

FINANCIAL ECONOMETRICS
SHANGHAI JIAO TONG UNIVERSITY
SUMMER SCHOOL – 2025
EMPIRICAL EXERCISE 3B
With Solutions

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Reference: Stock, J.H. and Watson, M.W., Introduction to Econometrics (4th Edition, Global Edition)

Data: Growth.csv from <http://www.pearsonglobaleditions.com>

Empirical Exercise 5.2. Using the data set **Growth** described in Empirical Exercise 4.1, but excluding the data for Malta, run a regression of *Growth* on *TradeShare*.

- (a) Is the estimated regression slope statistically significant? That is, can you reject the null hypothesis $H_0 : \beta_1 = 0$ vs. a two-sided alternative hypothesis at the 10%, 5%, or 1% significance level?
- (b) What is the p -value associated with the coefficient's t -statistic?
- (c) Construct a 90% confidence interval for β_1 .

Solution:

Key Formulas for Hypothesis Testing

t-statistic for testing $H_0 : \beta_1 = 0$:

$$t = \frac{\hat{\beta}_1}{SE(\hat{\beta}_1)}$$

Critical values for two-sided tests (large sample approximation):

Significance Level	Critical Value	Reject H_0 if
10% ($\alpha = 0.10$)	1.645	$ t > 1.645$
5% ($\alpha = 0.05$)	1.96	$ t > 1.96$
1% ($\alpha = 0.01$)	2.576	$ t > 2.576$

p-value: The probability of observing a test statistic as extreme as (or more extreme than) the one computed, assuming H_0 is true.

$$p\text{-value} = P(|t| > |t^{\text{observed}}|) = 2 \times P(t > |t^{\text{observed}}|)$$

Confidence Interval:

$$\hat{\beta}_1 \pm t_{\alpha/2, df} \times SE(\hat{\beta}_1)$$

For a 90% CI with $df = 62$: $t_{0.05, 62} \approx 1.67$

Data Overview

	With Malta	Without Malta
Sample Size	65	64
Mean Growth	1.94%	—
Mean Trade Share	0.56	—

Recall from E4.1: Malta was identified as an outlier with an unusually high trade share (≈ 2.0) due to its role as a transshipment hub. The exercise asks us to analyze the trade-growth relationship *excluding* this influential observation.

Regression: Growth on TradeShare (Excluding Malta)

	Coefficient	Std. Error	t-statistic	p-value
TradeShare	1.6809	0.9874	1.70	0.094
Constant	0.9574	0.5804	1.65	0.104
$n = 64, \quad R^2 = 0.0447, \quad \text{Root MSE} = 1.79, \quad df = 62$				

Estimated Regression Equation:

$$\widehat{\text{Growth}} = 0.957 + 1.681 \times \text{TradeShare}$$

(a) Is the Slope Statistically Significant?

Hypothesis Test Setup

- $H_0 : \beta_1 = 0$ (Trade share has no effect on growth)
- $H_1 : \beta_1 \neq 0$ (Trade share has an effect on growth)

Test Statistic

$$t = \frac{\hat{\beta}_1}{SE(\hat{\beta}_1)} = \frac{1.6809}{0.9874} = 1.70$$

Decision at Each Significance Level

Level	Critical Value	$ t = 1.70$ vs. CV	Decision
10%	1.645	$1.70 > 1.645$	Reject H_0
5%	1.96	$1.70 < 1.96$	Fail to reject H_0
1%	2.576	$1.70 < 2.576$	Fail to reject H_0

Answer: The slope is statistically significant at the **10% level**, but **NOT** at the 5% or 1% levels.

At the 10% level, we reject H_0 and conclude there is weak evidence of a relationship between trade share and growth. At the 5% level, we cannot reject the null hypothesis that trade share has no effect on growth.

(b) The p-value

The p-value is the probability of observing a t-statistic as extreme as 1.70 (in absolute value) if the null hypothesis were true.

$$p\text{-value} = P(|t_{62}| > 1.70) = 2 \times P(t_{62} > 1.70)$$

From the regression output:

p-value = 0.0937 (approximately 9.4%)
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Interpretation

- If H_0 were true (no relationship between trade and growth), there would be a 9.37% chance of observing a t-statistic as large as 1.70 by random chance alone.
- Since $p = 0.094$:
 - $p < 0.10 \Rightarrow$ Reject H_0 at 10% level
 - $p > 0.05 \Rightarrow$ Fail to reject H_0 at 5% level
 - $p > 0.01 \Rightarrow$ Fail to reject H_0 at 1% level
- The evidence against H_0 is **weak**—sometimes called “marginally significant.”

