Financial Econometrics
Shangai Jiao Tong University
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Problem Set - Lecture 1
With Solutions

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Reference: Stock, J.H. and Watson, M.W., Introduction to Econometrics (4th Edition, Global Edition)

Review of Probability

Question 1. The following table gives the joint probability distribution between employment status and college graduation among those either employed or looking for work (unemployed) in the working-age population of South Africa.

	Unemployed ($Y = 0$)	Employed ($Y = 1$)	Total
Non-college grads ($X = 0$)	0.078	0.673	0.751
College grads ($X = 1$)	0.042	0.207	0.249
Total	0.12	0.88	1.000

(a) Compute $E(Y)$.

(b) The unemployment rate is the fraction of the labor force that is unemployed. Show that the unemployment rate is given by $1 - E(Y)$.

(c) Calculate $E(Y|X = 1)$ and $E(Y|X = 0)$.

(d) Calculate the unemployment rate for (i) college graduates and (ii) non-college graduates.

(e) A randomly selected member of this population reports being unemployed. What is the probability that this worker is a college graduate? A non-college graduate?

(f) Are educational achievement and employment status independent? Explain.

Solution:

The table shows the joint probability distribution with $\Pr(X = 0) = 0.751$, $\Pr(X = 1) = 0.249$, $\Pr(Y = 0) = 0.12$, and $\Pr(Y = 1) = 0.88$.

(a) **Computing $E(Y)$:**

Since Y takes values 0 and 1:

$$E(Y) = 0 \times \Pr(Y = 0) + 1 \times \Pr(Y = 1) = 0 \times 0.12 + 1 \times 0.88 = \boxed{0.88}$$

(b) **Unemployment rate equals $1 - E(Y)$:**

The unemployment rate is defined as:

$$\text{Unemployment rate} = \frac{\text{Number unemployed}}{\text{Labor force}} = \Pr(Y = 0) = 0.12$$

Since $E(Y) = \Pr(Y = 1) = 0.88$, we have:

$$1 - E(Y) = 1 - 0.88 = 0.12 = \text{Unemployment rate} \quad \checkmark$$

Intuition: When Y is a binary variable (employed $= 1$, unemployed $= 0$), its expected value equals the employment rate. Hence, $1-E(Y)$ gives the unemployment rate.

(c) **Conditional expectations:**

First, calculate conditional probabilities:

$$\Pr(Y = 1|X = 1) = \frac{\Pr(Y = 1, X = 1)}{\Pr(X = 1)} = \frac{0.207}{0.249} = 0.831$$
$$\Pr(Y = 0|X = 1) = \frac{\Pr(Y = 0, X = 1)}{\Pr(X = 1)} = \frac{0.042}{0.249} = 0.169$$
$$\Pr(Y = 1|X = 0) = \frac{\Pr(Y = 1, X = 0)}{\Pr(X = 0)} = \frac{0.673}{0.751} = 0.896$$
$$\Pr(Y = 0|X = 0) = \frac{\Pr(Y = 0, X = 0)}{\Pr(X = 0)} = \frac{0.078}{0.751} = 0.104$$

The conditional expectations are:

$$E(Y|X=1) = 0 \times \Pr(Y=0|X=1) + 1 \times \Pr(Y=1|X=1) = 0.831$$
$$E(Y|X=0) = 0 \times \Pr(Y=0|X=0) + 1 \times \Pr(Y=1|X=0) = 0.896$$

(d) **Unemployment rates by education:**

(i) College graduates: $1 - E(Y|X=1) = 1 - 0.831 = \boxed{16.9\%}$

(ii) Non-college graduates: $1 - E(Y|X=0) = 1 - 0.896 = \boxed{10.4\%}$

Note: Surprisingly, college graduates have a *higher* unemployment rate in this data. This could reflect structural issues in the South African labor market or sample selection.

