Financial Econometrics | Lecture 10

Volatility, Systems and the Long Run: GARCH, VAR and Cointegration

In Class Group Activity

· SJTU, Summer School 2026 · Fatih Kansoy

How this block runs. The last one. You work in your home group of three or four, textbook open. Roles rotate: **Scribe**, **Reporter**, **Sceptic**, **Timekeeper**. All money today is in US dollars ($), pounds sterling (£) and euros (€).

- **Phase 1, Concept checks (~10 min).** Vote, talk, vote again.
- **Phase 2, Desk problem (~20 min).** Run a GARCH recursion and read a cointegrating relationship.
- **Phase 3, Case (~25 min).** One real story, ending hidden.

A short word list. **GARCH(1,1)**: $\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta\sigma_{t-1}^2$; **persistence** $\alpha+\beta$ (near 1 means a volatility shock dies slowly), the long-run variance is $\omega/(1-\alpha-\beta)$. **Value-at-risk** (VaR) scales with today's conditional σ_t. The **leverage effect**: a fall raises tomorrow's volatility more than an equal rise. A **VAR** is a system of equations; **Granger** causality there is predictability, not causation. A series is **I(1)** if its level is a random walk. Two I(1) series are **cointegrated** if a linear combination is stationary; an **error-correction model** (ECM) gives the speed of adjustment back to that equilibrium.

Phase 1 – Concept checks ~10 min

ConcepTest 1. A bank fits GARCH(1,1) to daily equity returns: $\alpha = 0.165$, $\beta = 0.805$, so $\alpha+\beta = 0.97$. A junior analyst says "persistence so close to 1 means the model is unstable and the volatility forecast will blow up." The right reading is:

(a) The analyst is right; $\alpha+\beta$ near 1 makes the variance explode, so the model is unusable.

(b) Persistence near 1 means a volatility shock fades slowly (a long half-life), but the model is stable because $\alpha+\beta < 1$; risk forecasts are useful *because* turbulence persists.

(c) Persistence near 1 means returns themselves are now forecastable, contradicting efficient markets.

(d) Persistence has nothing to do with how long a shock lasts; it only sets the average level of volatility.

ConcepTest 2. On a calm day a desk's GARCH model gives a 1-day 99% VaR of about 1.2% of the book. The desk hard-codes that 1.2% as its loss limit for the whole year "to keep things simple." Markets then enter a crisis and the conditional volatility multiplies. What has the desk done wrong?

(a) Nothing; 99% VaR is a fixed property of the portfolio, so one number a year is correct.

(b) It used the long-run volatility when it should have used an even lower number, so the limit is too tight.

(c) It froze VaR at the calm-day level; because VaR $= 2.326\,\sigma_t$ scales with today's conditional volatility, the true VaR rises sharply in the crisis, so the frozen limit badly understates the danger.

(d) VaR is irrelevant for equities; only the leverage effect matters in a crisis.

ConcepTest 3. The 10-year and 3-month Treasury rates each look like a random walk (DF-GLS does not reject a unit root: −1.08 and −1.41). You regress the 10-year on the 3-month in levels and get a big t and a high R^2. A classmate says "both rates trend together, so they are obviously cointegrated." When is the levels regression actually valid?

(a) Always; a high R^2 and a big t prove the relationship is real.

(b) Only if the residual from that regression is itself stationary (I(0)); that is what makes them cointegrated rather than spuriously correlated. Here the ADF on the residual is −3.69, beyond the 5% Engle-Granger value −3.34, so it *is* valid.

(c) Never; you can never regress two I(1) series, so the result is meaningless.

(d) Whenever both series are I(1) and move in the same general direction; shared trending is enough.

Phase 2 – The desk problem ~20 MIN

A GARCH recursion, and a cointegrated spread

The setting. You are on the risk desk for a FTSE 100 tracker. You fit GARCH(1,1) to daily percentage returns (units of per-cent-squared): $\omega = 0.05$, $\alpha = 0.13$, $\beta = 0.84$. Yesterday's conditional variance was $\sigma_{t-1}^2 = 1.00$ (a 1% day), and yesterday the market fell sharply, $u_{t-1} = -4.0\%$, so $u_{t-1}^2 = 16.0$.

