Swaps

Lecture 7

https://fatih.ai/sjtu/derivatives/

a strip of FRAs — **one swap** — a pair of bonds

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Outline

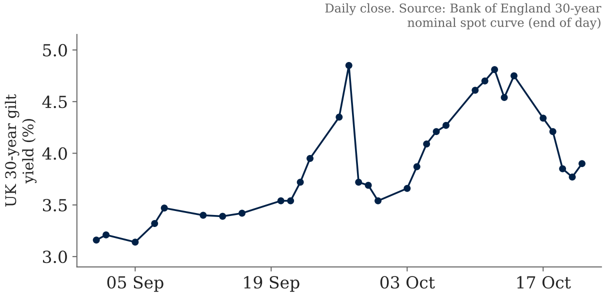
1. The interest rate swap: mechanics and uses
2. Pricing and valuing a swap
3. Currency swaps, clearing, and the 2022 gilt crisis

The hedge that was winning

In September 2022 a UK pension hedge that was **winning** still nearly collapsed the pension system. The instrument was the **interest-rate swap**.

Interest-rate contracts are about **79%** of the $846 trillion OTC notional (end-June 2025): the largest financial market by notional.

BIS OTC derivatives statistics, end-June 2025.



> Two questions, vote on both — 1. How can a hedge that is *winning* nearly fail?
>
> 2. The pensions were getting *safer* as yields rose, so what actually went wrong?

First build the instrument, what a swap is and how it is priced and valued. Then this crisis becomes readable.

The interest rate swap: mechanics and uses

What a swap is

A firm has a ¥3bn loan resetting each year at a benchmark $+0.85\%$. It could lock each reset with a separate FRA (Lecture 4), but that means many contracts and many counterparties.

> THE INSTRUMENT — A **swap** exchanges one stream of payments for another over several periods under one agreement: a bundle of forward rate agreements with one shared fixed rate.

One contract, not a drawer of FRAs, and it will run as the lecture's example: the same ¥3bn loan, its floating cost turned fixed.

— *A swap is a bundle of forwards under one shared fixed rate.* —

The plain-vanilla swap, and what actually moves

notional ¥100M: recorded, never exchanged

- Payments **net**: each year one party pays the difference. With the year-1 benchmark set at 2.33%, the fixed payer pays $2.73 - 2.33 = 0.40$ per 100, a single **¥0.40M** transfer.
- The notional is recorded, never exchanged: both legs reference the same amount, so only the difference moves.
- Default exposure is therefore the *value* of the remaining net payments, never the notional.

Transforming a liability

A swap fixes the ¥3bn floating loan directly, without renegotiating it:

loan: pay benchmark $+ 0.85\%$	floating
swap: pay fixed 2.73%, receive benchmark	
net: the benchmarks cancel	**fixed 3.58%**

The floating legs cancel and leave a fixed cost of $2.73 + 0.85 = 3.58\%$: on ¥3bn, a known **¥107.4M** a year.

— *A swap changes the interest-rate character of a debt without touching the loan.* —

Transforming an asset, and who uses swaps

The same instrument hedges the fixed-rate bond portfolio of Lecture 4: a **pay-fixed** swap cancels the fixed coupons and leaves *receive benchmark*, a synthetic floater with duration near zero. This is the same exposure as Lecture 6's 64 bond futures, in OTC form. Who uses swaps:

- **corporates**: transform debt, as above;
- **banks**: manage repricing gaps, in China the LPR-linked mortgages that reset against shorter funding;
- **insurers and pension funds**: buy long duration without buying decades of bonds (this returns in Section C);
- **investors**: take a pure rate view with almost no balance-sheet footprint.

The comparative-advantage argument

Two borrowers face this menu (credit spreads differ more in fixed markets):

	fixed	floating
AAA borrower	2.80%	benchmark $+0.20$
weaker borrower	4.00%	benchmark $+0.85$
difference	1.20%	0.65%

Each borrows where it is *relatively* cheaper (the AAA fixed, the weaker borrower floating), then swaps. The total saving is $1.20 - 0.65 = \mathbf{0.55}\%$ a year, split between the two and the dealer.