(e) **Conditional probability of education given unemployment:**

Using Bayes' theorem:

$$\Pr(X=1|Y=0) = \frac{\Pr(Y=0|X=1)\Pr(X=1)}{\Pr(Y=0)} = \frac{0.042}{0.12} = \boxed{0.35}$$
$$\Pr(X=0|Y=0) = \frac{\Pr(Y=0|X=0)\Pr(X=0)}{\Pr(Y=0)} = \frac{0.078}{0.12} = \boxed{0.65}$$

Interpretation: Among the unemployed, 35% are college graduates and 65% are non-college graduates.

(f) **Independence test:**

X and Y are independent if and only if $\Pr(X=x, Y=y) = \Pr(X=x) \times \Pr(Y=y)$ for all x, y.

Check: If independent, then:

$$\Pr(X=1, Y=1) \stackrel{?}{=} \Pr(X=1) \times \Pr(Y=1) = 0.249 \times 0.88 = 0.219$$

But from the table: $\Pr(X=1, Y=1) = 0.207 \neq 0.219$.

Conclusion: Educational achievement and employment status are **not independent**. The probability of being employed depends on education level.

Question 2. X and Y are discrete random variables with the following joint distribution:

Value of X	Value of Y				
	2	4	6	8	10
3	0.04	0.09	0.03	0.12	0.01
6	0.10	0.06	0.15	0.03	0.02
9	0.13	0.11	0.04	0.06	0.01

That is, $\Pr(X = 3, Y = 2) = 0.04$, and so forth.

(a) Calculate the probability distribution, mean, and variance of Y.

(b) Calculate the probability distribution, mean, and variance of Y given $X = 6$.

(c) Calculate the covariance and correlation between X and Y.

Solution:

(a) **Marginal distribution of Y:**

Sum across rows to get the marginal probabilities:

$$\begin{aligned}
\Pr(Y = 2) &= 0.04 + 0.10 + 0.13 = 0.27 \\
\Pr(Y = 4) &= 0.09 + 0.06 + 0.11 = 0.26 \\
\Pr(Y = 6) &= 0.03 + 0.15 + 0.04 = 0.22 \\
\Pr(Y = 8) &= 0.12 + 0.03 + 0.06 = 0.21 \\
\Pr(Y = 10) &= 0.01 + 0.02 + 0.01 = 0.04
\end{aligned}$$

Mean of Y:

$$\begin{aligned}
E(Y) &= \sum_y y \cdot \Pr(Y = y) \\
&= 2(0.27) + 4(0.26) + 6(0.22) + 8(0.21) + 10(0.04) \\
&= 0.54 + 1.04 + 1.32 + 1.68 + 0.40 = \boxed{4.98}
\end{aligned}$$

Variance of Y:

$$\begin{aligned}
E(Y^2) &= 4(0.27) + 16(0.26) + 36(0.22) + 64(0.21) + 100(0.04) \\
&= 1.08 + 4.16 + 7.92 + 13.44 + 4.00 = 30.60
\end{aligned}$$

$$\mathrm{Var}(Y) = E(Y^2) - [E(Y)]^2 = 30.60 - (4.98)^2 = 30.60 - 24.80 = \boxed{5.80}$$

(b) **Conditional distribution of Y given $X = 6$:**

First, find $\Pr(X = 6) = 0.10 + 0.06 + 0.15 + 0.03 + 0.02 = 0.36$.

Conditional probabilities:

$$\begin{aligned}
\Pr(Y = 2|X = 6) &= 0.10/0.36 = 0.278 \\
\Pr(Y = 4|X = 6) &= 0.06/0.36 = 0.167 \\
\Pr(Y = 6|X = 6) &= 0.15/0.36 = 0.417 \\
\Pr(Y = 8|X = 6) &= 0.03/0.36 = 0.083 \\
\Pr(Y = 10|X = 6) &= 0.02/0.36 = 0.056
\end{aligned}$$