(c) 90% Confidence Interval for β_1

Formula

$$90\% \text{ CI} = \hat{\beta}_1 \pm t_{\alpha/2, df} \times SE(\hat{\beta}_1)$$

For a 90% confidence interval with $df = 62$:

$$t_{0.05, 62} = 1.6698$$

Calculation

$$\begin{aligned} 90\% \text{ CI} &= 1.6809 \pm 1.6698 \times 0.9874 \\ &= 1.6809 \pm 1.6487 \\ &= (0.0322, 3.3296) \end{aligned}$$

90% Confidence Interval: (0.032, 3.330)
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Interpretation

We are 90% confident that the true effect of trade share on growth (β_1) lies between 0.032 and 3.330 percentage points.

Key observation: The 90% CI **excludes zero**, which is consistent with rejecting H_0 at the 10% significance level.

Comparison: 90% vs. 95% Confidence Intervals

Confidence Level	CI	Width	Includes Zero?
90%	(0.032, 3.330)	3.30	No
95%	(−0.293, 3.655)	3.95	Yes

Key Observations:

1. The 90% CI is **narrower** than the 95% CI (less coverage $\Rightarrow$ smaller margin of error).
2. The 90% CI **excludes zero** $\Rightarrow$ Reject H_0 at 10% level.
3. The 95% CI **includes zero** $\Rightarrow$ Fail to reject H_0 at 5% level.
4. This is consistent with the p-value of 0.094, which falls between 0.05 and 0.10.

Comparison: With Malta vs. Without Malta

	With Malta ($n = 65$)	Without Malta ($n = 64$)
Slope ($\hat{\beta}_1$)	2.31	1.68
Std. Error	0.77	0.99
t-statistic	2.98	1.70
p-value	0.004	0.094
Significant at 10%?	Yes	Yes
Significant at 5%?	Yes	No
Significant at 1%?	Yes	No

Key Finding: Removing Malta substantially **weakens** the statistical evidence for a trade-growth relationship.

- The slope falls from 2.31 to 1.68 (27% reduction)
- The t-statistic drops from 2.98 to 1.70
- The p-value increases from 0.004 to 0.094
- Statistical significance changes from “highly significant” to “marginally significant”

This illustrates how a single influential observation can dramatically affect regression results and statistical conclusions.

Summary: Key Takeaways

Question	Answer
Estimated slope	$\hat{\beta}_1 = 1.68$
Standard error	$SE = 0.99$
t-statistic	$t = 1.70$
(a) Significant at 10%?	Yes ($1.70 > 1.645$)
(a) Significant at 5%?	No ($1.70 < 1.96$)
(a) Significant at 1%?	No ($1.70 < 2.576$)
(b) p-value	0.094
(c) 90% CI	(0.032, 3.330)

The Big Picture

This exercise illustrates several important concepts:

1. **Marginal Significance:** With $p = 0.094$ and $t = 1.70$, the evidence against H_0 is weak. The result is “marginally significant”—significant at 10% but not at 5%. This is a borderline case where different researchers might reach different conclusions.
2. **Relationship Between CIs and Hypothesis Tests:**
 - 90% CI excludes 0 $\Leftrightarrow$ Reject H_0 at 10% level
 - 95% CI includes 0 $\Leftrightarrow$ Fail to reject H_0 at 5% level
3. **Influence of Outliers:** Removing Malta—a single observation out of 65—changes the conclusion from “highly significant” ($p = 0.004$) to “marginally significant” ($p = 0.094$). This demonstrates how influential observations can affect statistical inference.
4. **Robustness:** When results depend heavily on one observation, they are not “robust.” The trade-growth relationship is much weaker once Malta is excluded, raising questions about how generalizable the finding is.
5. **Practical Implications:** From a policy perspective, the evidence that trade promotes growth is weaker than it first appeared. The 90% CI ranges from nearly zero (0.03) to substantial (3.33), reflecting considerable uncertainty about the true effect.

Calculations performed using Stata. See accompanying `E3B_solution.do` file.