The critique of comparative advantage

A persistent 0.55% saving at this scale invites scrutiny:

- the weaker borrower's floating spread (+0.85) is **not locked**; the loan rolls, and if its credit worsens the spread widens. The swap fixed the benchmark, not the firm's own spread.
- the AAA's fixed-rate advantage is partly the value of a long-term commitment at its credit. Much of the apparent saving is payment for the rollover risk that stays with the weaker borrower.

— *The saving is largely the price of an unlocked credit spread and rollover risk, not a free gain.* —

The documents behind a swap

A swap is held together by paper, not an exchange:

- one **ISDA master agreement** per pair of firms (onshore in China, the **NAFMII master agreement**), each trade a short confirmation under it;
- **close-out netting**: on default, all trades between the pair collapse to a single net amount, the netting of Lecture 2 in contract law;
- **collateral** (a credit support annex): marks are exchanged in cash, typically daily, the OTC form of variation margin, mandatory for dealers since 1 March 2017.

The OTC market rebuilt the clearing-house defences bilaterally; Section C completes the convergence with central clearing.

Netting and daily collateral are Lecture 2's defences, rebuilt between two firms.

Pricing and valuing a swap

Two decompositions

SAME OBJECT pay-fixed swap = pay 2.73% fixed, receive benchmark, for two years

VIEW 1: SLICE BY TIME

Strip of FRAs

year 1: known net exchange
year 2: FRA on the next benchmark reset

value each date, then add PVs

VIEW 2: SLICE BY INSTRUMENT

Pair of bonds

long a floating-rate bond
short a 2.73% fixed-rate bond

for a fixed payer: $V = B_{\text{float}} - B_{\text{fixed}}$

Sliced by **time**, the swap is a strip of rate forwards; sliced by **instrument**, it is a floating bond minus a fixed bond.

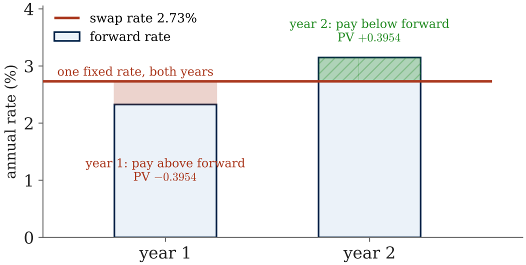
— *Two decompositions, one swap: the law of one price ties them together.* —

View 1: a strip of FRAs

Take the exchanges one year at a time (per ¥100):

- **year 1**: receive the benchmark set today (2.33%), pay 2.73: a known transfer;
- **year 2**: receive a rate set in one year, pay 2.73: an FRA (Lecture 4), struck at 2.73 not at the 3.15 forward.

The single fixed rate is the level that makes the early over-payment and the late under-payment cancel in present value.



View 2: a pair of bonds

Add ¥100 of principal to both legs at maturity, a net of zero since the notionals cancel, and re-read each leg (per ¥100):

- the **receive** leg becomes benchmark coupons plus 100 at the end: a **floating-rate bond**;
- the **pay** leg becomes 2.73 coupons plus 100 at the end: a **fixed-coupon bond**.

KEY EQUATION · THE SWAP AS A DIFFERENCE OF TWO BONDS

$$V_{\text{pay fixed}} = B_{\text{float}} - B_{\text{fixed}}.$$

Adding and subtracting one notional turns the swap into bonds that Lecture 4 already prices.

A floating-rate bond is worth par at a reset

> RESULT — At any reset date, a floating-rate bond is worth exactly its principal (par).