Conditional mean:

$$\begin{aligned}
E(Y|X = 6) &= 2(0.278) + 4(0.167) + 6(0.417) + 8(0.083) + 10(0.056) \\
&= 0.556 + 0.668 + 2.502 + 0.664 + 0.56 = \boxed{4.94}
\end{aligned}$$

Conditional variance:

$$\begin{aligned}
E(Y^2|X = 6) &= 4(0.278) + 16(0.167) + 36(0.417) + 64(0.083) + 100(0.056) \\
&= 1.112 + 2.672 + 15.012 + 5.312 + 5.60 = 29.71
\end{aligned}$$

$$\text{Var}(Y|X = 6) = 29.71 - (4.94)^2 = 29.71 - 24.40 = \boxed{5.31}$$

(c) **Covariance and correlation:**

First, find $E(X)$. The marginal distribution of X:

$$\begin{aligned}
&\Pr(X = 3) = 0.04 + 0.09 + 0.03 + 0.12 + 0.01 = 0.29 \\
&\Pr(X = 6) = 0.36 \\
&\Pr(X = 9) = 0.13 + 0.11 + 0.04 + 0.06 + 0.01 = 0.35
\end{aligned}$$

$$E(X) = 3(0.29) + 6(0.36) + 9(0.35) = 0.87 + 2.16 + 3.15 = 6.18$$

$$E(X^2) = 9(0.29) + 36(0.36) + 81(0.35) = 2.61 + 12.96 + 28.35 = 43.92$$

$$\text{Var}(X) = 43.92 - (6.18)^2 = 43.92 - 38.19 = 5.73$$

Calculate $E(XY)$:

$$
\begin{aligned}
E(XY) &= \sum_x \sum_y xy \cdot \Pr(X = x, Y = y) \\
&= 3 \times [2(0.04) + 4(0.09) + 6(0.03) + 8(0.12) + 10(0.01)] \\
&\quad + 6 \times [2(0.10) + 4(0.06) + 6(0.15) + 8(0.03) + 10(0.02)] \\
&\quad + 9 \times [2(0.13) + 4(0.11) + 6(0.04) + 8(0.06) + 10(0.01)] \\
&= 3(1.50) + 6(1.78) + 9(1.70) \\
&= 4.50 + 10.68 + 15.30 = 30.48
\end{aligned}
$$

Covariance:

$$\mathrm{Cov}(X, Y) = E(XY) - E(X)E(Y) = 30.48 - (6.18)(4.98) = 30.48 - 30.78 = \boxed{-0.30}$$

Correlation:

$$\mathrm{Corr}(X, Y) = \frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathrm{Var}(X)}\sqrt{\mathrm{Var}(Y)}} = \frac{-0.30}{\sqrt{5.73}\sqrt{5.80}} = \frac{-0.30}{5.76} = \boxed{-0.052}$$

Interpretation: The correlation is very close to zero, indicating almost no linear relationship between X and Y.

Review of Statistics

Question 3. In a poll of 500 likely voters, 270 responded that they would vote for the candidate from the democratic party, while 230 responded that they would vote for the candidate from the republican party. Let p denote the fraction of all likely voters who preferred the democratic candidate at the time of the poll, and let $\hat{p}$ be the fraction of survey respondents who preferred the democratic candidate.

(a) Use the poll results to estimate p.

(b) Use the estimator of the variance of $\hat{p}$, $\hat{p}(1 - \hat{p})/n$, to calculate the standard error of your estimator.

(c) What is the p-value for the test of $H_0 : p = 0.5$ vs. $H_1 : p \neq 0.5$?

(d) What is the p-value for the test of $H_0 : p = 0.5$ vs. $H_1 : p > 0.5$?

(e) Why do the results from (c) and (d) differ?

(f) Did the poll contain statistically significant evidence that the democratic candidate was ahead of the republican candidate at the time of the poll? Explain.

Solution:

Let $Y_i = 1$ if voter i prefers the democratic candidate and $Y_i = 0$ otherwise. Then Y_i is Bernoulli with probability p.

