Financial Econometrics | Lecture 6

Joint Hypothesis Tests: the F-statistic for Correlated Regressors

In Class Group Activity

· SJTU, Summer School 2026 · Fatih Kansoy

How this block runs. You work in your home group of three or four, textbook open. Roles rotate: **Scribe**, **Reporter**, **Sceptic**, **Timekeeper**. All money today is in US dollars ($) and pounds sterling (£).

- **Phase 1, Concept checks (~10 min).** Vote on your own, talk for 90 seconds, then vote again.
- **Phase 2, Desk problem (~20 min).** Two correlated factors, tested alone and together.
- **Phase 3, Case (~25 min).** One real story, ending hidden. You take a position, then we reveal what happened.

A short word list. A **joint hypothesis** sets q restrictions on the coefficients at once. The **F-statistic** tests them together against the **F-distribution** (the 5% critical value is 3.84 for $q = 1$, 3.00 for $q = 2$, 2.60 for $q = 3$). **Size distortion** is why you cannot just test coefficients one at a time: testing q true nulls each at 5% raises a false alarm with probability $1 - 0.95^q$. Two coefficients can be each **individually insignificant** yet **jointly significant** when the regressors are correlated. A test of $b_1 = b_2$ is a single restriction ($q = 1$). Always report the **heteroskedasticity-robust** F.

Phase 1 – Concept checks ~10 MIN

ConcepTest 1. You want to know whether age and sex *both* have zero coefficient in a wage regression. You test age alone at 5% and sex alone at 5%, and you reject the joint claim if *either* is significant. If both truly have zero effect and the two tests were independent, what is the chance of a false alarm?

(a) Exactly 5%, because each test is at 5%.

(b) About 9.75%, because $1 - (0.95)^2 = 0.0975$.

(c) About 2.5%, because you split the 5% across the two tests.

(d) Exactly 10%, because you add 5% and 5%.

ConcepTest 2. In a regression of a fund's return on a `value` factor and a `quality` factor, the value loading has $t = 1.4$ and the quality loading $t = 1.5$, so neither clears 1.96. The two factors correlate about +0.8. A colleague says: "Neither t is significant, so both factors are irrelevant; drop them both." Is the colleague right?

(a) Yes. If neither coefficient is individually significant, the two together cannot be significant either.

(b) No. Because the factors are highly correlated, each loading is imprecise on its own, yet the joint F-test can still reject; they may matter together.

(c) Yes, but only because the sample is small; more data would lift each t above 1.96.

(d) No, because two insignificant coefficients *always* become significant when tested jointly.

ConcepTest 3. You want to test whether a coup and an assassination have the *same* effect on growth, $H_0 : b_{\text{coups}} = b_{\text{assass}}$, in a regression containing both. How many restrictions is this, and how can you test it?

(a) One restriction ($q = 1$): a single F (against 3.84), or equivalently a t on the difference $b_{\text{coups}} - b_{\text{assass}}$.

(b) Two restrictions ($q = 2$), because two coefficients are involved; compare with 3.00.

(c) One restriction, but tested by adding the two t-statistics, $t_{\text{coups}} + t_{\text{assass}}$.

(d) It cannot be tested directly; drop one variable and compare R^2 by eye.

Phase 2 – The desk problem

~20 MIN

Two style factors that vanish one at a time

The setting. A quant desk regresses a fund's monthly excess return on two style factors over $n = 120$ months: a `value` factor and a `quality` factor, which correlate about +0.78. With robust standard errors:

	estimate	**robust SE**
b_{value}	0.182	0.140
b_{qual}	0.225	0.150

Robust joint F for "both zero" = 8.9 ($q = 2$). SE of the difference ($b_{\text{value}} - b_{\text{qual}}$) = 0.27. R^2 with both factors = 0.118, with neither = 0.010.

(a) Test each factor alone. Compute t_{value} and t_{qual} and compare with 1.96. Is either significant at 5%?

(b) Test them together. Compare the joint $F = 8.9$ with the $q = 2$ critical value (3.00 at 5%, 4.61 at 1%). What do you conclude? Explain the apparent contradiction with part (a), using the correlation between the two factors.

(c) Are the two loadings equal? Test $H_0 : b_{\text{value}} = b_{\text{qual}}$ as a single restriction ($q = 1$). Compute the difference and its t using $SE_{\text{diff}} = 0.27$. What does the result tell you about how well the data can separate the two loadings?

(d) The robust-vs-homoskedastic twist. Compute the homoskedasticity-only joint F from the R^2 form,

$$F = \frac{(R_u^2 - R_r^2)/q}{(1 - R_u^2)/(n - k - 1)}, \qquad q = 2,\ k = 2,\ n = 120.$$

Which F should you report on financial return data, and why?

Phase 3 – The case

~25 MIN

The two macro factors the stress test almost threw away

How to run this. Read the story with your group. The ending is hidden on purpose. Commit a written position before the reveal. Do not search the internet.

The story. A UK pension fund (£4.2bn) runs its annual regulatory stress test. The risk team models the fund's quarterly return on two macro factors from the government bond market: the `LEVEL` factor (a parallel shift in the whole yield curve) and the `SLOPE` factor (a steepening or flattening). Over $n = 80$ quarters, with robust standard errors:

	loading	**robust SE**	t
LEVEL	-0.46	0.34	-1.35
SLOPE	0.58	0.39	1.49

The two factors correlate about $+0.85$. A junior analyst writes: "Neither rate factor is significant at 5%. To keep the model parsimonious, I propose dropping both and reporting the fund as having no measurable interest-rate exposure." The head of risk is about to sign off. Sitting unused in the output: the joint F for "both zero" is **6.40** ($q = 2$).

The story stops here.

Four discussion questions (answer all four; take a position):

1. Confirm the analyst's claim: compute each t and check neither is significant at 5%.
2. Compare the joint $F = 6.40$ with the $q = 2$ critical value. What does it say about the proposal to drop both factors?
3. The two factors correlate about $+0.85$. In one sentence, why is each t small even though the pair is jointly significant?
4. The regulator cares about *total* interest-rate risk. If the team drops both factors, what happens to the reported exposure, and why is that dangerous?

End of activity handout. Solutions are distributed after the block.