A floating-rate bond is a sequence of one-period deposits, each refreshed at the market rate, so at every reset it is back to par. The one-line proof:

- at the last reset, in one period the bond pays $100\,(1+R)$ where R is the rate just set; discounting at that same rate gives $100(1+R)/(1+R)=100$;
- one reset earlier the bond will be worth 100 plus a just-set coupon in a period: the same argument. Induction reaches today.

The swap rate is the par yield

At signing the swap is worth zero, so

$$B_{\text{float}} = B_{\text{fixed}} \quad \Longrightarrow \quad 100 = B_{\text{fixed}}.$$

The swap rate is the fixed coupon that prices a bond at par: the **par yield** named in Lecture 4. In discount factors, per period,

KEY EQUATION · THE SWAP RATE AS THE PAR YIELD

$$s = \frac{100\,(1 - d_n)}{\sum_{i=1}^{n} d_i}.$$

— *The swap rate equals the par yield of the underlying curve.* —

The number 2.73, derived

Discount factors from the Lecture 4 curve ($z(1) = 2.3\%$, $z(2) = 2.7\%$):

$$d_1 = e^{-0.023} = 0.97726 \qquad \text{one-year discount factor}$$

$$d_2 = e^{-0.054} = 0.94743 \qquad \text{two-year discount factor}$$

$$s = \frac{100\,(1 - 0.94743)}{0.97726 + 0.94743} = \frac{5.257}{1.92469} = \mathbf{2.73}\% \qquad \text{the par yield}$$

The swap rate was already inside the bootstrapped curve: the same discount factors that priced the bonds of Lecture 4 price the swap.

The two views agree

Cross-check View 1: value each year at its forward rate against the fixed 2.73 (per ¥100):

year	forward rate	net to fixed payer	PV at d_i
1	2.33%	-0.4046	-0.3954
2	3.15%	$+0.4172$	$+0.3954$
		sum	**0.0000**

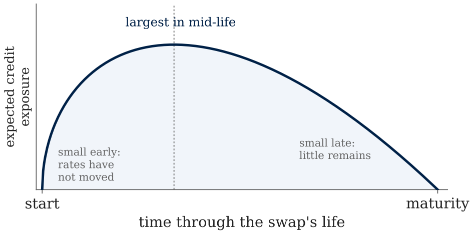
The known year-1 loss exactly funds the year-2 lock below its forward.

— *The FRA and bond decompositions give the same value, as the law of one price requires.* —

Valuing a seasoned swap

One year on, the year-1 payment has settled, one exchange remains, and $z(1)$ is now **3.4%**:

B_{float}: par at the reset	100.000
$B_{\text{fixed}} = 102.7313\, e^{-0.034}$	99.297
$V_{\text{pay fixed}} = B_{\text{float}} - B_{\text{fixed}}$	$+0.703$



On the ¥100M slice, +¥0.70M; on the ¥3bn programme, +¥21.1M.

FRA view, one line: the just-set benchmark is $e^{0.034} - 1 = 3.4585\%$, so $(3.4585 - 2.7313)\, e^{-0.034} = +0.703$. Both agree.

— *Credit exposure is the swap's value, not its notional, largest in mid-life.* —

Check yourself

PROBLEM A — A two-year annual swap has $d_1 = 0.9800$, $d_2 = 0.9500$. Compute the swap rate, and name the bond fact it represents. Answer: $s = 100(1 - 0.95)/1.93 = \mathbf{2.59}\%$, the two-year par yield.