(a) **Point estimate:**

$$\hat{p} = \frac{270}{500} = \boxed{0.54}$$

(b) **Standard error:**

The estimated variance of $\hat{p}$ is:

$$\widehat{\text{Var}}(\hat{p}) = \frac{\hat{p}(1-\hat{p})}{n} = \frac{0.54 \times 0.46}{500} = \frac{0.2484}{500} = 0.0004968$$

Standard error:

$$SE(\hat{p}) = \sqrt{0.0004968} = \boxed{0.0223}$$

(c) **Two-sided test p-value:**

Under $H_0 : p = 0.5$, the standard error is:

$$SE_0(\hat{p}) = \sqrt{\frac{0.5 \times 0.5}{500}} = \sqrt{0.0005} = 0.0224$$

Test statistic:

$$t = \frac{\hat{p} - 0.5}{SE_0(\hat{p})} = \frac{0.54 - 0.5}{0.0224} = \frac{0.04}{0.0224} = 1.79$$

Two-sided p-value (using standard normal approximation for large n):

$$p\text{-value} = 2 \times \Pr(Z > 1.79) = 2 \times (1 - \Phi(1.79)) = 2 \times 0.0367 = \boxed{0.0734}$$

(d) **One-sided test p-value:**

For $H_1 : p > 0.5$:

$$p\text{-value} = \Pr(Z > 1.79) = 1 - \Phi(1.79) = \boxed{0.0367}$$

(e) **Why the p-values differ:**

- Part (c) is a **two-sided test**: we reject H_0 if $\hat{p}$ is either significantly above *or* below 0.5. The p-value is the area in *both* tails.

- Part (d) is a **one-sided test**: we only reject H_0 if $\hat{p}$ is significantly above 0.5. The p-value is the area in the *right tail only*.

Since the one-sided test only considers extreme values in one direction, its p-value is exactly half of the two-sided p-value.

(f) **Statistical significance:**

For the one-sided test $H_0 : p = 0.5$ vs. $H_1 : p > 0.5$:

- p-value $= 0.0367 < 0.05$ (5% significance level)
- Test statistic $t = 1.79 > 1.645$ (5% one-sided critical value)

Conclusion: *Yes*, the poll contains statistically significant evidence at the 5% level that the democratic candidate was ahead. However, at the 1% level (p-value > 0.01), the evidence is not significant.

Practical interpretation: While statistically significant, the lead is only 4 percentage points with a margin of error of about $\pm 4.4\%$ (at 95% confidence), so the race remains competitive.

Question 4. A survey of 1000 registered voters is conducted, and the voters are asked to choose between candidate A and candidate B. Let p denote the fraction of voters in the population who prefer candidate A, and let $\hat{p}$ denote the fraction of voters in the sample who prefer candidate A.

(a) You are interested in the competing hypotheses $H_0 : p = 0.4$ vs. $H_1 : p \neq 0.4$. Suppose that you decide to reject H_0 if $|\hat{p} - 0.4| > 0.01$.

  (i) What is the size of this test?

  (ii) Compute the power of this test if $p = 0.45$.

(b) In the survey, $\hat{p} = 0.44$.

  (i) Test $H_0 : p = 0.4$ vs. $H_1 : p \neq 0.4$ using a 10% significance level.

  (ii) Test $H_0 : p = 0.4$ vs. $H_1 : p < 0.4$ using a 10% significance level.

  (iii) Construct a 90% confidence interval for p.

  (iv) Construct a 99% confidence interval for p.

  (v) Construct a 60% confidence interval for p.

(c) Suppose that the survey is carried out 30 times, using independently selected voters in each survey. For each of these 30 surveys, a 90% confidence interval for p is constructed.

(i) What is the probability that the true value of p is contained in all 30 of these confidence intervals?

(ii) How many of these confidence intervals do you expect to contain the true value of p?

