Financial Econometrics | Lecture 4

Testing a Regression Slope: t-statistics, Confidence Intervals and Robust Standard Errors

In Class Group Activity

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Lesson of the day. Last week you could estimate a slope; this week you test it. Attach a standard error, build a t-statistic and a confidence interval, and decide whether a slope is real, equal to an economically meaningful benchmark, or just sampling noise.

Phase 1 – Concept checks

ANSWERS

ConcepTest 1. **Answer.** Option **(b)**. The true beta is fixed; the interval is what is random, so the 95% describes the procedure across repeated samples, not this one interval. Option (a) is the common "probability of the truth" error; (c) confuses a confidence interval for the slope with the spread of the data; (d) wrongly ties interval width to significance (here 0 is far outside $[0.80, 1.20]$, so the beta *is* significant against 0).

ConcepTest 2. **Answer.** Option **(a)**. OLS picks the slope by minimising squared residuals, a step that never looks at the standard-error formula. Robust standard errors only change how the sampling variability of that same slope is computed. Options (b) and (c) imagine robust SEs re-estimate the coefficient; (d) imagines the point estimate responds to the heteroskedasticity.

In the lab, growth on trade share gives the slope 2.31 *under* both *formulas; only the SE moves (*$0.77 \to 0.66$*) and so the* t *(*$2.98 \to 3.48$*).*

ConcepTest 3. **Answer.** Option **(b)**. The huge t against 0 (22.5) says only that the stock moves with the market. The economically interesting test is against 1: $t = (0.90 - 1)/0.04 = -2.5$, so the beta is significantly below 1, a defensive stock. Option (a) collapses "different from 0" into "different from 1"; (c) eyeballs the gap instead of scaling it by the SE; (d) denies that non-zero nulls exist.

Phase 2 – The desk problem

WORKED SOLUTIONS

(a) $\text{CI} = 1.24 \pm 1.96(0.120) = 1.24 \pm 0.2352 = [1.00,\ 1.48]$ (precisely $[1.005, 1.475]$). *Means*: across many samples, about 95% of intervals built this way contain the true beta. *Does not mean*: there is a 95% chance the true beta is in this one interval.

(b) $t = \dfrac{1.24}{0.120} = 10.33$. Since $|10.33| > 1.96$, **reject** $H_0 : \beta = 0$: the stock clearly co-moves with the market (p far below 0.001). Consistent with (a), since 0 lies well outside $[1.00, 1.48]$. The interval and the two-sided test always agree.

(c) $t = \dfrac{1.24 - 1}{0.120} = \dfrac{0.24}{0.120} = 2.00$. Since $|2.00| > 1.96$ (just), **reject** $H_0 : \beta = 1$ at 5%: Riverstone is statistically more volatile than the market, an aggressive stock, though only marginally (the interval's lower edge sits essentially at 1).

The same slope and SE gave a crushing $t = 10.33$ *against* 0 *but a borderline* $t = 2.00$ *against* 1.

"Different from 0" and "different from 1" are different questions.

(d) With the homoskedastic $SE = 0.085$:

- the slope is **unchanged at** 1.24: the SE formula never touches the estimate;
- $H_0 : \beta = 0$: $t = 1.24/0.085 = 14.59$ (was 10.33), reject even more decisively;
- $H_0 : \beta = 1$: $t = 0.24/0.085 = 2.82$ (was 2.00), reject more comfortably;
- CI $= 1.24 \pm 1.96(0.085) = [1.07,\ 1.41]$, narrower than $[1.00, 1.48]$.

Against the benchmark $\beta = 1$, the homoskedastic SE makes the case look solid ($t = 2.82$); the honest robust SE makes it borderline ($t = 2.00$). The homoskedastic SE here is *too small*, so it overstates the evidence. Returns are heteroskedastic (volatility clusters), so the robust SE is the trustworthy one. Report the robust version.

Phase 3 – The case

REVEAL AND ANSWERS

1. "Defensive, beta 0.88" asserts the true beta is *below* 1. The relevant null is $H_0 : \beta = 1$ (against $\beta < 1$), not $\beta = 0$.

2. $t = \dfrac{0.88 - 1}{0.11} = \dfrac{-0.12}{0.11} = -1.09$. Since $|-1.09| < 1.96$ (and < 1.64 one-sided), **fail to reject** $\beta = 1$. The 95% interval is $0.88 \pm 1.96(0.11) = [0.66,\ 1.10]$, which *contains* 1. The data do not support "lower than the market".

3. Alpha: homoskedastic $t = 0.22/0.10 = 2.20$ (significant, just); robust $t = 0.22/0.13 = 1.69$ (< 1.96, **not** significant). The "significant alpha" rests entirely on the smaller, wrong standard error.

4. **Instructor reveal.** The slope never moved (0.88 either way), so robust SEs do not "fix" the beta; what moved was the SE, and that changed everything. Claim 1: the honest test is $\beta = 1$, the robust t is -1.09 and the interval $[0.66, 1.10]$ straddles 1, so the fund is statistically indistinguishable from a market tracker, not defensive. Claim 2: under the classical SE the alpha t is 2.20 and just clears the bar; under the robust SE it is 1.69 and does not. The small-type footnote was doing the heavy lifting. Financial returns cluster in volatility, so the robust SE is the trustworthy one. Neither headline claim survives an honest re-test; recommend against relying on the factsheet.

Reserve alternate (wide interval). An analyst reports a "precise" oil-factor loading of 1.6 for an airline, $n = 28$, robust $SE = 0.9$. Against 0, $t = 1.6/0.9 = 1.78$ (< 1.96, not significant), and the 95% interval is about $[-0.16, 3.36]$, so wide that the "precise" 1.6 is useless for hedging. With $n = 28$ the honest critical value is the $t_{26} \approx 2.06$, not 1.96.

End of worked solutions.