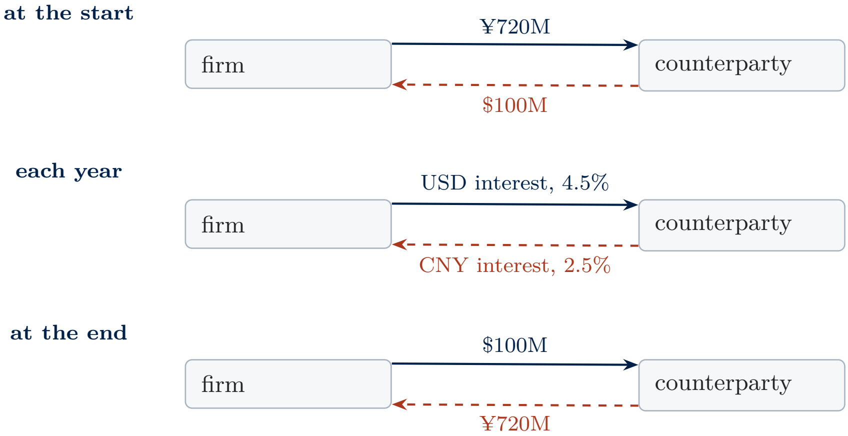
PROBLEM B — That swap, one year on: the one-year benchmark has just set at 3.20%, and $d = 0.9690$. Value the remaining exchange to the fixed payer, per ¥100. Answer: $(3.20 - 2.59) \times 0.9690 = +\mathbf{0.59}$.

Both reuse frames 15–18. Full step-by-step solutions are in appendix A11–A12.

Currency swaps, clearing, and the 2022 gilt crisis

The currency swap

Lecture 5 locked one dollar invoice. A firm that earns dollars for years borrows cheaply at home and expensively abroad, and would rather borrow where it is known and owe where it earns. A currency swap does this:



Principal moves, and marking the swap

The deep difference from an IRS: principal is exchanged at the start and **re-exchanged at the original rate** at maturity. Credit exposure therefore includes the principal at the final exchange. Mark it two years on, the yuan weakened to **7.45**.

$B_{\text{CNY}} = 18\,d_1 + 738\,d_2$	¥716.8M
$B_{\text{USD}} = 4.5\,e^{-0.045} + 104.5\,e^{-0.09}$	\$99.81M
$V = B_{\text{CNY}} - 7.45 \times B_{\text{USD}}$	**−¥26.8M**

CNY leg off the Lecture 4 curve; USD leg at 4.5%; one FX conversion at 7.45.

The firm receives the yuan bond and owes the dollar bond, so a dearer dollar marks against it.

A negative mark on a currency hedge can be the hedge working, because the dollar liability was chosen to match dollar revenue.

Benchmarks and central clearing

The floating leg references a benchmark, and standard swaps are now cleared:

- globally, new swaps reference **overnight rates compounded** (SOFR and its peers); onshore, the interbank RMB swap mainly references **FR007**, the 7-day repo fixing (about 96% of 2024 onshore rate-derivative notional); **Swap Connect** has let offshore investors trade onshore RMB swaps since 15 May 2023;
- standard swaps are **centrally cleared**: novation, default funds, daily margin (Lecture 2), with LCH globally and the **Shanghai Clearing House** mandatory for standard interbank RMB swaps since 1 July 2014.

— *With clearing comes margin, so a swap book faces the same daily cash demands as a futures book.* —

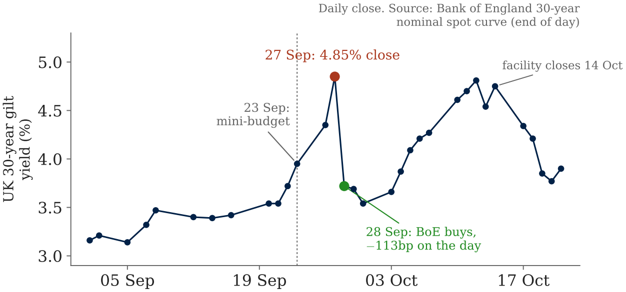
The longest liabilities in finance

Defined-benefit pension funds owe payments stretching decades, the longest-duration liabilities anywhere. When rates fall, the present value of those promises rises.

- The textbook hedge (Lectures 4 and 7): match the duration, with receive-fixed swaps and long-dated bonds.
- The implementation: **liability-driven investment (LDI)** funds holding bonds, swaps, and repo, levered roughly **two to four times** (median about 2.5 at the September 2022 peak), so a little capital hedges a large liability.