(d) In survey jargon, the "margin of error" is $1.96 \times SE(\hat{p})$. Suppose you want to design a survey that has a margin of error of at most 0.5%. How large should n be if the survey uses simple random sampling?

Solution:

Under the null hypothesis, $\hat{p} \sim N\left(0.4, \frac{0.4\times0.6}{1000}\right) = N(0.4, 0.00024)$, so $SE_0 = 0.0155$.

(a) **Size and Power:**

(i) **Size of the test:**

Size = Pr(Reject $H_0|H_0$ true) = $\Pr(|\hat{p} - 0.4| > 0.01|p = 0.4)$

Standardizing: $\frac{0.01}{0.0155} = 0.645$

$$\text{Size} = \Pr\left(|Z| > 0.645\right) = 2 \times [1 - \Phi(0.645)] = 2 \times 0.2595 = \boxed{0.519}$$

Note: This is a very poorly designed test—it rejects H_0 more than half the time even when H_0 is true!

(ii) **Power when $p = 0.45$:**

Under H_1: $\hat{p} \sim N\left(0.45, \frac{0.45\times0.55}{1000}\right)$, so $SE_1 = 0.0157$.

We reject when $\hat{p} > 0.41$ or $\hat{p} < 0.39$.

$$\begin{aligned}\text{Power} &= \Pr(\hat{p} > 0.41|p = 0.45) + \Pr(\hat{p} < 0.39|p = 0.45)\\ &= \Pr\left(Z > \frac{0.41 - 0.45}{0.0157}\right) + \Pr\left(Z < \frac{0.39 - 0.45}{0.0157}\right)\\ &= \Pr(Z > -2.55) + \Pr(Z < -3.82)\\ &= 0.9946 + 0.0001 = \boxed{0.995}\end{aligned}$$

(b) **Tests and confidence intervals with $\hat{p} = 0.44$:**

(i) **Two-sided test at 10% level:**

$t = \frac{0.44-0.4}{0.0155} = 2.58$

Critical value: $z_{0.05} = 1.645$. Since $|2.58| > 1.645$, **reject** H_0.

(ii) **One-sided test $H_1 : p < 0.4$:**

Since $\hat{p} = 0.44 > 0.4$, the data provides no evidence that $p < 0.4$.

$t = 2.58 > 0$, so we **fail to reject** H_0 (the test statistic is in the wrong direction).

(iii) **90% CI:** $\hat{p} \pm 1.645 \times SE(\hat{p})$

$SE(\hat{p}) = \sqrt{\frac{0.44 \times 0.56}{1000}} = 0.0157$

$0.44 \pm 1.645 \times 0.0157 = \boxed{[0.414, 0.466]}$

(iv) **99% CI:** $0.44 \pm 2.576 \times 0.0157 = \boxed{[0.400, 0.480]}$

(v) **60% CI:** $0.44 \pm 0.842 \times 0.0157 = \boxed{[0.427, 0.453]}$

(c) **Multiple confidence intervals:**

(i) Each 90% CI contains the true p with probability 0.9. With 30 independent surveys:

$$\Pr(\text{All 30 contain } p) = (0.9)^{30} = \boxed{0.042}$$

(ii) Expected number containing p: $30 \times 0.9 = \boxed{27}$

(d) **Sample size for margin of error $\leq 0.5\%$:**

Margin of error: $1.96 \times \sqrt{\frac{p(1-p)}{n}} \leq 0.005$

The maximum value of $p(1-p)$ is 0.25 (at $p = 0.5$). So:

$$1.96 \times \sqrt{\frac{0.25}{n}} \leq 0.005 \implies \sqrt{n} \geq \frac{1.96 \times 0.5}{0.005} = 196$$

$$n \geq 196^2 = \boxed{38,416}$$

Question 5. Data on fifth-grade test scores (reading and mathematics) for 400 school districts in Brussels yield average score $\bar{Y} = 712.1$ and standard deviation $s_Y = 23.2$.

