily margin protects counterparties, but in stress it can force cash sales and amplify liquidity pressure.

# Policy response and course synthesis

The tension: the trade supplies cheap hedging in calm and consumes liquidity in stress. One response is in train, the SEC's mandate to **central clearing** for Treasuries:

- cash Treasuries by **31 Dec 2026**;
- Treasury repo by **30 Jun 2027**.

Lecture 2's central-counterparty architecture, applied to the core of the system.

| | |
|---|---|
| L1 | the forward promise; the arbitrageur |
| L2 | margin, forced liquidation, clearing |
| L3 | the hedge ratio, via the CF |
| L4 | the curve, repo, duration |
| L5 | carry; arbitrage supplies a market |
| L6 | the menu, the CTD, the basis |

— The Treasury basis trade connects carry pricing, margining, hedging, and funding liquidity in one market.

# Summary

- Lecture 1: derivatives transfer risk. Lecture 2: clearing concentrates it. Lecture 3: hedging exchanges it.
- Lecture 4: every date has a price. Lecture 5: a forward price is the spot, carried. Lecture 6: the futures price is the spot price of the cheapest bond, carried and converted.

Arbitrage keeps the futures price aligned with the cash bond, but on borrowed cash. The risk-free curve is held accurate by leveraged balance sheets whose funding can fail in stress, which is why Lecture 2's clearing architecture is now being extended to the Treasury market.

— Price the futures through the CTD: dirty price, carry, strip coupons, divide by the conversion factor; the arbitrage needs funding.

# Appendix: Conventions, proofs, and solutions

# A1. The money-market convention: discount quotes

A 90-day US Treasury bill quoted at a discount rate of 4.00:

$$\text{price} = 100\left(1 - 0.0400 \times \frac{90}{360}\right) = \$99.00.$$

- The true 90-day return is $100/99 - 1 = 1.0101\%$; annualised on ACT/365 it is $\mathbf{4.10}\%$.
- The quoted 4.00 understates the return *twice*: it divides by the face value rather than the price paid, and it uses a 360-day year.

The same lesson as Lecture 4's deposit comparison, in a third market: convert to a common basis before comparing.

## A2. The mainland bond market

- Chinese government bonds (CGBs) mostly pay annual coupons (some semiannual); accrued interest uses actual/actual.
- Two venues: the **interbank market** (the larger, institutional, quoted through CFETS and valued daily by ChinaBond) and the **exchange market** (smaller, retail-accessible).
- Money-market legs (repo and the DR rates of Lecture 4) keep their own day-count conventions.

The futures contract of this lecture settles against these bonds.

# A3. 30/360 and ACT/ACT worked example

Same bond, 1 March to 1 May, quoted ¥100.40, ¥1.50 per half-year:

| | |
|---|---|
| ACT/ACT: $1.50 \times 61/184$ | ¥0.4973 |
| 30/360: $1.50 \times 60/180$ | ¥0.5000 |
| difference per ¥100 | ¥0.0027 |
| **difference per ¥1 billion face** | **≈ ¥27,000** |

30/360 counts March to May as 60 days (two 30-day months) over a 180-day half-year; ACT/ACT counts 61 actual days over the 184-day period. The same dates, two calendars, two cash prices.

## A4. Conversion factor for bond A

Bond A: 4% annual coupon, 8 years, valued at a flat 3%:

$$a(8, 3\%) = \frac{1 - 1.03^{-8}}{0.03} = 7.01969$$ present value of ¥1 a year for 8 years

$$\text{coupons} = 4 \times 7.01969 = 28.079$$ the coupon strip

$$\text{principal} = 100 \times 1.03^{-8} = 78.941$$ the redemption

$$\text{price} = 28.079 + 78.941 = 107.02$$ CF = 1.0702

Bond C (3% on the ruler) prices to par, CF $= 1$; bond B (2.4%, 9 years) to 95.33, CF $= 0.9533$.

# A5. The contract family

| contract | face | residual window | tick | margin |
|---|---|---|---|---|
| TS (2y) | **¥2,000,000** | 1.5–2.25y | 0.002 | 0.5% |
| TF (5y) | ¥1,000,000 | 4–5.25y | 0.005 | 1% |
| T (10y) | ¥1,000,000 | $\geq 6.5$y | 0.005 | 2% |
| TL (30y) | ¥1,000,000 | $\geq 25$y | 0.010 | 3.5% |

All four carry a 3% notional coupon. Each has an *original*-term cap too: TF original $\leq 7$ years, T $\leq 10$, TL $\leq 30$.

# A6. The four-step price in general symbols

Let $S_{\text{dirty}}$ be the CTD's dirty price today, $A_T$ its accrued at delivery, and $\text{FV}(\text{coupons})$ the delivery-date value of any coupons received before delivery:

KEY EQUATION · THE FUTURES PRICE ON THE CTD

$$F = \frac{S_{\text{dirty}}\, e^{rT} - A_T - \text{FV}(\text{coupons})}{\text{CF}}.$$

- This is Hull equation (6.1), $F_0 = (S_0 - I)e^{rT}$ on the CTD, with the conversion factor divided out and accrued stripped.
- $F$ depends on the CTD, and the CTD (frame 13) depends on $F$: desks iterate to a stable fixed point.