This is the course's own medicine, duration matching, plus leverage for capital efficiency.

September 2022, the gilt crisis



A mini-budget on 23 Sep 2022 (about £45bn of unfunded tax cuts) startled the market; 30-year yields rose about **1.2 percentage points in three trading days**. Then the loop:

- yields up → LDI marks move against the funds;
- collateral calls in cash → forced gilt sales into a falling market;
- which pushes yields up again.

The Bank of England intervened on 28 Sep with a facility of up to **£65bn** capacity, of which about **£19.3bn** was used; the facility closed 14 Oct.

— *Margining turned a winning hedge into a scramble for cash.* —

The lesson of the LDI episode

The decisive point: the hedge was *winning*. Higher rates cut the present value of the pension liabilities more than the assets fell, so funded status **improved** through the crisis (Breeden: liabilities -36% versus assets -20%).

- The funds were right on solvency and nearly failed on **liquidity**: margin on a winning position still demands cash immediately, as with Tsingshan, Metallgesellschaft, and Lecture 6's basis trade.
- The reform: LDI funds must hold liquidity buffers sized to large yield moves, the cash line treated as part of the hedge.

— *A correctly hedged position can still fail for liquidity, because margin demands cash before the hedge pays off.* —

Summary

- Lecture 1: transfer. Lecture 2: concentrate. Lecture 3: exchange. Lecture 4: every date a price. Lecture 5: the present, carried. Lecture 6: the cheapest bond sets the price.
- Lecture 7: a swap **redraws who owes whom**, creating no new payoff.

Every swap today came apart into pieces already known: FRAs, bonds, and forwards. Its power and its risk live in the redrawing, which benchmark, which currency, which counterparty, which dates the cash is due.

— *A swap redraws who owes whom; no new payoff, but new cash-flow timing.* —

A practical summary

the debt, fixed	a pay-fixed swap across the ¥3bn programme; all-in 3.58% on the worked tenor; the firm's own credit spread still floats at each loan roll
the mark	+¥0.70M per ¥100M slice as rates rose; collateral received daily against it
the dollars	funding matched to dollar revenue via the currency swap; the −¥26.8M mark is the hedge working; principal at the final exchange is a real exposure
the plumbing	cleared where standard (SHCH); margin lines sized to the LDI lesson, with the hedge's cash demands treated as part of the hedge

In-class problems

A. A two-year annual swap with $d_1 = 0.9800$, $d_2 = 0.9500$. Compute the swap rate, and state the bond fact it represents. $s = 2.59\%$

B. That swap, one year on: the one-year benchmark has just set at 3.20%, and $d = 0.9690$. Value the remaining exchange to the fixed payer, per ¥100. $V = +0.59$

C. Borrowing menu: firm X fixed 3.2% / benchmark $+0.3$; firm Y fixed 4.1% / benchmark $+0.7$. Size the comparative-advantage saving, propose a split, and name the risk firm Y still carries. saving 0.5%

Step-by-step solutions to all three are in the appendix; try them before you look.

The opening vote: how can a winning hedge nearly fail?

The funds were right on solvency and nearly failed on liquidity: margin demands cash before the hedge pays off.

Every swap today came apart into pieces that could be priced and checked: FRAs, bonds, and forwards. Lecture 8: bundles built so that they could *not* be taken apart: securitisation and the 2008 credit crisis.

Homework: Hull Ch. 7, the mechanics and transformation problems, the comparative-advantage set, and valuation by both methods. Field task: pull the current FR007 swap curve from CFETS, bootstrap one discount factor, and check the two-year par-yield identity.