(a) Construct a 90% confidence interval for the mean test score in the population.

(b) When the districts were divided into districts with small classes ($<$ 20 students per teacher) and large classes ($\geq$ 20 students per teacher), the following results were found:

Class Size	Average Score ($\bar{Y}$)	Std. Dev. (s_Y)	n
Small	721.8	24.4	150
Large	710.9	20.6	250

Is there statistically significant evidence that the districts with smaller classes have higher average test scores? Explain.

Solution:

(a) **90% CI for population mean:**

Standard error: $SE(\bar{Y}) = \frac{s_Y}{\sqrt{n}} = \frac{23.2}{\sqrt{400}} = \frac{23.2}{20} = 1.16$

90% CI: $\bar{Y} \pm 1.645 \times SE(\bar{Y}) = 712.1 \pm 1.645 \times 1.16 = 712.1 \pm 1.91$

$$\boxed{[710.19, 714.01]}$$

(b) **Difference in means test:**

Let μ_S = mean for small classes, μ_L = mean for large classes.

$H_0 : \mu_S - \mu_L = 0$ vs. $H_1 : \mu_S - \mu_L > 0$ (one-sided, since we're testing if small classes are better)

Difference in sample means: $\bar{Y}_S - \bar{Y}_L = 721.8 - 710.9 = 10.9$

Standard error of the difference:

$$SE(\bar{Y}_S - \bar{Y}_L) = \sqrt{\frac{s_S^2}{n_S} + \frac{s_L^2}{n_L}} = \sqrt{\frac{24.4^2}{150} + \frac{20.6^2}{250}} = \sqrt{3.969 + 1.698} = \sqrt{5.667} = 2.38$$

Test statistic:

$$t = \frac{10.9 - 0}{2.38} = 4.58$$

The p-value for the one-sided test: $\Pr(Z > 4.58) \approx 0$ (essentially zero).

Conclusion: Since $t = 4.58 \gg 1.645$ (10% critical value) and $\gg 2.33$ (1% critical value), we **strongly reject** H_0.

There is **highly statistically significant evidence** that districts with smaller classes have higher average test scores. The difference of 10.9 points is substantial (about 0.45 standard deviations).

Caution: This is an observational study, not a randomized experiment. The observed correlation between class size and test scores may be due to confounding factors (e.g., wealthier districts may have both smaller classes and higher-achieving students).

Question 6. Suppose $Y_i \sim$ i.i.d. $N(\mu_Y, \sigma_Y^2)$ for $i = 1, \ldots, n$. With σ_Y known, the t-statistic for testing $H_0 : \mu_Y = 0$ vs. $H_1 : \mu_Y > 0$ is $t = (\bar{Y} - 0)/SE(\bar{Y})$, where

$SE(\bar{Y}) = \sigma_Y/\sqrt{n}$. Suppose $\sigma_Y = 10$ and $n = 100$, so that $SE(\bar{Y}) = 1$. Using a test with a size of 5%, the null hypothesis is rejected if $t > 1.64$.

(a) Suppose $\mu_Y = 0$, so the null hypothesis is true. What is the probability that the null hypothesis is rejected?

(b) Suppose $\mu_Y = 2$, so the alternative hypothesis is true. What is the probability that the null hypothesis is rejected?

(c) Suppose that in 90% of cases the data are drawn from a population where the null is true ($\mu_Y = 0$) and in 10% of cases the data come from a population where the alternative is true and $\mu_Y = 2$. Your data came from either the first or the second population, but you don't know which.

  (i) You compute the t-statistic. What is the probability that $t > 1.64$—that is, that you reject the null hypothesis?

  (ii) Suppose you reject the null hypothesis; that is, $t > 1.64$. What is the probability that the sample data were drawn from the $\mu_Y = 0$ population?