(a) One step of the recursion (worked for you).

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta\sigma_{t-1}^2 = 0.05 + 0.13(16.0) + 0.84(1.00) = 0.05 + 2.08 + 0.84 = 2.97,$$

so today's volatility is $\sigma_t = \sqrt{2.97} = 1.72\%$. One bad day pushed volatility from 1% to 1.72%, mostly through the αu^2 term (2.08 of the 2.97).

(b) Persistence, long run, half-life. Compute $\alpha + \beta$, the long-run variance $\omega/(1 - \alpha - \beta)$ and its square root, and the half-life of a volatility shock $\ln(0.5)/\ln(\alpha + \beta)$. Is today's σ_t above or below the long-run level?

(c) Value-at-risk. Using 1-day 99% VaR $= 2.326\,\sigma_t$, compute the VaR on a calm day (take σ_t at the long-run level) and on today's turbulent day. By how much does it rise?

(d) The cointegration twist. The desk also trades a spread between two UK rates, both I(1), with long-run relation $\text{Rate10}_t = 1.50 + 0.90\,\text{Rate3}_t + z_t$ and an ECM whose z_{t-1} coefficient is −0.05 ($t = -3.4$). The 10-year currently sits 0.80 points *above* its long-run relation ($z_{t-1} = +0.80$).

(1) What change does the ECM correction term predict for the 10-year next period?

(2) What fraction of the gap closes per period, and is that fast or slow?

(3) Why is regressing the levels valid *here*, when Lecture 8 said never to regress two trending series?

Phase 3 – The case ~25 MIN

The limit that did not move: a risk desk in March 2020

How to run this. Read the story with your group. The ending is hidden on purpose. Commit a written position before the reveal. Do not search the internet.

The story. You run market risk at the Meridian US Equity Fund, a $2 billion book tracking the S&P 500. Through 2017–2019 the market is calm; daily conditional volatility hovers near 0.75–0.85%. Your board approved a simple rule three years ago: the 1-day 99% VaR limit is set *once a year* from the trailing one-year average (unconditional) volatility. For 2020 the limit was fixed in January at about 1.2% of the book, roughly $24 million. The quant team proposed a GARCH-based conditional VaR that updates daily; the board rejected it as "too jumpy." The GARCH(1,1) on your desk: $\omega \approx 0.04$, $\alpha \approx 0.165$, $\beta \approx 0.805$, persistence 0.97.

Late February 2020, COVID headlines spread. On 24 February the S&P falls about 3.4%; on 27 February about 4.4%. Squared returns jump, and the conditional volatility, if you ran the recursion, would already be climbing fast. But the official limit is still 1.2%, set in January, so on paper the desk is "within limit." Then the second week of March: −7.6% on the 9th, −9.5% on the 12th, and −**12.77**% on Monday 16 March, the worst day in your whole sample. The conditional 99% VaR has run to about 17 to 18% of the book, around $350 million, against a frozen limit of $24 million.

The story stops here.

Four discussion questions (answer all four; take a position):

1. On 27 February, with a −4.4% shock ($u^2 = 19.4$) hitting after a calm $\sigma^2 \approx 0.70$, run the recursion: roughly what is σ_t^2 and the conditional 99% VaR? Is the frozen 1.2% limit protecting anyone?
2. Persistence is 0.97. Is this spike a one-off, or is it forecast to last? Estimate the half-life in trading days.
3. In one sentence, what would you put in front of the board to argue that a *constant* VaR limit is the dangerous choice?
4. **Decide.** On the morning of 16 March, conditional VaR about 17–18% versus a $24m limit: do you (i) keep trading because the rule says you are compliant, (ii) override and cut the book hard, or (iii) freeze new risk and escalate to the board? Commit and give your one-line reason.

End of the activity, and of the course. Solutions are distributed after the block. Well done.