Appendix: proofs, mechanics, and solutions

A1. The scale of the swap market

- BIS OTC statistics, end-June 2025: total notional outstanding **\$846 trillion**; gross market value **\$21.8 trillion**; interest-rate derivatives are about **79%** of all OTC notional (roughly \$668tn).
- Notional far exceeds value because notional counts the face amounts of promises, dealers run largely offsetting books, and only net differences ever move: the notional-versus-value lesson of Lecture 1, at market scale.
- **Trade compression** retires offsetting trades in bulk, so notional totals can fall even as activity rises.

A2. Procter & Gamble versus Bankers Trust, 1994

A swap from Bankers Trust whose floating leg was not a benchmark but a **leveraged formula** of US interest rates, which produced a large loss when rates rose in 1994.

- P&G took a pre-tax charge of about **$157 million** (about $102m after-tax) on two leveraged swaps; the case settled in 1996 with P&G absorbing about $35m of the roughly $200m claimed.
- The label on a contract is not the risk inside it: the confirmation defines the payoff.
- Complexity is a price, usually paid to whoever designed the formula.

A3. The dealer in the middle

A firm rarely finds an exactly offsetting counterparty, so each side faces a market-making bank:

- the dealer quotes two-way, for example receive fixed at 2.71, pay fixed at 2.75; the bid-ask spread compensates it for warehousing the mismatch;
- the residual risk while it waits for the other side is duration, which it hedges with the bond futures of Lecture 6.

The worked 2.73 sits inside a 2.71/2.75 quote. Later diagrams omit the dealer, but the spread is always present.

A4. The floating-rate bond at par, in full

Let R be the rate set at the last reset, one period before maturity:

$$\begin{aligned} \text{at the last reset}: \frac{100\,(1+R)}{1+R} &= 100 && \text{coupon plus principal, discounted at } R \\ \text{one reset earlier}: \frac{100+(\text{just-set coupon})}{1+R'} &= 100 && \text{value in a period is 100, by the line above} \\ \text{by induction}: \text{worth 100 at every reset} & && \text{the argument repeats back to today} \end{aligned}$$

Between resets, the bond is worth the next known coupon plus 100, discounted from the next reset date: par grows to par-plus-coupon and is pulled back.

A5. The swap rate is the par yield, derived

At signing $B_{\text{fixed}} = 100$. Write the fixed bond's price with per-period coupon s (per 100) and discount factors d_i:

$$s \sum_{i=1}^{n} d_i + 100\, d_n = 100 \qquad \text{coupons plus principal, priced at par}$$

$$s \sum_{i=1}^{n} d_i = 100\,(1 - d_n) \qquad \text{move the principal term across}$$

$$s = \frac{100\,(1 - d_n)}{\sum_{i=1}^{n} d_i} \qquad \text{the par yield}$$

The par yield named in Lecture 4 is exactly the swap rate: the same object, read off the same curve.

A6. The market runs it in reverse

In class the curve gives the swap rate; on a dealing desk the direction is reversed:

- quoted **swap rates** for every tenor are among the most liquid rate prices, so banks **bootstrap the curve from swaps** (Lecture 4's method, fed by this market);
- the "swap curve" is, for practitioners, the par-yield curve of the deepest market, and is the curve used for discounting.

The instrument is priced off the curve, and the curve is read off the instrument: one self-consistent object.

A7. The currency swap, decomposed

Slice the currency swap by time: each line of the frame-20 diagram is an exchange of currencies at a fixed future rate, an **FX forward** (Lecture 5):

- by time: a strip of FX forwards, each priced by covered interest parity;
- by instrument: long a bond in one currency, short a bond in the other, exactly the marking of frame 21.

The marking arithmetic: $B_{\mathrm{CNY}} = 18\,d_1 + 738\,d_2 = 716.8$M (the 2.5% CNY coupons plus the ¥720M principal, off the Lecture 4 curve);
$B_{\mathrm{USD}} = 4.5\,e^{-0.045} + 104.5\,e^{-0.09} = 99.81$M; then $V = 716.8 - 7.45 \times 99.81 = -¥26.8$M.