(d) It is hard to discover a new effective drug. Suppose 90% of new drugs are ineffective and only 10% are effective. Let Y denote the drop in the level of a specific blood toxin for a patient taking a new drug. If the drug is ineffective, $\mu_Y = 0$ and $\sigma_Y = 10$; if the drug is effective, $\mu_Y = 2$ and $\sigma_Y = 10$.

  (i) A new drug is tested on a random sample of $n = 100$ patients. What is the probability that the drug is ineffective given that $t > 1.64$ (i.e., the false positive rate)?

  (ii) Suppose the one-sided test uses the 0.5% significance level. What is the false positive rate?

Solution:

Given: $\sigma_Y = 10$, $n = 100$, so $SE(\bar{Y}) = 10/\sqrt{100} = 1$.

(a) **Rejection probability under H_0 (size):**

When $\mu_Y = 0$: $t = \bar{Y}/1 = \bar{Y} \sim N(0, 1)$

$$\Pr(t > 1.64|\mu_Y = 0) = 1 - \Phi(1.64) = \boxed{0.05}$$

This is the **size** of the test by construction.

(b) **Rejection probability under H_1 (power):**

When $\mu_Y = 2$: $t = \bar{Y}/1 \sim N(2, 1)$

$$\Pr(t > 1.64|\mu_Y = 2) = \Pr\left(Z > \frac{1.64 - 2}{1}\right) = \Pr(Z > -0.36) = \Phi(0.36) = \boxed{0.64}$$

This is the **power** of the test.

(c) **Mixed population scenario:**

Let A = "data from null population ($\mu_Y = 0$)" and B = "data from alternative population ($\mu_Y = 2$)".

Given: $\Pr(A) = 0.9$, $\Pr(B) = 0.1$.

(i) **Total probability of rejection:**

By the law of total probability:

$$\begin{aligned}\Pr(t > 1.64) &= \Pr(t > 1.64|A) \cdot \Pr(A) + \Pr(t > 1.64|B) \cdot \Pr(B) \\ &= 0.05 \times 0.9 + 0.64 \times 0.1 \\ &= 0.045 + 0.064 = \boxed{0.109}\end{aligned}$$

(ii) **Probability of false positive (from null population given rejection):**

By Bayes' theorem:

$$\Pr(A|t > 1.64) = \frac{\Pr(t > 1.64|A) \cdot \Pr(A)}{\Pr(t > 1.64)} = \frac{0.05 \times 0.9}{0.109} = \frac{0.045}{0.109} = \boxed{0.41}$$

Key insight: Even though the test has only 5% size, 41% of significant results come from the null population! This is because the null is much more common (90% of cases).

(d) **Drug testing application:**

This is the same setup as part (c), applied to drug testing.

(i) **False positive rate at 5% level:**

From part (c)(ii): $\Pr(\text{Ineffective}|t > 1.64) = \boxed{0.41}$ or 41%.

Interpretation: Even when a drug "passes" the statistical test, there's a 41% chance it's actually ineffective!

(ii) **False positive rate at 0.5% level:**

At 0.5% level, the critical value is $z_{0.005} = 2.576$.

New calculations:

- $\Pr(t > 2.576|\text{Ineffective}) = 0.005$
- $\Pr(t > 2.576|\text{Effective}) = \Pr(Z > 2.576 - 2) = \Pr(Z > 0.576) = 0.282$

$$\Pr(t > 2.576) = 0.005 \times 0.9 + 0.282 \times 0.1 = 0.0045 + 0.0282 = 0.0327$$

$$\Pr(\text{Ineffective}|t > 2.576) = \frac{0.0045}{0.0327} = \boxed{0.14} \text{ or } 14\%$$

Conclusion: Using a stricter significance level (0.5% instead of 5%) reduces the false positive rate from 41% to 14%. This illustrates why medical trials often use very stringent significance levels.

General Lesson: The "false discovery rate" depends not just on the test's significance level but also on the **base rate** (prior probability) of true effects. When true effects are rare, even stringent tests can have high false positive rates among "significant" results.