Financial Econometrics
Shangai Jiao Tong University Summer School 2025-2026
Problem Set - Lecture 4

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Reference: Stock, J.H. and Watson, M.W., Introduction to Econometrics (4th Edition, Global Edition)

Chapter 6: Linear Regression with Multiple Regressors

Question 1. Data were collected from a random sample of 200 home sales from a community in 2013. Let $Price$ denote the selling price (in \$1000s), BDR denote the number of bedrooms, $Bath$ denote the number of bathrooms, $Hsize$ denote the size of the house (in square feet), $Lsize$ denote the lot size (in square feet), Age denote the age of the house (in years), and $Poor$ denote a binary variable that is equal to 1 if the condition of the house is reported as "poor." An estimated regression yields:

$$\begin{aligned}\widehat{Price} &= 109.7 + 0.567 \cdot BDR + 26.9 \cdot Bath + 0.239 \cdot Hsize \\ &\quad + 0.005 \cdot Lsize + 0.1 \cdot Age - 56.9 \cdot Poor\end{aligned}$$

with $R^2 = 0.85$ and $SER = 45.8$.

(a) Suppose that a homeowner converts part of an existing family room in her house into a new bathroom. What is the expected increase in the value of the house?

(b) Suppose that a homeowner adds a new bathroom to her house, which increases the size of the house by 80 square feet. What is the expected increase in the value of the house?

(c) What is the loss in value if a homeowner lets his house run down so that its condition becomes "poor"?

(d) Compute the $\bar{R}^2$ for the regression.

Question 2. (Requires calculus) Consider the regression model:

$$Y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

for $i = 1, \ldots, n$. (Notice that there is no constant term in the regression.) Following analysis like that used in Appendix 4.2:

(a) Specify the least squares function that is minimized by OLS.

(b) Compute the partial derivatives of the objective function with respect to β_1 and β_2.

(c) Suppose that $\sum_{i=1}^n X_{1i}X_{2i} = 0$. Show that $\hat{\beta}_1 = \sum_{i=1}^n X_{1i}Y_i / \sum_{i=1}^n X_{1i}^2$.

(d) Suppose that $\sum_{i=1}^n X_{1i}X_{2i} \neq 0$. Derive an expression for $\hat{\beta}_1$ as a function of the data (Y_i, X_{1i}, X_{2i}), $i = 1, \ldots, n$.

(e) Suppose that the model includes an intercept: $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$. Show that the least squares estimators satisfy $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}_1 - \hat{\beta}_2 \bar{X}_2$.

(f) As in (e), suppose that the model contains an intercept. Also suppose that $\sum_{i=1}^n (X_{1i} - \bar{X}_1)(X_{2i} - \bar{X}_2) = 0$. Show that $\hat{\beta}_1 = \sum_{i=1}^n (X_{1i} - \bar{X}_1)(Y_i - \bar{Y}) / \sum_{i=1}^n (X_{1i} - \bar{X}_1)^2$. How does this compare to the OLS estimator of β_1 from the regression that omits X_2?