A8. The floating benchmark after LIBOR

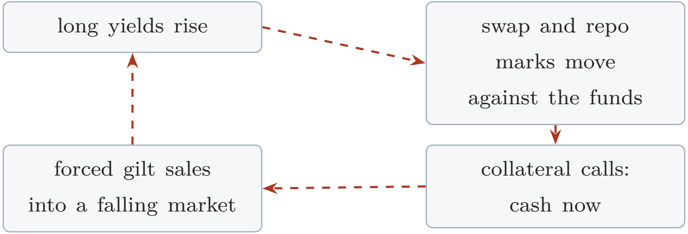
- Globally, new swaps reference **overnight rates compounded** (SOFR and its peers); the swap curve is built from these.
- Onshore, the interbank RMB swap market mainly references **FR007**, the 7-day repo fixing (transaction-anchored, the market cousin of DR007), about **96%** of 2024 onshore rate-derivative notional, with Shibor tenors second.
- Since 15 May 2023, **Swap Connect** (northbound) lets offshore investors trade onshore RMB swaps through Hong Kong.

A9. Central clearing detail

- **Novation**: the clearing house steps between the two firms and becomes buyer to the seller and seller to the buyer (Lecture 2).
- **Default funds and daily margin**: initial margin sized to a stress move, variation margin exchanged in cash each day.
- **LCH** clears standard swaps globally; the **Shanghai Clearing House** has been mandatory for standard interbank RMB swaps since 1 July 2014.

The clearing arc from Lecture 2 now reaches swaps, and is being extended to the Treasury market of Lecture 6.

A10. The LDI feedback loop and reform, in full



Leverage of about 2.5 times meant a large notional hedge on a little capital, so a 1.2-percentage-point yield move drained cash faster than the funds could raise it. The reform: liquidity buffers sized to large yield moves, the same lesson as Tsingshan, Metallgesellschaft, and Lecture 6's basis trade.

A11. Solution to Problem A

A two-year annual swap with $d_1 = 0.9800$, $d_2 = 0.9500$:

$$\sum d_i = 0.9800 + 0.9500 = 1.9300 \qquad \text{the annuity factor}$$

$$100\,(1 - d_2) = 100\,(1 - 0.9500) = 5.00 \qquad \text{the numerator}$$

$$s = \frac{5.00}{1.9300} = \mathbf{2.59\%} \qquad \text{the two-year par yield}$$

The bond fact: a par bond has coupon equal to its yield, so this coupon prices B_{fixed} at 100, and at signing $B_{\text{float}} = B_{\text{fixed}} = 100$.

A12. Solution to Problem B

The same swap one year on: the one-year benchmark has just set at 3.20%, one exchange remains, and the one-year discount factor is $d = 0.9690$. Value it by the FRA view (per ¥100):

$$\text{net to fixed payer} = 3.20 - 2.59 = 0.61 \qquad \text{receive just-set benchmark, pay the swap rate}$$

$$V = 0.61 \times 0.9690 = +\mathbf{0.59} \qquad \text{discount the single net payment}$$

The fixed payer is ahead: the benchmark set above the swap rate, so the one remaining exchange is a receipt.

A13. Solution to Problem C

Firm X: fixed 3.2% / benchmark $+0.3$. Firm Y: fixed 4.1% / benchmark $+0.7$.

$$\text{fixed gap} = 4.1 - 3.2 = 0.9\% \qquad \text{Y pays 0.9 more fixed}$$

$$\text{floating gap} = 0.7 - 0.3 = 0.4\% \qquad \text{Y pays 0.4 more floating}$$

$$\text{total saving} = 0.9 - 0.4 = \mathbf{0.5}\% \qquad \text{split, say 0.25 each}$$

X (relatively better in fixed) borrows fixed; Y (relatively better in floating) borrows floating; they swap. The risk Y still carries: its own floating credit spread ($+0.7$) is not locked and **resets at every loan roll** (frame 9).