

Macroeconomics | Activity 2, Lecture 3

Why Are Some Countries So Rich? Production and Productivity

Worked solutions

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Lesson of the day. Piling up capital runs into diminishing returns, so it can take a country only so far. What mainly separates rich countries from poor ones is total factor productivity: how well they turn inputs into output, which comes from technology, ideas and institutions.

Phase 1 – Concept checks

ANSWERS

ConceptTest 1. Answer. (b). On a ratio scale, equal vertical distances stand for equal *percentage* changes, so a straight line is a constant growth *rate* and its slope is that rate. A constant dollar increase, as in (c), would bend downward on a ratio scale.

ConceptTest 2. Answer. (b). The exponents $1/3$ and $2/3$ add to 1, so the function has constant returns to scale: $A(2K)^{1/3}(2L)^{2/3} = 2^{1/3+2/3}AK^{1/3}L^{2/3} = 2Y$. Doubling both inputs exactly doubles output. This is the replication idea: build a second identical factory and you get a second identical output.

ConceptTest 3. Answer. (a), one-third. With competitive markets each input is paid its marginal product, and for a Cobb–Douglas function the income share of each input equals its exponent. Capital's exponent is $1/3$, so capital earns one-third of income and labour earns two-thirds. The level of A does not change the shares.

ConceptTest 4. Answer. (b), 2 times. With $y = Ak^{1/3}$ and the same A , an 8-fold gap in capital raises output per worker by $8^{1/3} = 2$. Diminishing returns are exactly why 8 times the capital gives only 2 times the output.

Phase 2 – The desk problem

FULL WORKING

(a) What capital alone predicts. $(k_R/k_P)^{1/3} = (400,000/50,000)^{1/3} = 8^{1/3} = 2$. With equal productivity the rich country should be only twice as rich per worker.

(b) What is actually true. The actual ratio is $y_R/y_P = 100,000/12,500 = 8$. The rich country is eight times richer per worker, four times more than capital alone can explain.

(c) The missing piece. The TFP ratio is

$$\frac{A_R}{A_P} = \frac{\text{actual}}{\text{capital-predicted}} = \frac{8}{2} = 4.$$

The rich country gets four times as much output from the *same* inputs. (Check directly from $y = Ak^{1/3}$: $A_R = 100,000/400,000^{1/3} \approx 1,357$ and $A_P = 12,500/50,000^{1/3} \approx 339$, a ratio of 4.)

(d) Split the gap. $8 = \underbrace{2}_{\text{capital}} \times \underbrace{4}_{\text{TFP}}$. Capital contributes a factor of 2, TFP a factor of 4. In the textbook's logarithmic split, capital accounts for $\ln 2 / \ln 8 = 1/3$ of the gap and TFP for $2/3$.

Being rich is mainly about *productivity*, how well a country uses its inputs, not simply about owning more machines.

The same income gap, read two ways: capital explains a factor of 2 (one-third of the gap), productivity explains a factor of 4 (two-thirds). Machines matter, but productivity matters more.

Phase 3 – The case

WORKED ANSWERS

1. Doubling capital per worker with A fixed raises output per worker by $2^{1/3} \approx 1.26$, that is about **26 per cent**, not 100 per cent. Diminishing returns: each fresh dose of capital adds less than the last, so growth bought purely with investment gets steadily harder.
2. A country can lift output for a while by raising its investment rate, but the extra capital earns less and less. To keep output growing at the same pace it would have to raise the investment rate again and again, which cannot go on forever. Without rising TFP, diminishing returns eventually stall the economy, however much it invests.
3. It did not. The projected crossover never arrived; in Samuelson's own textbook the expected date was pushed later from one edition to the next. Soviet growth slowed through the 1970s, stagnated in the 1980s, and the Soviet Union dissolved in 1991, never close to United States output.
4. Investment helps, but it cannot by itself make a country rich. Sustained prosperity needs rising productivity, from technology, better ideas, and institutions that let an economy use its inputs well. The same warning applies to any fast-investing economy today, China included: heavy investment raised incomes quickly, but closing the rest of the gap depends on TFP.

Economists once read fast Soviet growth as a permanent rise. It was input accumulation, more capital and more labour, with little gain in productivity (Krugman, "The Myth of Asia's Miracle", 1994). Diminishing returns did the rest. This is the desk problem as history: capital explains about a third, productivity the rest.

End of solutions. Sources: Jones, *Macroeconomics* 6e, Ch. 3–4; P. Krugman, "The Myth of Asia's Miracle" (1994). All figures verified.