## A7. Why low yields favour low-duration bonds

Take bonds A (4%, 8y) and C (3%, 7y) from the stylised flat-curve world, and compare converted prices $P_i(y)/\mathrm{CF}_i$:

| flat yield $y$ | A: $P/\mathrm{CF}$ | C: $P/\mathrm{CF}$ |
|---|---|---|
| 1.75% | 109.0 | **108.2** |
| 3.00% | 100.0 | 100.0 |
| 5.00% | **87.4** | 88.4 |

Below the 3% ruler the shorter, lower-duration bond (C) has the lower converted price and is cheaper to deliver; above it the longer bond (A) is. The CTD switches at the ruler.

## A8. The short's timing options

Beyond choosing which bond, the short chooses *when* in the delivery month, which is a second option:

- **wild card**: the settlement price is struck in the afternoon, but the short can announce delivery later that day; if bond prices fall after the fixing, the short buys cheap and delivers at the fixed invoice;
- **end-of-month**: the timing choice across the delivery days.

Both options belong to the short, so both push the futures price *down*: the buyer pays less for a contract whose counterparty holds the choices. Pricing them is Part II.

## A9. Where the convexity adjustment comes from

Daily settlement links the futures cash flows to the very rates that produced them:

- rates rise $\Rightarrow$ the short's gain is banked and reinvested at the new high rate; rates fall $\Rightarrow$ the loss is financed cheaply. The correlation is mechanical and always in the short's favour.
- so the futures rate exceeds the forward rate by $\frac{1}{2}\sigma^2 T_1 T_2$, growing with volatility and maturity.

| horizon $(T_1, T_2)$ | $\sigma = 1.2\%$ | adjustment |
|---|---|---|
| 6 months (0.5, 0.75) | $0.5\sigma^2 T_1 T_2$ | $\approx 0.3$bp |
| 5 years (5, 5.25) | $0.5\sigma^2 T_1 T_2$ | $\approx 19$bp |

## A10. The Eurodollar contract

- For four decades the three-month Eurodollar future was among the most heavily traded financial contracts in the world: a bet on three-month US-dollar LIBOR, $25 per basis point, listed years out.
- LIBOR's cessation (Lecture 4) left a large open interest referencing a rate that would disappear.
- On 14 April 2023 CME converted every open position to the equivalent SOFR contract, about 7.5 million contracts, at the fixed ISDA fallback spread of 26.161 bp.

The spread is the priced credit gap between unsecured LIBOR and secured SOFR, fixed once so no position gained or lost from the switch itself.

## A11. The basis trade in stress: a desk summary

| | |
|---|---|
| the funding | repo haircuts on Treasuries run about 2% or lower, at times near zero, so leverage is often bounded by futures margin, not the repo lender |
| the fragility | a volatility spike raises futures margin (L2) and repo haircuts at once, forcing cash the position does not hold |
| the unwind | sell the cash bonds, buy back the futures; at scale this sells the Treasury market into the panic (March 2020; concerns recurred in 2025) |
| the practical rule | a hedge's exit liquidity depends on leveraged basis positions; in stress expect a wider basis and higher margins, and pre-fund |

## A12. Solution to Problem A

A 4% semiannual bond (¥2.00 per period) is quoted at 102.30, with 77 days elapsed of a 182-day period:

$$\begin{aligned} \text{accrued} &= 2.00 \times \tfrac{77}{182} && \text{actual/actual, per ¥100} \\ &= 2.00 \times 0.42308 = 0.846 && \text{the seller's share of the coupon} \\ \text{cash price} &= 102.30 + 0.846 = \mathbf{103.146} && \text{clean plus accrued} \\ \text{per ¥500M face} &= 103.146 \times 5\text{M} = \mathbf{¥515.73}\text{M} && \text{scale by 5 million hundreds} \end{aligned}$$

## A13. Solution to Problem B

Futures at 104.20. Bond X: quote 109.85, CF 1.0540. Bond Y: quote 99.40, CF 0.9520. Delivery cost is $\text{quote} - F \times \text{CF}$:

$$X = 109.85 - 104.20 \times 1.0540 = 109.85 - 109.826 = \mathbf{0.024} \qquad \text{cheapest}$$

$$Y = 99.40 - 104.20 \times 0.9520 = 99.40 - 99.198 = 0.20 \qquad \text{dearer to deliver}$$

Bond X is the cheapest to deliver, with a basis of about **0.02** per ¥100. Bond Y costs roughly ten times as much to deliver.

## A14. Four-contract SOFR hedge

A borrower fears rising dollar rates and **sells four** SOFR contracts at 97.05. Rates set at 3.40%:

$$\text{settlement} = 100 - 3.40 = 96.60 \qquad \text{price is 100 minus the rate}$$

$$\text{move} = 97.05 - 96.60 = 0.45 = 45\text{bp} \qquad \text{the short gains as the price falls}$$

$$\text{P\&L} = 45\text{bp} \times \$25 \times 4 = \mathbf{\$4{,}500} \qquad \text{\$25 per bp, four contracts}$$

$$\text{locked rate} = 100 - 97.05 = \mathbf{2.95}\% \qquad \text{the rate sold}$$

The $4,500 gain offsets the higher financing, locking the borrowing at 2.95%.